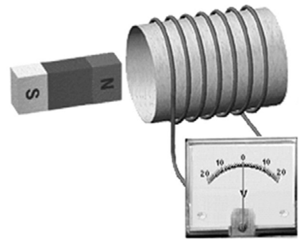
CHAPTER - 2

Electromagnetism

Faradays Law of Induction



Learning objectives

- Magnetic flux lines
- Force on current carrying conductor
- Biot Savart's law
- Magnetomotive force
- Permeability and relative permeability
- Reluctance
- Comparison of electric circuits and magnetic circuits
- Composite series magnetic circuits
- Leakage coefficient
- Electromagnetic induction
- Faraday's laws
- Lenz law
- Dynamically and statically induced emf
- Self and mutual inductance
- Coefficient of coupling
- Energy in a magnetic field

Coloumb first determined experimentally the quantitative expression for the magnetic force between two isolated poles. In reality magnetic poles cannot exist in isolation. Thus, the concept is purely theoretical. However, poles of a long thin magnet may be assumed to be isolated poles. The force between two magnetic poles placed in a medium is

(i) directly proportional to their pole strengths m,
(ii) inversely proportional to the square of the distance d between them
(iii) inversely proportional to the absolute permeability of the medium.

$$\vec{F} \propto \frac{m_1 m_2}{\mu d^2} \quad \text{or} \quad F = \frac{K m_1 m_2}{\mu d^2}$$

In SI system of units the value of K is $\frac{1}{4\pi}$

$$\vec{F} = \frac{m_1 m_2}{4\pi \mu d^2} = \frac{m_1 m_2}{4\pi \mu_0 \mu_r d^2} N \tag{2.1}$$

where m_1, m_2 are the pole strengths, d is the distance in m, μ_0 is permeability of free space $= 4\pi \times 10^{-7}$H/m, μ_r is relative permeability of the medium.

Thus, theoretically a unit magnetic pole may be defined as that pole which when placed in vaccum at a distance of one meter from a similar and equal pole repels it with a force of $\frac{1}{4\pi\mu_0} N$. Oersted discovered in 1820 that a magnetic field is produced around a current carrying conductor.

2.2.1. Biot-Savart Law

The expression for the magnetic field $d\vec{B}$ produced at a point P by an elemental length $d\vec{l}$ of a conductor carrying a current of I amperes is given by *Biot-Savart's law*. Referring to Fig. 2.2.

$$d\vec{B} = \frac{\mu I dl \sin\theta}{4\pi r^2} \text{Wb/m}^2$$

or

$$d\vec{B} = \frac{\mu I d\vec{l} \times \vec{a}_r}{4\pi r^2} \text{Wb/m}^2 \tag{2.2}$$

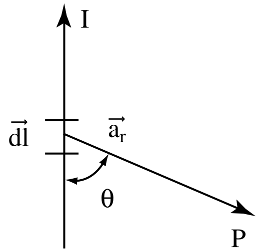


Figure 2.2 *Biot-Savart's law.*

where $\vec{a}_r$ is the unit vector along lines joining dl to P. The direction of $d\vec{B}$ is perpendicular to the plane containing both $d\vec{l}$ and $\vec{a}_r$. The field at a distance r due to an infinitely long straight conductor carrying a current I amperes is given by

$$\vec{B} = \frac{\mu I}{2\pi r} \text{Wb/m}^2.$$

The flux lines are in the form of concentric circles around the conductor. If the conductor is held with the thumb pointing in the direction of the current, the encircling fingers give the direction of the magnetic field.

2.2.2. Force on a current carrying conductor

It was further observed that another current carrying conductor experiences a force when placed in the field. Now we can recollect that current is nothing but flow of electrons (charges!). Thus magnetic fields are produced by moving charges (current carrying conductor) and exert a force on moving charges. The characteristics of this magnetic force on a moving charge are as follows:

- Its magnitude is proportional to the magnitude of the charge.
- The magnitude of the force is proportional to the magnitude or strength of the field.
- The magnetic force depends on the particle's (charge's) velocity $\vec{v}$. This is different from the electric field force which is the same whether the charge is moving or not. A charged particle at rest experiences no magnetic force.
- By experiment it is found that the force is always perpendicular to both the magnetic field $\vec{B}$ and the velocity $\vec{v}$.

The above characteristics can be put compactly as,

$$\vec{F} = q\vec{v} \times \vec{B} \tag{2.3}$$

Similarly the force experienced by a current carrying conductor in a magnetic field is found to be proportional to the magnetic field $\vec{B}$, the current I and the length of the conductor and is perpendicular to the field and the length of the conductor. Thus,

$$\vec{F} = I\vec{l} \times \vec{B} \tag{2.4}$$

Since, the direction of the conductor, fixes the direction of the current (in space) (2.4) is more commonly written as

$$\vec{F} = l\vec{I} \times \vec{B} \tag{2.5}$$

Let $\vec{F}$ be the force in Newtons, $\vec{I}$ the current in amperes and l the length of the conductor in meters, at right amperes to the magnetic field. Then the magnetic field $\vec{B}$ or *flux density* is the density of a magnetic field such that a conductor carrying a current of 1 ampere at right angles to the field has a force of 1 newton per meter acting upon it. The unit is Tesla (T), after the scientist Nikola Tesla. The force on a current carrying conductor is given by,

$$\vec{F} = lIB\sin\theta \tag{2.6}$$

where θ is the angle between the magnetic field and the current carrying conductor. Thus a current carrying conductor experiences a force in the presence of a magnetic field. This principle is used in all electric motors.

The direction of the force may be found from Fleming's left-hand rule as shown in Fig. 2.3.

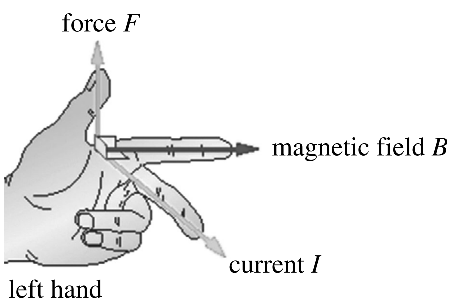


Figure 2.3

Hold out your left hand with the fore finger, middle finger and thumb at right angles to each other. If the fore finger represents the direction of the field and the middle finger the direction of the current, the thumb gives the direction of the force on the conductor. From (2.5) it is obvious that no force is exerted on the conductor when it is parallel to the magnetic field ($\theta = 0°$).

2.2.3. Force between two current carrying conductors

Consider two conductors carrying currents I_1 and I_2 respectively, separated by a distance of dm. The force between the conductors is attractive if the currents flow in the same direction and repulsive if the currents flow in opposite directions. Let us consider the force on the second current due to the first. The field produced by conductor 1 is given by

$$B = \frac{\mu I_1}{2\pi d}\ T$$

The force experienced by conductor 2, from (2.5) is given by

$$F = \frac{\mu l I_2 I_1}{2\pi d}\ N$$

or the force per unit length is given by

$$F = \frac{\mu I_1 I_2}{2\pi d}\ \text{N/m.}$$

2.2.4. Magnetic flux

For a magnetic field having a cross-sectional area $A\ m^2$ and a uniform flux density of B Teslas, the total flux in Webers (Wb) passing through a plane at right angles to the flow is given by

$$\phi = BA$$

$$\text{(Webers)} = \text{(Tesla)} \times (m^2)$$

or

$$B = \frac{\phi}{A} \tag{2.7}$$

Hence the unit of B is also Wb/m^2

$$1\text{Tesla} = 1\text{Wb/m}^2$$

Example 2.1 A conductor carries a current of 500A at right angles to a magnetic field having a density of 0.4T. Calculate the force per unit length on the conductor. What would be the force if the conductor makes an angle of 45° to the magnetic field?

Solution:

$$F = l\vec{I} \times \vec{B} = lIB\sin\theta$$

When conductor is at right angles to the magnetic field, $\theta = 90°$.

$$F = (1\text{m})(500\text{A})(0.4\text{T}) = 200\text{N/m}.$$

When $\theta = 45°$,

$$\begin{aligned} F &= (1\text{m})(500\text{A})(0.4\text{T}) \times \sin 45° \\ &= 141.42\text{N/m}. \end{aligned}$$

Example 2.2 A rectangular coil 100mm by 150mm is mounted so that it rotates about the mid points of the 150mm sides. The axis of rotation is at right angles to a magnetic field with a flux density of 0.02T. Calculate the flux in the coil when

(i) Maximum flux links with the coil. What is the position at which this occurs?

(ii) The flux through the coil when the 150mm sides make an angle of 30° to the direction of flux.

Solution:

(i) This is shown in Fig. 2.4(a). The maximum flux passes through the coil when the plane of the coil is at right angles to the direction of the flux.

$$\begin{aligned} \phi &= BA = 0.02 \times (100 \times 10^{-3}) \times (150 \times 10^{-3}) \\ &= 0.3\text{mWb}. \end{aligned}$$

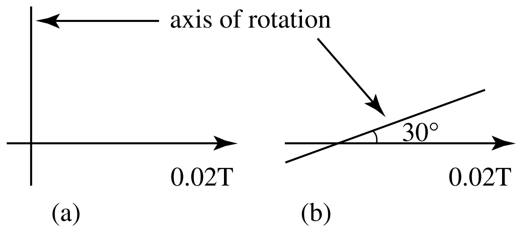


Figure 2.4 *Example 2.1.*

(ii) This is shown in Fig. 2.4(b).

$$\phi = BA \sin\theta = (0.3 \times 10^{-3}) \times \sin 30^\circ$$
$$= 0.15\text{mWb}.$$

2.3. Magnetomotive force and magnetic field strength

The magnetic flux is present in a magnetic circuit due to the existence of a *magnetomotive force (mmf)*, caused by a current flowing through one or more turns. It is analogous to emf in an electric circuit which is responsible for the electric current.

$$mmf = NI \tag{2.8}$$

where N is the number of turns. N is a dimensionless quantity. Hence, the unit of mmf is actually Ampere, though more commonly the unit is said to be ampere-turns (AT).

Consider a coil as shown in Fig. 2.5. If the magnetic circuit is homogeneous and has a uniform cross sectional area, the mmf per metre length of the magnetic circuit is called the *magnetic field strength H*.

$$H = \frac{NI}{l} \text{AT/m} \tag{2.9}$$

The unit of H in SI units is A/m.

The ratio B/H is the *permeability*, μ_0, in free space

$$\mu_0 = \frac{B}{H} \tag{2.10}$$

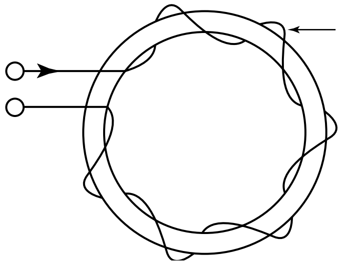


Figure 2.5 *Coil with N turns on a toroid.*

This value is almost same when the conductor is placed in free space, air or in any other non-magnetic material like water, wood, oil etc.

$$\mu_0 = 4\pi \times 10^{-7} \text{H/m} \tag{2.11}$$

For magnetic materials,

$$H = \frac{B}{\mu}$$
$$\mu = \mu_0 \mu_r \tag{2.12}$$

where μ_r is the relative permeability.

The relative permeability is defined as the ratio of the flux density produced in a material to the flux density produced in vacuum by the same magnetic field strength.

The relative permeability of non-magnetic materials is close to 1. The relative permeability of magnetic materials is very high, as shown in Table 2.1.

Table 2.1 Relative permeability of magnetic materials.

Material	μ_r	Application
Ferrite U60	8	UHF chokes
Ferrite M33	750	Resonant circuit RM cores
Nickel (99% pure)	600	
Ferrite N41	3,000	Power circuits
Ferrite T38	10,000	Broadband transformers
Silicon steel	40,000	Dynamos, mains transformer

When working with non magnetic materials, the permeability is close to μ_0, making it difficult to characterize them by permeability. We make use of *magnetic susceptibility* defined as

$$\psi_m = \mu_r - 1 \tag{2.13}$$

Example 2.3 A coil of 100 turns is wound uniformly over a wooden ring having a mean circumference of 500mm and a uniform cross sectional area of 500mm^2. If the current through the coil is 2.0A calculate

(i) the magnetic field strength

(ii) the flux density

(iii) the flux

(iv) mmf

Solution:

(i) Mean circumference = 500mm = 0.5m.

$$H = \frac{NI}{l} = \frac{100 \times 2}{0.5} = 400\text{AT/m or A/m}$$

(ii) $B = \mu_0 H = 4\pi \times 10^{-7} \times 400 = 502.65\,\mu\text{T}$

(iii)

$$\begin{aligned}\phi = BA &= 502.65 \times 10^{-6} \times 500 \times 10^{-6} \\ &= 0.2513\,\mu\text{Wb}\end{aligned}$$

(iv) mmf $= NI = 100 \times 2 = 200$AT.

Example 2.4 Calculate the mmf required to produce a flux of 0.01Wb across an airgap 2mm long, having an effective area of 100cm^2.

Solution:

$$A = 100\text{cm}^2 = 100 \times 10^{-4} = 0.01\text{m}^2$$
$$B = \frac{\phi}{A} = \frac{0.01}{0.01} = 1\text{T}$$
$$H = \frac{B}{\mu_0} = \frac{1}{4\pi \times 10^{-7}}\text{AT/m}$$
$$l = 2\text{mm} = 2 \times 10^{-3}\text{m}$$
$$mmf = H \times l = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7}} = 1591.55\text{AT}$$

2.4. Reluctance

Consider the toroid shown in Fig. 2.5, with a cross-sectional area A m^2 and a mean circumference of l metres, with N turns carrying a current I amperes. We know

$$\phi = BA$$
$$mmf = Hl$$
$$\therefore \frac{\phi}{mmf} = \frac{BA}{Hl} = \mu_r \mu_0 \frac{A}{l}$$

or

$$\phi = \frac{mmf}{l/\mu_r \mu_0 A} = \frac{mmf}{S}$$

where

$$S = \frac{l}{\mu_0 \mu_r A} \tag{2.14}$$

S is the reluctance of the magnetic circuit and is indicative of the opposition of a magnetic circuit to creation of magnetic flux through it. From (2.14) we can write

$$mmf = \phi S \tag{2.15}$$

similar to Ohm's law. Unit of S is AT/Wb. It is analogous to resistance in electric circuits. The reciprocal of reluctance is called the *permeance* of the magnetic circuit. Its unit is Wb/AT. It is analogous to conductance in electric circuits.

2.5. Comparison of magnetic and electric circuits

Table 2.2 gives the analogous quantities between electric circuits and magnetic circuits.

Table 2.2 Comparison of magnetic and electric circuits.

	Magnetic circuit	Electric circuit
1	ϕ-Flux (Webers)	I-Current (Amperes)
2	B-Flux density (Wb/m^2)	J-Current density (A/m^2)
3	S-Reluctance $= \dfrac{l}{\mu_0 \mu_r A}$ (At/Wb)	R-Resistance $= \dfrac{\rho l}{A}$ (Ohms)
4	P-Permeance $= \dfrac{1}{S}$ (Wb/AT)	G-Conductance $= \frac{1}{R}$ (mhos)
5	mmf $= \phi S$ (AT)	emf $= IR$ (Volts)
6	Permeability (μ)	Conductivity (σ)
7	Reluctivity $\left(\dfrac{1}{\mu}\right)$	Resistivity (ρ)

There are however some differences between electric circuits and magnetic circuits:

- The flux does not flow through the magnetic circuit like the current does in an electric circuit.
- In electric circuits if the temperature is maintained a constant, the resistance is a constant and independent of the current. In a magnetic circuit, the reluctance depends on the flux established through it. The reluctance is small for small values of B and larger for larger values of B. This is because the B–H curve is not a straight line.
- Flow of electric current requires continuous expenditure of energy but in a magnetic circuit energy is expanded only in creating the magnetic flux but not in maintaining it.
- A magnetic circuit stores energy in its field, while an electric circuit dissipates its energy as heat.

2.6. Composite magnetic circuits

We will now discuss composite magnetic circuits; circuits connected in series and parallel.

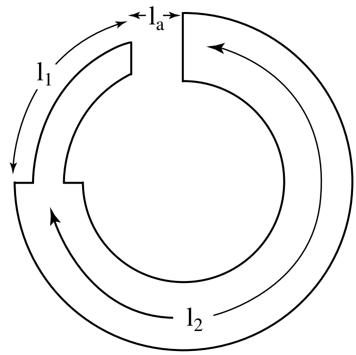


Figure 2.6 *Composite series magnetic circuit.*

2.6.1. Composite series circuits

Consider the composite circuit of Fig. 2.6, made up of two different sections in series and an air gap. Each section is made of a different material and has its own reluctance. The total reluctance is sum of the individual reluctances

$$S_T = S_1 + S_2 + S_a$$

$$= \frac{l_1}{\mu_0 \mu_{r1} A_1} + \frac{l_2}{\mu_0 \mu_{r2} A_2} + \frac{l_a}{\mu_0 A_a} \quad (2.16)$$

$$= \frac{mmf}{S_T} \quad (2.17)$$

To find the ampere-turns or the total mmf,

- Find the H of each section using

$$H = \frac{B}{\mu_0} \quad \text{(if it is air gap)}$$

$$= \frac{B}{\mu_0 \mu_r} \quad \text{(for magnetic material)}$$

- Find the mmf (AT) for each section by

$$AT = Hl$$

- Add these ampere-turns to get the total ampere turns. (Similar to adding emf's in series!)

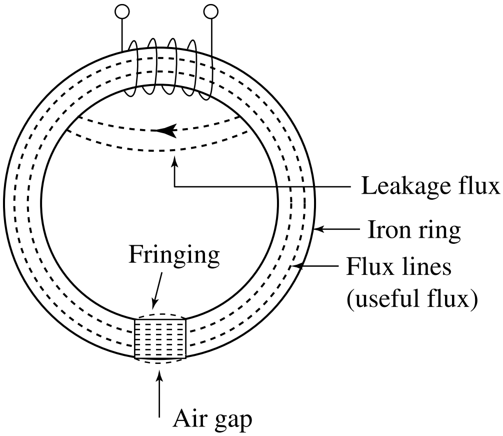


Figure 2.8 *Leakage flux and fringing.*

Solution: The circuit is shown in Fig. 2.9.

$$\phi = 0.6 \times 10^{-3}\text{Wb}$$

$$A = 10 \times 10^{-4}\text{m}^2$$

$$B = \frac{\phi}{A} = \frac{0.6 \times 10^{-3}}{10 \times 10^{-4}}$$

$$= 0.6\text{Wb/m}^2$$

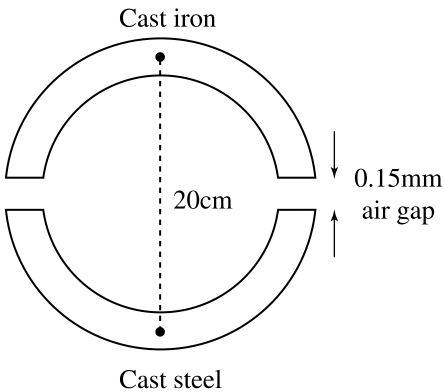


Figure 2.9 *Example 2.5.*

Air gap

$$H = \frac{B}{\mu_0} = \frac{0.6}{4\pi \times 10^{-7}} = 4.77 \times 10^5 \text{AT/m}$$

$$\text{Total air gap length} = 0.15 + 0.15 = 0.3\text{mm}$$

$$= 0.3 \times 10^{-3}\text{m}$$

$$\text{AT required to set up flux in air gap} = Hl$$

$$= 4.77 \times 10^5 \times 0.3 \times 10^{-3}$$

$$= 143.1AT$$

Cast iron section

$$H = \frac{B}{\mu_0\mu_r} = \frac{0.6}{4\pi \times 10^{-7} \times 800} = 596.83\text{AT/m}$$

$$\text{length of the path} = \pi r = \frac{\pi D}{2} = \frac{\pi \times 20 \times 10^{-2}}{2} = 0.314\text{m}$$

$$\text{AT required} = Hl = 596.83 \times 0.314 = 187.41\text{AT}$$

Cast steel section

$$H = \frac{B}{\mu_0\mu_r} = \frac{0.6}{4\pi \times 10^{-7} \times 166} = 2876.29\text{AT/m}$$

$$\text{length of the path} = \frac{\pi D}{2} = 0.314\text{m}$$

$$\text{AT required} = 2876.29 \times 0.314 = 903.16\text{AT}$$

$$\text{Total AT required} = 143.1 + 187.41 + 903.16 = 1233.67\text{AT}.$$

Example 2.6 A mild steel ring having a cross sectional area of 600mm^2 and a mean circumference of 500mm has a coil of 300 turns wound uniformly around it. Calculate

(i) the reluctance of the ring

(ii) the current required to produce a flux of 800μWb in the ring, if the relative permeability is 400.

Solution:

(i)

$$S = \frac{l}{\mu_0 \mu_r A} = \frac{500 \times 10^{-3}}{4\pi \times 10^{-7} \times 400 \times 600 \times 10^{-6}}$$
$$= 1.658 \times 10^6 \text{AT/Wb}$$

(ii)

$$mmf = \phi \times S$$
$$= 800 \times 10^{-6} \times 1.658 \times 10^6$$
$$= 1326.4\text{AT}$$
$$I = \frac{mmf}{N} = \frac{1326.4}{300} = 4.42A$$

Example 2.7 A magnetic circuit comprises three parts in series as follows:
(a) A length of 60mm with a cross section area of 50mm^2
(b) A length of 30mm with a cross section area of 80mm^2
(c) An air gap of length 0.3mm and cross section area of 150mm^2.

A coil of 2500 turns is wound on part (b) and the flux density in air gap is 0.3T. Assuming that there is no leakage, and the relative permeability $\mu_r = 1500$, estimate the current required in the circuit to produce the flux density.

Solution:

$$\phi = B_c A_c = 0.3 \times 150 \times 10^{-6} = 45 \times 10^{-6}\text{Wb}$$
$$F_a = mmf_a = \phi S_a = \frac{\phi l_a}{\mu_0 \mu_r A_a} = \frac{45 \times 10^{-6} \times 60 \times 10^{-3}}{4\pi \times 10^{-7} \times 1500 \times 50 \times 10^{-6}}$$
$$= 28.6\text{AT}$$
$$F_b = mmf_b = \phi S_b = \frac{\phi l_b}{\mu_0 \mu_r A_b} = \frac{45 \times 10^{-6} \times 30 \times 10^{-3}}{4\pi \times 10^{-7} \times 1500 \times 80 \times 10^{-6}}$$
$$= 8.95\text{AT}$$

$$F_c = mmf_c = \phi S_c = \frac{\phi l_c}{\mu_0 \mu_r A_c} = \frac{45 \times 10^{-6} \times 0.3 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 150 \times 10^{-6}}$$

$$= 71.62\text{AT}$$

$$F = F_a + F_b + F_c = 28.6 + 8.95 + 71.62 = 109.17\text{AT}$$

$$I = \frac{F}{N} = \frac{109.17}{2500} = 0.04367A = 43.67\text{mA}.$$

(* Note the large mmf required to set up the flux through air gap as compared to a magnetic material.)

Example 2.8 A wooden ring has a circular cross section of 200 sq mm and a mean diameter of 200mm. It is uniformly wound with 600 turns. If the $\mu_r = 1$, find (1) the field strength produced by a current of 2A (ii) magnetic flux density (iii) current required to produce a flux density of 0.015Wb/m^2.

Solution:

(i)

$$mmf = NI = 600 \times 2 = 1200\text{AT}$$

$$\text{Mean length} = \pi d = \pi \times 200 \times 10^{-3} = 0.628m$$

$$H = \frac{NI}{l} = \frac{1200}{0.628} = 1910.83\text{AT/m}$$

(ii)

$$B = \mu_0 \mu_r H = 4\pi \times 10^{-7} \times 1 \times 1910.83 = 2.4 \times 10^{-3}\text{Wb/m}^2$$

(iii) The flux density is proportional to the current. A current of 2A produces 0.0024Wb/m^2. Therefore current required to produce 0.015Wb/m^2 is

$$\frac{2 \times 0.015}{0.0024} = 12.5\text{A}.$$

Example 2.9 A magnetic core in the form of a closed circular ring has a mean length of 20cm and a cross sectional area of 1cm^2. The relative permeability of the material is 2200. What current is needed in the coil of 2000 turns wound uniformly around the ring to create a flux of 0.15mWb in the iron? If an air gap of 1mm is cut through the core in a direction perpendicular to the direction of this flux, what current is now needed to maintain the same flux in the air gap?

Solution: Reluctance of core

$$S = \frac{l}{\mu_0 \mu_r A} = \frac{20 \times 10^{-2}}{4\pi \times 10^{-7} \times 2200 \times 1 \times 10^{-4}}$$

$$= 723431.5 \text{ AT/Wb}$$

$$\phi = 0.15 \times 10^{-3} \text{Wb}$$

$$mmf = \phi S = 0.15 \times 10^{-3} \times 723431.5 = 108.5\text{AT}$$

$$mmf = NI$$

$$I = \frac{mmf}{N} = \frac{108.5}{2000} = 0.05425A$$

$$\text{Reluctance of 1mm air gap} = \frac{1 \times 10^{-3}}{4\pi \times 10^{-7} \times 1 \times 1 \times 10^{-4}} = 7957747.1 \text{ AT/Wb}$$

$$mmf \text{ required to set up flux in air gap} = 0.15 \times 10^{-3} \times 7957747.1 = 1193.66\text{AT}$$

$$\text{Total } mmf = 108.5 + 1193.66$$

$$= 1302.16\text{AT}$$

$$\text{Current required} = \frac{1302.16}{2000} = 0.65108A$$

Example 2.10 An iron ring of mean diameter 20cm, having a cross section area of 3 sq cm is required to produce a flux of 0.45mWb. If $\mu_r = 1800$ find the *mmf* required. If an air gap of 1mm is made in the ring, how many extra ampere turns are required to maintain the same flux?

Solution:

$$\begin{aligned}
\text{Length of the mean path} &= \pi d = \pi \times 0.2 \\
&= 0.6283\text{m} \\
\phi &= 0.45 \times 10^{-3}\text{Wb} \\
B &= \phi/A = 0.45 \times 10^{-3}/3 \times 10^{-4} = 1.5\text{Wb/m}^2 \\
H &= \frac{B}{\mu_0\mu_r} = \frac{1.5}{4\pi \times 10^{-7} \times 1800} = 663.154\text{AT/m} \\
mmf &= Hl = 663.154 \times 0.6283 = 416.65\text{AT}
\end{aligned}$$

An air gap of 1mm will need extra *mmf*.

$$\begin{aligned}
H_g &= \frac{B_g}{\mu_0} = \frac{1.5}{4\pi \times 10^{-7}} \\
mmf &= H_g \times l_g = \frac{1.5 \times 1 \times 10^{-3}}{4\pi \times 10^{-7}} = 1,193.66\text{AT}
\end{aligned}$$

So additional *mmf* required when a 1mm air gap is cut is 1,193.66AT.

2.7. Electromagnetic induction—Faraday's law

In 1831, Michael Faraday discovered the principle of *electromagnetic induction*. Two of his experiments are well known. The first experimental setup is shown in Fig. 2.10(a).

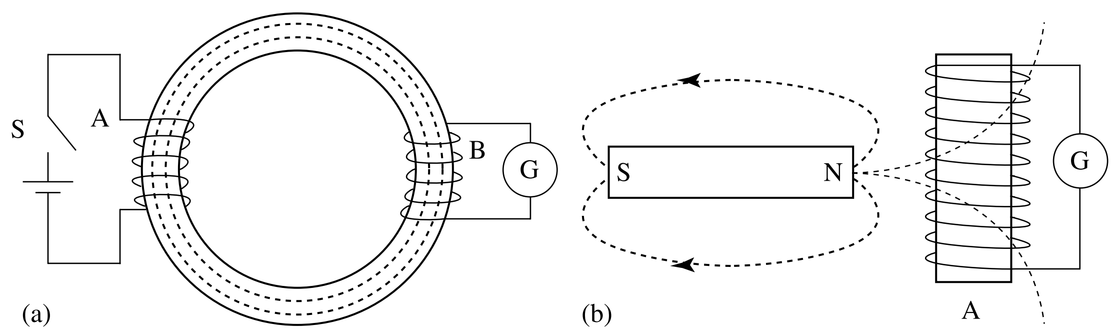


Figure 2.10 *Electromagnetic induction.*

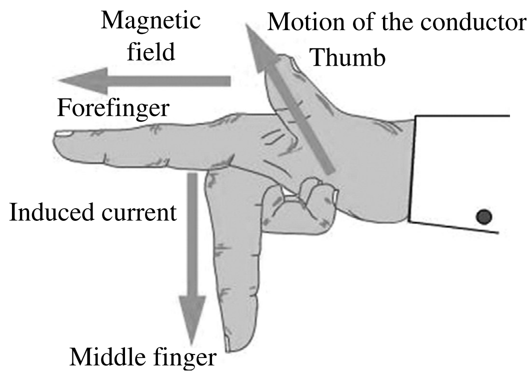


Figure 2.11 *Fleming's right hand rule.*

Consider a coil of N turns. Let the flux linking the coil change from ϕ_1 Wb to ϕ_2 Wb in t seconds. Now,

$$\text{Initial flux linkage } \psi_1 = N\phi_1 \text{ Wb turns}$$
$$\text{Final flux linkage } \psi_2 = N\phi_2 \text{ Wb turns}$$
$$\text{emf induced, } e = \frac{N\phi_2 - N\phi_1}{t} \text{volts}$$
$$= \frac{N(\phi_2 - \phi_1)}{t}$$

The above relationship can be written as

$$e = N\frac{d\phi}{dt} = \frac{d\psi}{dt} \text{volts} \tag{2.19}$$

To incorporate Lenz's law we often write $e = -N\frac{d\phi}{dt}$, meaning the induced emf is set up in a direction such that it opposes the rate of change of flux.

The emf can be induced in two ways

(i) Dynamically induced emf (or motional emf)
(ii) Statically induced emf (or transformer emf)

Average induced emf in the coil is $\frac{LI}{t}$ volts. This current also produces a change in flux, from 0 to ϕWb whose average rate is given by $\frac{\phi}{t}$. From Faraday's law the induced emf $= \frac{N\phi}{t}$. Thus, we have

$$\frac{LI}{t} = \frac{N\phi}{t}$$

or

$$L = \frac{N\phi}{I} = \frac{\psi}{I} \tag{2.21}$$

Thus inductance is nothing but the flux linkage per ampere. An alternative expression is

$$L = N\frac{d\phi}{dI}$$

We can also define 1H as the inductance of a coil when a current of 1 ampere through the coil produces a flux linkage of 1Wb turn. Now from (2.15)

$$\phi = \frac{mmf}{\text{reluctance}} = \frac{NI}{S} = \frac{NI}{l/\mu_0\mu_r A}$$

$$\therefore\ L = \frac{N\phi}{I} = N\frac{NI}{I(l/\mu_0\mu_r A)} = \frac{N^2\mu_0\mu_r A}{l}H \tag{2.22}$$

Equation (2.22) gives the expression for L from the geometric parameters of the coil.

2.7.7. Mutually induced emf

Consider two coils A and B placed as shown in Fig. 2.12. When the switch S is closed, some flux produced by A, also links with B. This produces an induced emf in coil B and a current flows through circuit of B, as indicated by the galvanometer deflection. Since a change of current in one coil is accompanied by a change of flux linked with the other coil inducing an emf in it, it is called *mutually induced emf*. The two coils are said to have a *mutual inductance*.

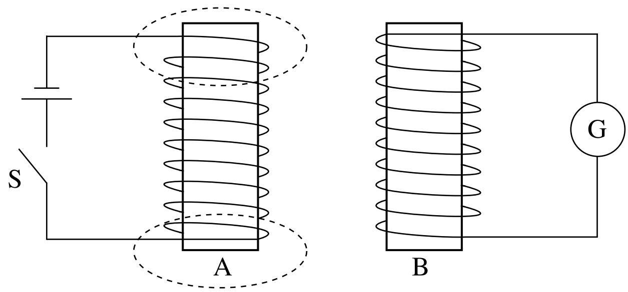


Figure 2.12 *Mutually induced emf.*

2.7.8. Mutual inductance

The unit of mutual inductance is also Henry. Two coils have a mutual inductance of 1H if an emf of 1volt is induced in one coil when the current through the other coil varies uniformly at the rate of 1A/s.

Again consider the two coils of Fig. 2.12.

Let I_1 be the current flowing through coil A, which produces a flux ϕ_1. All this flux does not link with coil B. The flux ϕ_{11} which links only with coil A is called the *leakage flux*. The flux ϕ_{12} which also links with coil B is called the *mutual flux*.

$$\phi_1 = \phi_{11} + \phi_{12} \tag{2.23}$$

Coefficient of coupling, 'K' is defined as the ratio of mutual flux to total flux.

$$K_1 = \frac{\phi_{12}}{\phi_1} \tag{2.24}$$

$K \leq 1$. The induced emf in coil B is given by

$$e_2 = M\frac{dI_1}{dt}.$$

Also

$$e_2 = N_2\frac{d\phi_{12}}{dt}$$

$$\therefore\ M = N_2\frac{d\phi_{12}}{dI_1}$$

where L_1, L_2 are the self inductances of coil A and coil B respectively. If $K_1 \neq K_2$, then in (2.27) we use the geometric mean of K_1 and K_2;

$$K = \sqrt{K_1 K_2}$$

$0 \leq K \leq 1$. Larger values of coefficient of coupling are obtained with coils which are physically closer, which are wound or oriented to provide a larger common magnetic flux or which are provided with a common path through a material which serves to concentrate and localize the magnetic flux. Coils with K close to unity are said to be *tightly coupled*.

Example 2.11 A coil consists of 750 turns. A current of 10A in the coil gives rise to a magnetic flux of 1200μWb. Determine the inductance of the coil and the average induced emf in the coil when the current is reversed in 0.01sec.

Solution:

$$N = 750; \quad I = 10\text{A}; \quad \phi = 1200 \times 10^{-6}\text{Wb}$$

$$L = \frac{N\phi}{I} = \frac{750 \times 1200 \times 10^{-6}}{10} = 0.09\text{H}$$

Current reverses from 10A to -10A.

$$\therefore \; dI = 10 - (-10) = 20\text{A}$$

$$e = L\frac{dI}{dt} = 0.09 \times \frac{20}{0.01} = 180\text{V}.$$

Example 2.12 An air cored solenoid has a length of 50cm and diameter of 2cm. Calculate its inductance if it has 1000 turns.

Solution:

$$L = \frac{N^2 \mu_0 \mu_r A}{l}$$

$$A = \frac{\pi d^2}{4} = \frac{\pi \times (2 \times 10^{-2})^2}{4} = 3.14 \times 10^{-4}\text{m}^2$$

$$l = 50 \times 10^{-2}\text{m}$$

$$L = \frac{(1000)^2 \times 4\pi \times 10^{-7} \times 1 \times 3.14 \times 10^{-4}}{50 \times 10^{-2}}$$

$$= 78.9 \times 10^{-5} H = 0.7892\text{mH}$$

2.8. Energy stored in magnetic field

In an electric field energy is continuously dissipated. The energy cannot be stored. In a magnetic field on the other hand, we need energy only to set up the initial flux and no energy is required to maintain it. Magnetic field stores the energy which has been used to create the flux.

Let the current flowing through a coil of constant inductance L Henrys grow at an uniform rate from zero to I amperes in t seconds. The average value of current is $\frac{I}{2}$ and the emf induced in the coil is $\left(L \times \frac{I}{t}\right)$ volts. The average power absorbed by the magnetic field is

$$\frac{1}{2}I \times \frac{LI}{t}\text{Watts}$$

and the total energy absorbed is

$$\text{average power} \times \text{time} = \frac{1}{2}I \times \frac{LI}{t} \times t$$

or

$$W = \frac{1}{2}LI^2\text{Joules} \tag{2.28}$$

Now lets consider a more general case where the instantaneous current i increases in a coil having a constant inductance LH. The rate of increase can be uniform or non-uniform. If the current increases by di amperes in dt seconds, the induced emf is given by

$$e = L\frac{di}{dt}\text{Volts}$$

The energy absorbed is

$$W = i\left(L\frac{di}{dt}\right)dt = Li \cdot di \text{ Joules.}$$

The total energy absorbed by the magnetic field when the current increases from 0 to I amperes is given by

$$W = \int_0^I Li \cdot di = L \times \frac{1}{2}[i^2]_0^I = \frac{1}{2}LI^2 \text{ Joules}$$

From (2.22) $L = N^2\mu\frac{A}{l}H$. The energy per cubic meter W_f is

$$W_f = \frac{1}{2}I^2N^2\frac{\mu}{l^2} = \frac{1}{2}\mu H^2 = \frac{1}{2}BH = \frac{1}{2}\frac{B^2}{\mu}\text{Joules} \tag{2.29}$$

Equation (2.29) can be used only if μ_r is a constant.

Now when the inductive circuit is opened, the current has to reduce to zero and the stored energy released. If there is no resistor in the circuit the energy will be mostly dissipated in the arc across the switch. If there is a resistor, the energy is dissipated as heat in the resistor.

2.9. Dot convention

In an inductor, which is a two terminal element, if the current enters the terminal used as positive reference, the induced voltage $L\frac{di}{dt}$ is positive, as shown in Fig. 2.13(a). If it enters the coil at the terminal used as negative reference, it is negative, as in Fig. 2.13(b).

The *dot convention* is used to fix the polarity of the voltage induced in a coil due to mutual inductance. A dot is placed on each of the coils. The sign of the voltage due to mutual inductance is as follows:

A current *entering* the *dotted* terminal of one coil produces a voltage with a positive reference at the dotted terminal of the second coil.

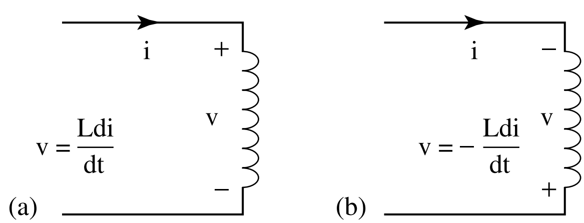


Figure 2.13 *Induced voltage.*

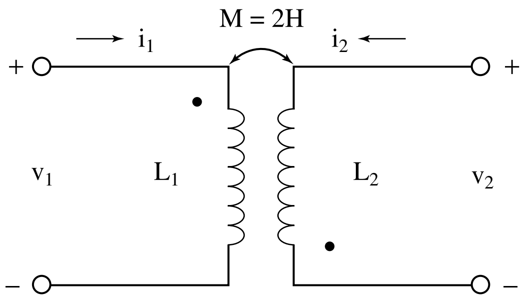


Figure 2.16 *Example 2.13*

Solution: (i) i_2 enters undotted terminal of L_2. Hence, mutually induced emf is positive at the undotted terminal of L_1. However, v_1 is referenced positive at the dotted terminal. Therefore,

$$v_1 = -M\frac{di_2}{dt} = -(2)(314)(10\cos 314t)$$
$$= -6280\cos 314t\,\mathrm{V}$$

Since $i_1 = 0$, there is no self induced emf.

(ii) i_1 enters the dotted terminal of L_1. Hence, v_2 is positive at the dotted terminal of L_2. However, it is referenced positive at the undotted terminal.

$$\therefore\ v_2 = -M\frac{di_1}{dt} = -(2)(-1)(-8e^{-t})$$
$$= -16e^{-t}\,\mathrm{V}.$$

2.10. Inductance in series

Consider two inductances connected in series as shown in Fig. 2.17(a).

Each of the coils has self induced emf and mutually induced emf.

$$v_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$$
$$i_1 = i_2 = i$$
$$\therefore\ v_1 = \frac{di}{dt}(L_1 + M)$$

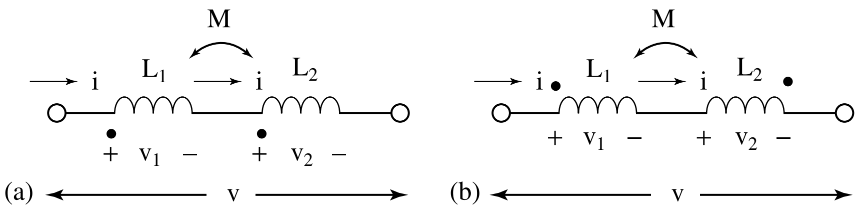


Figure 2.17 *Inductances in series.*

Similarly

$$v_2 = \frac{di}{dt}(L_2 + M)$$

$$v = v_1 + v_2 = \frac{di}{dt}(L_1 + L_2 + 2M)$$

$$= L_{eq}\frac{di}{dt}$$

$$L_{eq} = L_1 + L_2 + 2M \tag{2.30}$$

This is called *series-aiding connection*, where the mutual flux and leakage flux aid each other.

In Fig. 2.17(b) M is negative

$$v_1 = \frac{di}{dt}(L_1 - M)$$

$$v_2 = \frac{di}{dt}(L_2 - M)$$

$$v = v_1 + v_2 = \frac{di}{dt}(L_1 + L_2 - 2M)$$

$$L_{eq} = L_1 + L_2 - 2M \tag{2.31}$$

This is called *series-opposing connection.*

The instantaneous energy stored in a coupled circuit is given by

$$W = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M i_1 i_2 \tag{2.32}$$

As derived earlier $M = K\sqrt{L_1 L_2}$.

Example 2.15 The equivalent of two inductances connected in series is 0.6H or 0.1H, depending on the connection. If $L_1 = 0.2H$ find (i) M (ii) K.

Solution:

$$L_1 + L_2 + 2M = 0.6 \quad \text{(i)}$$
$$L_1 + L_2 - 2M = 0.1 \quad \text{(ii)}$$
$$4M = 0.5 \quad \text{or} \quad M = 0.125H$$
$$L_1 = 0.2H$$

From (i) $L_2 = 0.6 - 0.2 - 0.125 \times 2 = 0.15H$

$$K = M/\sqrt{L_1 L_2} = \frac{0.125}{\sqrt{0.2 \times 0.15}} = 0.722.$$

Example 2.16 Two identical air cored solenoids have 200 turns, length of 25cm and cross section area of 3cm^2 each. The mutual inductance between them is 0.5μH. Find the self inductance of each coil and the coefficient of coupling.

Solution:

$$L = \frac{N^2 \mu_0 \mu_r A}{l} = \frac{(200)^2 (4\pi \times 10^{-7})(1)3 \times 10^{-4}}{0.25} = 60.318\mu\text{H}$$
$$L_1 = L_2 = L$$
$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.5 \times 10^{-6}}{\sqrt{(60.318 \times 10^{-6})^2}} = 8.289 \times 10^{-3}.$$

Example 2.17 A closed iron ring of mean diameter 12cm is made from round iron bar of 2cm diameter. It has a winding of 1000 turns. Calculate the current required to produce a flux density of 1.5Wb/m^2 given the relative permeability is 1250. Hence find the self inductance.

(i)

$$\mu = \frac{B}{H} = \frac{0.5}{416.67} = 1.2 \times 10^{-3}$$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{1.2 \times 10^{-3}}{4\pi \times 10^{-7}} = 954.9$$

(ii)

$$L = \frac{N\phi}{I}; \qquad \phi = B \times A = 0.5 \times \frac{\pi(2 \times 10^{-2})^2}{4}$$

$$= 1.57 \times 10^{-4}\text{Wb}$$

$$L = \frac{250 \times 1.57 \times 10^{-4}}{0.5} = 0.0785\text{H}$$

(iii)

$$\phi_1 = BA = 1.57 \times 10^{-4}\text{Wb}$$

$$\phi_2 = 10\% \text{ of } \phi_1 = 0.157 \times 10^{-4}\text{Wb}$$

$$d\phi = \phi_1 - \phi_2 = 1.413 \times 10^{-4}\text{Wb}$$

$$e = N\frac{d\phi}{dt} = 250 \times \frac{1.413 \times 10^{-4}}{0.001} = 35.325\text{V}$$

Example 2.19 When a voltage of 220V is applied to a coil with a resistance of 50Ω, the flux linking with the coil is 0.005Wb. If the coil has 1000 turns find the inductance of the coil and the energy stored in the magnetic field.

Solution:

$$\text{Current} = \frac{V}{R} = \frac{220}{50} = 4.4\text{A}$$

$$L = \frac{N\phi}{I} = \frac{1000 \times 0.005}{4.4} = 1.136\text{H}$$

$$\text{Energy stored} = \frac{1}{2}LI^2 = \frac{1}{2} \times 1.136 \times 4.4^2 = 11\text{J}$$

Example 2.20 A mild steel ring has a mean diameter of 160mm and a cross section area of 300mm^2. Calculate
(a) the mmf to produce a flux of 333μWb
(b) reluctance
(c) relative permeability.

The B-H data is given in table below.

B(T)	0.9	1.1	1.2	1.3
H(AT/m)	260	450	600	820

Solution:

$$B = \frac{\phi}{A} = \frac{400 \times 10^{-6}}{333 \times 10^{-6}} = 1.2\text{T}$$

(a) From Table, $H = 600$AT/m

$$mmf = Hl = 600 \times \pi \times (160 \times 10^{-3})$$
$$= 301.59AT$$

(b) $mmf = \phi S$

$$S = \frac{mmf}{\phi} = \frac{301.59}{333 \times 10^{-6}} = 9.057 \times 10^5 \text{AT/Wb}$$

(c)

$$\mu = \frac{B}{H} = \frac{1.2}{600} = 2 \times 10^{-3}$$
$$\mu_r = \frac{\mu}{\mu_0} = \frac{2 \times 10^{-3}}{4\pi \times 10^{-7}} = 1591.5.$$

Example 2.21 A steel circuit has a uniform cross sectional area of 5cm^2 and a length of 25cm. A coil of 120 turns is wound uniformly over it. When the current in the coil is 1.5A, the total flux is 0.3mWb. Find (i) H (ii) μ_r.

Solution:

$$mmf = NI = 120 \times 1.5 = 180\text{AT}$$
$$mmf = Hl$$
$$\therefore\ H = \frac{NI}{l} = \frac{180}{2.5 \times 10^{-2}} = 720\text{AT/m}$$
$$B = \frac{\phi}{A} = \frac{0.3 \times 10^{-3}}{5 \times 10^{-4}} = 0.6\text{Wb/m}^2$$
$$B = \mu_0 \mu_r H \quad \text{or} \quad \mu_r = \frac{B}{\mu_0 H} = \frac{0.6}{4\pi \times 10^{-7} \times 720}$$
$$= 663.145$$

Example 2.22 A steel ring has a mean circumference of 750mm and a cross sectional area of 500mm^2. It is wound with 120 turns (a) Using the table of example 2.20 find the current required to set up a magnetic flux of 630μWb in the ring (b) If the air gap in a magnetic circuit is 1.1mm long and 2000mm^2 in cross section, calculate the reluctance of the air gap and the mmf required to send a flux of 700μWb across the air gap.

Solution:

(a)

$$B = \frac{\phi}{A} = \frac{630 \times 10^{-6}}{500 \times 10^{-6}} = 1.26$$

From table, using interpolation, $H = 732$AT/m.

$$I = \frac{Hl}{N} = \frac{732 \times 750 \times 10^{-3}}{120} = 4.575\text{A}$$

(b)

$$B = \frac{\phi}{A} = \frac{700 \times 10^{-6}}{2000 \times 10^{-6}} = 0.35\text{Wb/m}^2$$
$$H = \frac{B}{\mu_0} = \frac{0.35}{4\pi \times 10^{-7}} = 2.785 \times 10^5\text{AT/m}$$

(b)

$$L_1 = \frac{N_1\phi_1}{I_1} = \frac{12,000 \times 0.05 \times 10^{-3}}{5} = 0.12\text{H}$$

$$L_2 = \frac{N_2\phi_2}{I_2} = \frac{15,000 \times 0.075 \times 10^{-3}}{5} = 0.225\text{H}$$

(c)

$$K = \frac{M}{\sqrt{L_1 L_2}} = \frac{0.0675}{\sqrt{0.12 \times 0.225}} = 0.411$$

Example 2.25 Two coupled coils of self inductances 0.6H and 0.16H have a coefficient of coupling 0.8. Find mutual inductance and turns ratio.

Solution:

$$M = K\sqrt{L_1 L_2} = 0.8\sqrt{0.6 \times 0.16}$$
$$= 0.248\text{H}$$

$$I_1 = \frac{N_1\phi_1}{L_1}$$

$$M = \frac{N_2 K\phi_1}{I_1} = \frac{N_2 K\phi_1}{N_1\phi_1/L_1} = \frac{N_2 L_1 K}{N_1}$$

$$\therefore \frac{N_2}{N_1} = \frac{M}{K L_1} = \frac{0.248}{0.8 \times 0.6} = 0.516$$

Questions

(1) Define magnetic flux.

(2) What is Biot-Savart's law?

(3) What is the force between two current carrying conductors?

(4) Explain mmf and its analogy with emf.