CHAPTER 2

# The First Law and Its Applications

## 2.1 INTRODUCTION

The first law of thermodynamics was formulated on the basis of Paddle Wheel Experiment conducted by Joule. (Fig. 2.1) A number of experiments were conducted by him wherein a paddle wheel was rotated by different forms of inputs. His findings were that the work expended was proportional to increase in thermal energy.

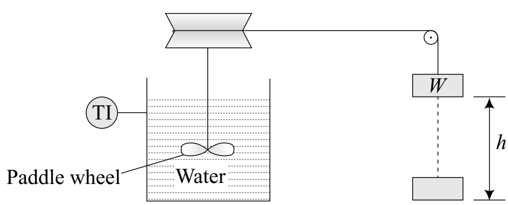



**Fig. 2.1** Joule's paddle wheel experiment

$$Q \propto W$$

or $$Q = \frac{W}{J}$$

where, $J$ = proportionality constant called mechanical equivalent.

In SI system,

$$J = 1$$

The first law of thermodynamics states that work and heat are mutually convertible. The present tendency is to include all forms of energy.

The first law can be stated in many ways:

**Case I** For a closed system, there is no flow energy (FE)

$$\therefore \quad q_{1-2} - w_{1-2} = g(Z_2 - Z_1) + \frac{V_2^2 - V_1^2}{2} + (u_2 - u_1)$$

This is called non-flow energy equation.

**Case II** When there is no change in the potential energy of a closed system, i.e., $PE_2 = PE_1$

$$\therefore \quad q_{1-2} - w_{1-2} = (KE_2 - KE_1) + (u_2 - u_1) = \frac{V_2^2 - V_1^2}{2} + (u_2 - u_1)$$

**Case III** Closed or non-flow thermodynamic system, when there is no change of *PE* and also there is no flow of mass into or out of a system, i.e.,

$$PE_2 = PE_1 \text{ and } KE_2 = KE_1$$

$$\therefore \quad q_{1-2} - w_{1-2} = u_2 - u_1$$

**Case IV** Isolated system,

$$q_{1-2} = 0; \quad w_{1-2} = 0$$

$$e_2 = e_1 \quad \text{and} \quad u_2 = u_1$$

This shows that the first law of thermodynamics is the law of conservation of energy.

**Case V** Cyclic process

There is no change in the internal energy and the stored energy is zero.

$$\oint \delta q = \oint \delta w.$$

## 2.4 FIRST LAW ANALYSIS OF PROCESSES FOR IDEAL GAS

For an ideal gas,

$$du = Cv\, dT$$

$$dh = Cp\, dT$$

$$pv = RT.$$

These equations are valid for all processes.

### 2.4.1 Constant Volume Process

The first law equation for a non-flow process when kinetic energy and potential energy are negligible:

$$\delta q - \delta w = du.$$

For a reversible process,

$$\delta w = p\, dv \text{ and } du = Cv\, dT$$

$$\therefore \quad \delta q - p\,dv = Cv\,dT$$

For constant volume process

$$\delta w = p\,dv = 0$$

$$\therefore \quad \delta q = Cv\,dT = du$$

or
$$q_{1-2} = u_2 - u_1 = \int_1^2 Cv\,dT$$

If the value of *Cv* as a function of *T* is known, integral can be evaluated.

### 2.4.2 Constant Pressure Process

The first law equation is:

$$\delta q - \delta w = du$$

or
$$\delta q = du + p\,dv$$

$$\delta q = d(u + pv) \qquad [\because \; p = \text{constant}]$$

$$= dh = Cp\,dT$$

$$\therefore \quad q_{1-2} = \int_1^2 Cp\,dT = h_2 - h_1$$

If the value of *Cp* as a function of temperature is known, integral can be evaluated.

### 2.4.3 Constant Temperature Process

The first law equation is:

$$\delta q - \delta w = du$$

$$du = Cv\,dT = 0 \qquad (\because \; T \text{ is constant})$$

$$\therefore \quad \delta q = p\,dv$$

$$q_{1-2} = \int_1^2 p\,dv$$

For an ideal gas,

$$pv = RT$$

$$\therefore \quad p = \frac{RT}{v}$$

$$\therefore \qquad q_{1-2} = w_{1-2} = \int_1^2 \frac{RT}{v} dv$$

$$= RT \ln \frac{v_2}{v_1}$$

$$= p_1 v_1 \ln \frac{v_2}{v_1}$$

Also, $$w_{1-2} = p_1 v_1 \ln \frac{p_1}{p_2} \qquad (\because \quad p_1 v_1 = p_2 v_2)$$

### 2.4.4 Adiabatic Process

The first law equation is,

$$\delta q - \delta q = du.$$

No heat leaves or enters the system.

$$\therefore \qquad 0 - \delta w = du$$

$$0 - p\,dv = Cv\,dT$$

$$\therefore \qquad dT = \frac{-pdv}{Cv} \qquad \text{...(a)}$$

But, $$pv = RT$$

Differentiating,

$$pdv + vdp = RdT$$

$$\therefore \qquad dT = \frac{pdv + vdp}{R} = \frac{pdv + vdp}{(Cp - Cv)} \qquad [\because R = Cp - Cv] \quad \text{...(b)}$$

Equating (a) and (b) for $dT$

$$\frac{-pdv}{Cv} = \frac{pdv + vdp}{(Cp - Cv)}$$

$$\therefore \qquad \frac{(C_p - Cv)}{Cv} = \frac{pdv + vdp}{-pdv} = -1 - \frac{vdp}{pdv}$$

$$\therefore \qquad \frac{Cp}{Cv} - 1 = -1 - \left[\frac{v}{dv} \times \frac{dp}{p}\right]$$

or $$\gamma = -\left[\frac{v}{dv} \times \frac{dp}{p}\right] \qquad \left[\because \frac{C_p}{Cv} = \gamma\right]$$

∴ $$\gamma\left[\frac{dv}{v}\right] = -\frac{dp}{p}$$

or $$\gamma\frac{dv}{v} + \frac{dp}{p} = 0$$

Integrating,

$$\gamma \ln v + \ln p = \text{constant}$$

$$\ln pv^{\gamma} = \ln(\text{constant})$$

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} = pv^{\gamma} = \text{constant}$$

$$\frac{p_1}{p_2} = \left(\frac{v_2}{v_1}\right)^{\gamma}$$

$$\frac{v_1}{v_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}}$$

But $$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

or $$\frac{v_1}{v_2} = \frac{T_1}{T_2} \times \frac{p_2}{p_1}$$

∴ $$\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} = \frac{T_1}{T_2} \times \frac{p_2}{p_1}$$

or $$\frac{T_1}{T_2} = \left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \times \frac{p_2}{p_1}$$

$$= \left(\frac{p_1}{p_2}\right)^{-\frac{1}{\gamma}+1}$$

## 2.4.6 Free Expansion Process

Free expansion process is unrestricted expansion of a gas without any work output. As the gas cannot be compressed back to its initial state without the use of external work, the process of free expansion is highly irreversible process. The process is also called constant internal energy process as there is no exchange of heat and work between the system and the surrounding. The process can be explained by a simple example.

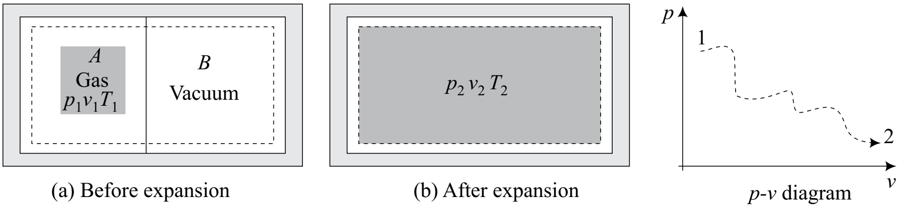



**Fig. 2.3** Free expansion process

There are two chambers $A$ and $B$ completely insulated and separated by a membrane. Chamber $A$ has gas at parameters $p_1 v_1 T_1$ and chamber $B$ is completely evacuated. The state is plotted as point 1 on $p$-$v$ diagram. The partition is removed and the gas occupies the total volume of chambers $A$ and $B$. The final conditions of the system are $p_2 v_2 T_2$ are shown by point 2 on the $p$-$v$ diagram. The free expansion is shown by a dotted line as the process is uncontrolled, irreversible and unspecified.

1. The work done is zero as there is no expansion of boundary

$$W_{1-2} = 0$$

2. The system is insulated and there is no exchange of heat with the surrounding.

$$Q_{1-2} = 0$$

3. Applying the first law of thermodynamics to the closed system,

$$Q_{1-2} - W_{1-2} = U_2 - U_1$$

$$0 - 0 = U_2 - U_1$$

$$\therefore \quad U_2 = U_1$$

The internal energy of the system during free expansion remains constant.

4. $$U_2 - U_1 = m\,Cv\,(T_2 - T_1) = 0$$

$$\therefore \quad T_2 = T_1.$$

The process of free expansion is isothermal process.

5. $$H_2 - H_1 = m\,Cp\,(T_2 - T_1) = 0$$

The enthalpy of the system remains constant.

### 2.4.7 Summary of Thermodynamic Relations for Non-flow Processes

The thermodynamic relations for an ideal gas in different non-flow processes are summarised in Table 2.1. The following relationships have been tabulated:

1. Governing equation or *p*-*v*-*T* relationship
2. Work done
3. Change of internal energy
4. Change of enthalpy
5. Heat exchanged.

The values given are for unit mass. The mechanical work in a steady flow process is compared with that in non-flow process in Table 2.2.

**Example 2.1** Three kg of air kept at 100 kPa and 300 K is compressed polytropically to 1500 kPa and 500 K. Calculate:

(i) Index $n$
(ii) Final volume
(iii) Work done
(iv) Heat exchanged

**Solution:**

(i) *Index n*

$$p_1 = 100 \text{ kPa} = 100 \times 10^3 \text{ N/m}^3$$

$$T_1 = 300 \text{ K}$$

$$p_2 = 1500 \text{ kPa} = 1500 \times 10^3 \text{ N/m}^2$$

$$T_2 = 500 \text{ K}$$

$$p_1 V_1^n = p_2 V_2^n$$

and $$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$\therefore$ $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\therefore \quad \frac{n-1}{n} = \frac{\ln \dfrac{T_2}{T_1}}{\ln \dfrac{p_2}{p_1}} = \frac{\ln \dfrac{500}{300}}{\ln \dfrac{1500}{100}} = \frac{0.5108}{2.708} = 0.1886.$$

$$\therefore \quad n = 1.23$$

(ii) *Final volume*

$$p_1 V_1 = m\,RT_1$$

Take $R = 287$ J/kg-K for air

$$V_1 = \frac{m\,RT_1}{p_1} = \frac{3 \times 287 \times 300}{100 \times 10^3} = 2.583 \text{ m}^3$$

$$p_2 V_2 = m\,RT_2$$

$$V_2 = \frac{m\,RT_2}{p_2} = \frac{3 \times 287 \times 500}{1500 \times 10^3} = \mathbf{0.287\ m^3} \quad \textbf{Ans.}$$

(iii) *Work done*

$$W_{1-2} = \frac{p_1 V_1 - p_2 V_2}{n-1} = \frac{m\,R(T_1 - T_2)}{n-1} = \frac{3 \times 287\,(300 - 500)}{1.23 - 1}$$

$$= -748696 \text{ J} = -748.7 \text{ kJ}$$

Work is done on the air during compression.

(iv) *Heat exchanged*

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1}\, W_{1-2} = \frac{1.4 - 1.23}{1.4 - 1}\,(-748.7)$$

$$= -318.2 \text{ kJ}.$$

Heat is rejected by the system.

## 2.5 THROTTLING PROCESS

The expansion of gas through an obstruction in the form of a partly opened valve or orifice is called throttling.

There is reduction in pressure and increase in the volume of the fluid. The process is adiabatic as there is no exchange of heat but it is irreversible. No work output occurs during throttling. It is not possible to compress the fluid back to initial pressure without the aid of external work. Therefore, the process is irreversible.

4. The change in velocity is negligible

$$V_1 \approx V_2$$

$$KE_1 = KE_2$$

Applying steady flow energy equation to unit mass flow

$$h_1 + \frac{v_1^2}{2} + gz_1 + q_{1-2} = h_2 + \frac{v_2^2}{2} + gz_2 + w_{1-2}$$

$$\therefore \quad h_1 = h_2$$

Therefore, throttling process is a constant enthalpy process

$$h_1 = h_2 = h_3 = h_4 = h_5 = ...$$

If the readings of pressure and temperature of the experiment are plotted on *T*-*p* diagram, a constant enthalpy line is obtained.

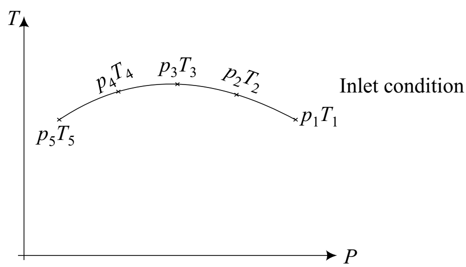



**Fig. 2.5** Constant enthalpy process

The slope of the constant enthalpy curve is called Joule-Thomson coefficient.

$$\mu = \left(\frac{dT}{dp}\right)_h$$

For perfect gas, $\mu = 0$.

**Applications of Throttling**

Although throttling is a energy loss process, it is used for the following:

1. To find out the dryness fraction of steam in a throttling calorimeter.
2. The speed of steam turbine is controlled in throttle governing.
3. Refrigeration effect (cooling) is obtained by throttling the refrigerant in a valve or capillary tube at inlet to evaporator.

$$\because \quad h_1 = h_2$$

$$\therefore \quad h_1 - h_2 = 0$$

$$Cp(T_1 - T_2) = 0$$

$$\therefore \quad T_1 = T_2$$

$$Cv(T_1 - T_2) = 0$$

$$\therefore \quad u_1 = u_2$$

For an ideal gas, throttling takes place at:

1. Constant enthalpy
2. Constant internal energy
3. Constant temperature.

However, in a real gas or fluid (refrigerant),

$$T_1 \neq T_2.$$

## 2.6 STEADY FLOW ENERGY EQUATION

According to the first law of thermodynamics, the total energy entering a system must be equal to total energy leaving the system. For unit mass,

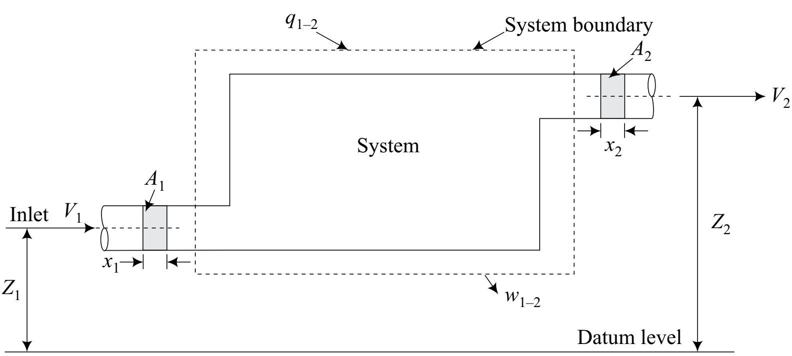



**Fig. 2.6** Steady flow process

$$e_1 = e_2$$

$$u_1 + p_1 v_1 + \frac{V_1^2}{2} + gz_1 + q_{1-2} = u_2 + p_2 v_2 + \frac{V_2^2}{2} + gz_2 + w_{1-2}$$

where, suffix 1 is for inlet and 2 for outlet.

$u$ = Specific internal energy
$pv$ = Flow work
$V$ = Fluid velocity
$Z$ = Height
$q_{1-2}$ = Heat exchange
$w_{1-2}$ = Work exchange

Now, $h = u + pv$

$$\therefore \quad h_1 + \frac{V_1^2}{2} + qz_1 + q_{1-2} = h_2 + \frac{V_2^2}{2} + qz_2 + w_{1-2}$$

This is called steady flow energy equation. This equation may also be written as follows:

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + q(z_2 - z_1)$$

In differential form,

$$dq - dw = dh + d(ke) + d(pe)$$

**Case I.** If the effect of gravity can be neglected, i.e., $z_2 \approx z_1$

$\therefore$ $$d(pe) = 0$$

$$dq - dw = dh + (dke)$$

or $$q_{1-2} - w_{1-2} = (h_2 - h_1) + \left(\frac{V_2^2 - V_1^2}{2}\right)$$

**Case II.** If gravity can be neglected and the change in velocity is negligible, i.e., $V_2 \approx V_1$

$$d(pe) = 0$$

$$d(ke) = 0$$

$\therefore$ $$q_{1-2} - w_{1-2} = (h_2 - h_1).$$

**Case III.** Applying the steady flow energy equation to a closed system (non-flow process)

$$d(pe) = 0$$

$$d(ke) = 0$$

$$p_1 v_1 = 0$$ (Flow energy or displacement energy at inlet and outlet is zero)

$$p_2 v_2 = 0$$

$\therefore$ $$h_1 = u_1$$

$$h_2 = u_2$$

$\therefore$ $$q_{1-2} - w_{1-2} = u_2 - u_1$$

This is called energy equation for a non-flow process.

## 2.6.1 Equation of Continuity

The mass flow rate ($\dot{m}$) of the working substance entering the system is same as leaving the system. The steady flow energy equation will be:

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + qZ_1 + q_{1-2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2} + qZ_2 + W_{1-2}\right)$$

or $$\dot{Q}_{1-2} - \dot{W}_{1-2} = \dot{m}(h_2 - h_1) + \frac{\dot{m}}{2}(V_1^2 - V_1^2) + \dot{m}g(z_2 - z_1)$$

$$-60 - \dot{W}_{1-2} = 0.5\left[(860-976)+\frac{180^2-200^2}{2\times10^3}+\frac{9.81\times60}{10^3}\right]$$

$$= 0.5\,[-116 - 3.8 + 0.5886] = -59.6$$

$$\therefore \qquad \dot{W}_{1-2} = -60 + 59.6 = \mathbf{0.4\ kW} \quad \textbf{Ans.}$$

Work is done on the system.

## 2.7 APPLICATIONS OF STEADY FLOW ENERGY EQUATION

The steady flow energy equation can be applied to various energy systems and devices such as boilers, condensers, evaporators, nozzles, turbines, compressors, etc.

### 2.7.1 Heat Exchanger

A heat exchanger is a device in which heat is transferred from one fluid to another. There are two steady flow streams, one of heating fluid and other fluid to be heated. The flow through a heat exchanger is characterized by,

1. No work exchange,

$$W_{1-2} = 0$$

2. No change in potential energy

$$Z_2 = Z_1$$

3. No change in kinetic energy

$$V_2 \approx V_1$$

4. Normally, no external heat interaction, if heat exchanger is insulated.
   Heat gained by cold fluid = Heat lost by hot fluid

$$\dot{m}_c(h_1 - h_2) = \dot{m}_h(h_4 - h_3)$$

where, $\dot{m}_c$ and $\dot{m}_h$ are flow rate of cold fluid and hot fluid respectively.

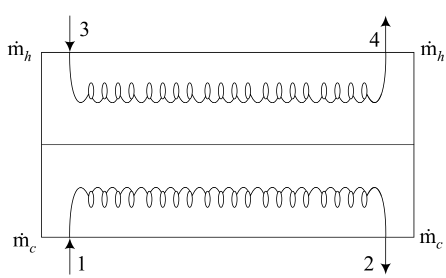



**Fig. 2.7** Heat exchanger

1. Work is done on the system and, hence, it is negative.

   $\dot{W}_{1-2}$ is –ve.
2. Potential energy and kinetic energy can normally be neglected.

$$Z_2 \approx Z_1$$
$$V_2 \approx V_1$$

3. Heat is lost from the compressor either by radiation or through a coolant (air or water).

   The heat exchange is –ve.

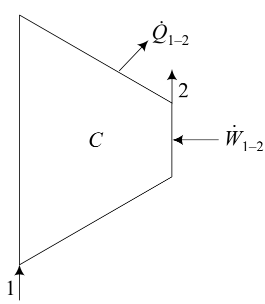
   $\dot{Q}_{1-2}$ is –ve or zero.

$$-\dot{Q}_{1-2} - (-\dot{W}_{1-2}) = \dot{m}(h_2 - h_1)$$

$$\therefore \qquad \dot{W}_{1-2} = \dot{Q}_{1-2} + \dot{m}(h_2 - h_1)$$

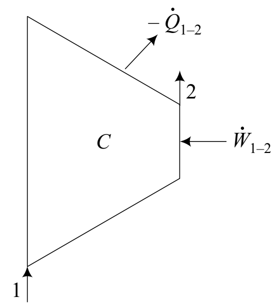
Therefore, work is done on the system to increase the enthalpy of the fluid.

**Example 2.4** 0.8 kg of air flows through a compressor under steady state conditions. The properties of air at entry are: pressure 1 bar, velocity 10 m/s, specific volume 0.95 m$^3$/kg and internal energy 30 kJ/kg. The corresponding values at exit are: 8 bar, 6 m/s, 0.2 m$^3$/kg and 124 kJ/kg. Neglecting the change in potential energy, determine the power input and pipe diameter at entry and exit.




**Solution:** The data given,

$$\dot{m} = 0.8 \text{ kg/s}$$
$$p_1 = 1 \text{ bar} = 100 \text{ kPa}$$
$$V_1 = 10 \text{ m/s}$$
$$v_1 = 0.95 \text{ m}^3/\text{kg}$$
$$u_1 = 0.9 \text{ kJ/kg}$$
$$p_2 = 8 \text{ bar} = 800 \text{ kPa}$$




**Solution:**

1. *Inlet conditions*

$$p_1 = 0.1 \text{ MPa} = 0.1 \times 10^3 \text{ kN/m}^2$$
$$T_1 = 27°\text{C} + 273 = 300 \text{ K}$$
$$V_1 = 40 \text{ m/s}$$
$$A_1 = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$$

2. *Outlet conditions*

$$p_2 = 10 \times 0.1 \times 10^3 \text{ kN/m}^2 = 10^3 \text{ kN/m}^2$$
$$V_2 = 100 \text{ m/s}$$
$$A_2 = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$$

3. *Equation of continuity*

The equation of state,

$$p_1 V_1 = mRT_1$$

or $$p_1 v_1 = RT_1$$

$\therefore$ $$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 300}{100}$$

$$= 0.861 \text{ m}^3\text{/kg} \quad [R = 0.287 \text{ kJ/kg-K for air}]$$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(100 \times 10^{-4}) \times 40}{0.861} = 0.4646 \text{ kg/s}$$

$$\dot{m} = \frac{A_2 V_2}{v_2}$$

$\therefore$ $$v_2 = \frac{A_2 V_2}{\dot{m}} = \frac{(20 \times 10^{-4}) \times 100}{0.4646} = 0.43 \text{ m}^3\text{/kg}$$

$$p_2 v_2 = RT_2$$

$$T_2 = \frac{p_2 v_2}{R} = \frac{10^3 \times 0.43}{0.287} = 1498 \text{ K}$$

4. *Steady flow energy equation*

$$Q_{1-2} - W_{1-2} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3}(Z_2 - Z_1)\right]$$

$$-0.05\, W_{1-2} - W_{1-2} =$$

$$0.43\left[1.005(1498 - 300) + \frac{(100)^2 - (40)^2}{2 \times 10^3} + 0\right]$$

$$W_{1-2} = \mathbf{546.86\ kW} \quad \textbf{Ans.}$$

### 2.7.3 Gas Turbine

A gas turbine converts the heat energy of hot gases into mechanical work. A compressor driven by the gas turbine compresses air or gas to a higher pressure. The high pressure air or gas is heated by combustion of fuel. The high pressure and high temperature air or gas is admitted to a gas turbine. Power is produced in the turbine at the expense of enthalpy drop of the gas. The main characteristics of the system are:

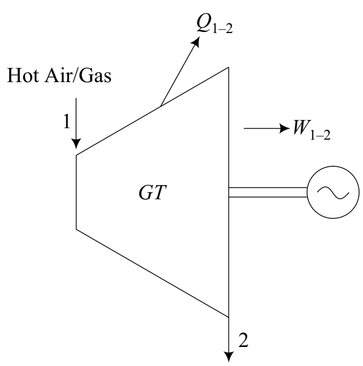



**Fig. 2.9** Gas turbine

1. The heat loss by radiation to the surrounding is –ve.
2. Normally, *ke* and *pe* are neglected.

$$V_2 \approx V_1$$

$$Z_2 = Z_1$$

$$\dot{W}_{1-2} = Q_{1-2} + \dot{m}(h_1 - h_2)$$

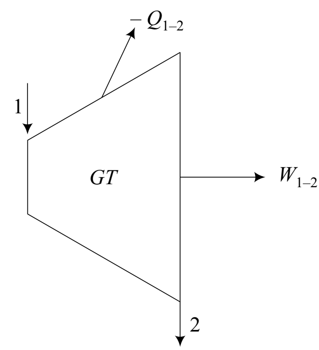
**Example 2.6** Air passes through a gas turbine system at the rate of 4.5 kg/s. It enters the turbine system with a velocity of 90 m/s and a specific volume of 0.85 $m^3/kg$. It leaves the turbine system with a specific volume of 1.45 $m^3/kg$. The exit area of the turbine system is 0.38 $m^2$. In its passage through the turbine, the specific enthalpy of air is reduced by 200 kJ/kg and there is a heat loss of 40 kJ/kg. Determine:

(i) The inlet area of turbine.

(ii) The exit velocity of air in m/s.

(iii) Power developed by the turbine system in kW.

**Solution:**




1. *Inlet conditions*

$$\dot{m} = 4.5 \text{ kg/s}$$
$$V_1 = 90 \text{ m/s}$$
$$v_1 = 0.85 \text{ m}^3/\text{kg}$$

2. *Outlet conditions*

$$v_2 = 1.45 \text{ m}^3/\text{kg}$$
$$A_2 = 0.38 \text{ m}^2$$
$$Q_{1-2} = -40 \text{ kJ/kg}$$
$$\Delta h = 200 \text{ kJ/kg}$$

3. *Equation of continuity*

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

$$\therefore \quad A_1 = \frac{\dot{m} V_1}{v_1} = \frac{4.5 \times 0.85}{90} = \mathbf{0.0425\ m^2} \quad \textbf{Ans.}$$

$$V_2 = \frac{\dot{m} V_2}{A_2} = \frac{4.5 \times 1.45}{0.38} = \mathbf{17\ m/s} \quad \textbf{Ans.}$$

4. *Steady flow energy equation*

$$Q_{1-2} - W_{1-2} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3}(Z_2 - Z_1)\right]$$

$$-40 \times 4.5 - W_{1-2} = 4.5\left[200 + \frac{(17)^2 - (90)^2}{2 \times 10^3} + 0\right]$$

$$W_{1-2} = \mathbf{702.43\ kW} \quad \textbf{Ans.}$$

## 2.7.4 Steam Turbine

High pressure and high temperature steam from a steam generator or a boiler is admitted through a steam turbine. During expansion of steam through the turbine, there is enthalpy drop. The turbine gives out positive mechanical work. Although it is insulated, there can be some heat loss due to radiation. The main characteristics of the system are:

1. There is enthalpy drop through the turbine.
2. Change of potential energy is usually neglected

$$Z_2 = Z_1$$

3. If $V_2 = V_1$, change of kinetic energy is neglected.
4. The heat loss due to radiation is –ve.

$$W_{1-2} = \dot{m}(h_1 - h_2) + Q_{1-2}$$

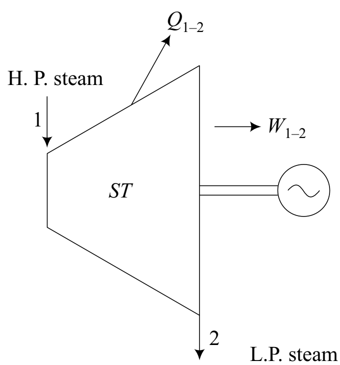



**Fig. 2.10** Steam turbine

**Example 2.11:** The following details refer to a steam turbine:

Steam flow rate = 1 kg/sec

Inlet velocity of steam = 100 m/sec

Exit velocity of steam = 150 m/sec

Enthalpy at inlet = 2900 kJ/kg

Enthalpy at outlet = 1600 kJ/kg

Write the steady flow energy equation. Assuming that the change in potential energy is negligible, determine the power available from the turbine.

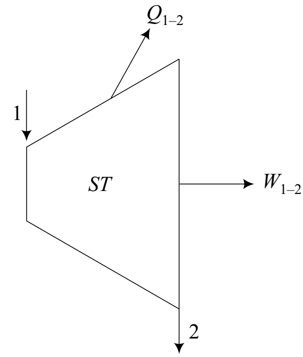
**Solution:**

1. *Inlet Data*

$\dot{m}$ = 1 kg/sec

$V_1$ = 100 m/s

**Solution:**




1. *Inlet data,*

$$\dot{m} = 3600 \text{ kg/hour} = 1 \text{ kg/sec}$$
$$V_1 = 80 \text{ m/s}$$
$$Z_1 = 10 \text{ m}$$
$$h_1 = 3276 \text{ kJ/kg}$$

$$Q_{1-2} = -36 \text{ MJ/hour} = -\frac{36 \times 10^3}{3600} \text{ kJ/sec}$$

$$= -10 \text{ kJ/sec}$$

2. *Outlet conditions,*

$$V_2 = 150 \text{ m/sec}$$
$$Z_2 = 3 \text{ m}$$
$$h_2 = 2465 \text{ kJ/kg}$$

3. *Steady flow energy equation,*

$$Q_{1-2} - W_{1-2} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3}(Z_2 - Z_1)\right]$$

$$-10 - W_{1-2} = 1\left[(2465 - 3276) + \frac{(150)^2 - (80)^2}{2 \times 10^3} + \frac{9.81}{10^3}(3 - 10)\right]$$

$$-10 - W_{1-2} = -811 + 8.05 - 0.06867$$

$$W_{1-2} = \mathbf{793\ kW} \quad \textbf{Ans.}$$

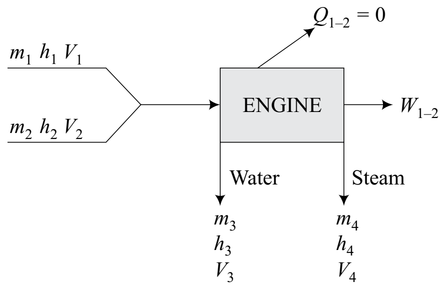
**Example 2.9** The steam supply to an engine comprises two streams which mix before entering the engine. One stream is supplied at the rate of 0.01 kg/s with an enthalpy of 2950 kJ/kg and a velocity of 20 m/s. The other stream is supplied at the rate of 0.1 kg/s with an enthalpy of 2665 kJ/kg and a velocity of 120 m/s. At the exit from the engine the fluid leaves as two streams, one of water at the rate of 0.001 kg/s with an enthalpy of 421 kJ/kg and the other of steam. The fluid velocities at the exit are negligible. The engine develops a shaft power of 25 kW. The heat transfer is negligible. Evaluate the enthalpy of the second exit stream.

**Solution:**

1. *Inlet data,*

$$m_1 = 0.01 \text{ kg/s}$$

$$h_1 = 2950 \text{ kJ/kg}$$




$V_1 = 20$ m/s

$m_2 = 0.1$ kg/s

$h_2 = 2665$ kJ/kg

$V_2 = 120$ m/s

Total inlet energy

$$E_i = m_1 h_1 + \frac{m_1 V_1^2}{2 \times 10^3} + \frac{g}{10^3} Z_1 + Q_{1-2} + m_2 h_2 + \frac{m_1 V_2^2}{2 \times 10^3} + \frac{g}{10^3} Z_2$$

$$= 0.01 \times 2950 + \frac{0.01 \times (20)^2}{2 \times 10^3} + 0 + 0 + 0.1 \times 2665 + \frac{0.1 \times 120^2}{2 \times 10^3} + 0$$

$E_i = 29.5 + 0.002 + 266.5 + 0.72 = 296.722$ kJ

$m_i$ =1 $0.01 + 0.1 = 0.11$ kg/s

2. *Outlet data*

$m_3 = 0.001$ kg/s

$h_3 = 421$ kJ/kg

$V_3 = 0$

$Z_3 = 0$

$m_4 = m_i - m_3 = 0.11 - 0.001 = 0.109$ kg/sec

Total energy,

$E_e = m_3 h_3 + m_4 h_4 + W_{1-2}$

$= 0.001 \times 421 + 0.109 h_4 + 25$

$= 0.109 h_4 + 25.421$

For steady flow conditions,

$$E_i = E_e$$

$$\therefore \quad 296.722 = 0.109 h_4 + 25.421$$

$$\therefore \quad h_4 = \frac{296.722 - 25.421}{0.109} = \mathbf{2489\ kJ/kg} \quad \textbf{Ans.}$$

### 2.7.5 Nozzle

The nozzle converts the pressure energy of a stream into its kinetic energy and as a result the velocity of the stream increases. The enthalpy drop of the fluid is used to accelerate the flow. Nozzles are used in steam turbines, gas turbines, pumps, etc. The operating characteristics of a nozzle are:

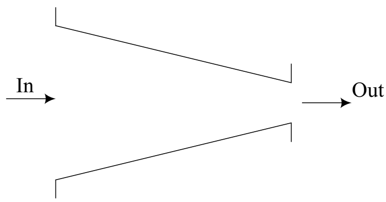



**Fig. 2.11** Nozzle

1. There is no work output,

   $$W_{1-2} = 0$$

2. The heat loss is normally absent,

   $$Q_{1-2} = 0$$

3. The change of potential energy is negligible,

   $$Z_2 = Z_1$$

4. The steady flow energy equation,

   $$\frac{V_2^2 - V_1^2}{2} = (h_1 - h_2)$$

   $$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

   If $V_2 >> V_1$ and $V_1$ can be neglected,

   $$V_2 = \sqrt{2(h_1 - h_2)}$$

   The mass flow rate,

   $$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

The area of nozzle at the inlet and outlet can be estimated.

**Example 2.10** In an isentropic flow through a nozzle, air flows at the rate of 600 kg/hr. At inlet to nozzle, pressure is 2 MPa and temperature is 127°C. The exit pressure is 0.5 MPa. If the initial air velocity is 300 m/s, determine:

(i) Exit velocity of air, and

(ii) Inlet and exit area of nozzle.

**Solution:**

1. *Inlet conditions,*

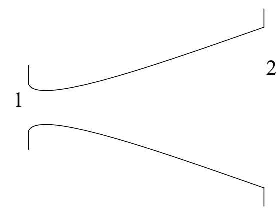
$$p_1 = 2 \text{ MPa} = 2 \times 10^3 \text{ kPa}$$

$$T_1 = 127°\text{C} + 273 = 400 \text{ K}$$

$$\dot{m} = 600 \text{ kg/hr} = \frac{600}{3600} = \frac{1}{6} \text{ kg/s}$$

$$V_1 = 300 \text{ m/s}$$

$$Q_{1-2} = 0$$




2. *Outlet conditions,*

$$p_2 = 0.5 \text{ MPa} = 0.5 \times 10^3 \text{ kPa}$$

The flow through nozzle is isentropic,

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma = 1}{\gamma}}$$

For air $\gamma = 1.4$

$$\therefore \quad T_2 = 400 \left(\frac{0.5}{2}\right)^{\frac{1.4-1}{1.4}} = 269.18 \text{ K}$$

$$W_{1-2} = 0$$

3. *Steady flow energy equation,*

$$Q_{1-2} - W_{1-2} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3}(Z_2 - Z_1)\right]$$

$$h_2 - h_1 = C_p(T_2 - T_1)$$

$$\therefore \quad 0 - 0 = \frac{1}{6}\left[1.005(400 - 269.18) + \frac{V_2^2 - (300)^2}{2 \times 10^3} + 0\right]$$

$$\therefore \quad 0 = 131.474 + \frac{V_2^2 - (300)^2}{2 \times 10^3}$$

$$\therefore \quad V_2 = \sqrt{131.474 \times 2 \times 10^3 + 300^2}$$

$$= \mathbf{594\ m/s} \quad \textbf{Ans.}$$

4. *Equation of continuity*

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

From equation of state,

$$p_1 v_1 = RT_1$$

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 400}{2 \times 10^3} = 0.0574 \text{ m}^3/\text{kg}$$

For air, $R = 0.287$ kJ/kg-K

$$\therefore \quad A_1 = \frac{\dot{m} V_1}{v_1} = \frac{\frac{1}{6} \times 0.0574}{300} = 3.1888 \times 10^{-5} \text{ m}^2$$

$$= 31.88 \text{ mm}^2$$

$$v_2 = \frac{RT_2}{p_2} = \frac{0.287 \times 269.18}{0.5 \times 10^3} = 0.1545 \text{ m}^3/\text{kg}$$

$$\therefore \quad A_2 = \frac{\dot{m} v_2}{V_2} = \frac{\frac{1}{6} \times 0.1545}{594} = 4.335 \times 10^{-5} \text{ m}^2$$

$$= \mathbf{43.35\ mm^2} \quad \textbf{Ans.}$$

The exit area of the nozzle is more than inlet area. Therefore, the nozzle is a convergent-divergent nozzle.

### 2.7.6 Diffuser

A diffuser has varying cross-section and reduces the velocity of the flowing fluid. There are two types of diffusers:

1. *Subsonic Diffuser*

   The velocity of the fluid is less then sonic speed and the area of cross-section of diffuser increases from inlet to exit.

2. *Supersonic Diffuser*

   The velocity of fluid is more than sonic velocity and the area of diffuser decreases.

The operating characteristics of a diffuser are similar to that for a nozzle.

**Example 2.11** Water vapour at 90 kPa and 150°C enters a subsonic diffuser with a velocity of 150 m/s and leaves the diffuser at 190 kPa with a velocity of 55 m/s and during the process 1.5 kJ/kg of heat is lost to the surrounding. Determine:

(i) The final temperature

(ii) The mass flow rate

(iii) The exit diameter, assuming the inlet diameter as 10 cm and steady flow.

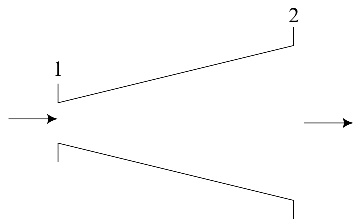
**Solution:**

1. *Inlet condition,*

$$p_1 = 90 \text{ kN/m}^2$$

$$T_1 = 150°\text{C} + 273 = 423 \text{ K}$$

$$V_1 = 150 \text{ m/s}$$




2. *Outlet condition,*

$$p_2 = 190 \text{ kN/m}^2$$

$$V_2 = 55 \text{ m/s}$$

For water vapours take Cp = 2.1 kJ/kg-K

3. *Steady flow energy equation,*

$$Q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^1 - V_1^2}{2 \times 10^3} + \frac{g}{10^3}(Z_2 - Z_1)$$

For the given diffuser,

$$W_{1-2} = 0$$

$$Z_2 = Z_1$$

$$\therefore \quad q_{1-2} = (h_2 - h_1) + \frac{V_2^1 - V_1^2}{2 \times 10^3}$$

$$-1.5 = Cp(T_2 - T_1) + \frac{(55)^2 - (150)^2}{2 \times 10^3}$$

$$Cp(T_2 - T_1) = -1.5 + 9.7375 = 8.2375$$

$$T_2 = \frac{8.2375}{2.1} + 423 = 427 \text{ K}$$

or $\quad t_2 = 427 - 273 = \mathbf{154°C}$ **Ans.**

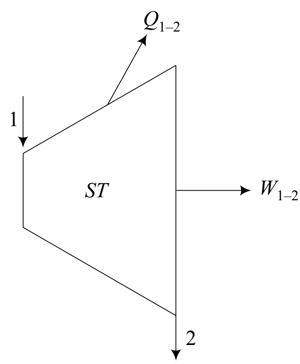
4. *Mass flow rate,*

$$-0.05\,W_{1-2} - W_{1-2} = 0.43\left[1.005(1498-300) + \frac{(100)^2-(40)^2}{2\times 10^3} + 0\right]$$

$$W_{1-2} = \mathbf{546.86\ kW} \quad \textbf{Ans.}$$

**Example 2.13** A steam turbine operating under steady flow conditions receives 3600 kg of steam per hour. The steam enters the turbine at a velocity of 80 m/s, an elevation of 10 m and specific enthalpy of 3276 kJ/kg. It leaves the turbine at a velocity of 150 m/sec, an elevation of 3 m and a specific enthalpy of 2465 kJ/kg. Heat losses from the turbine to the surrounding amount to 36 MJ/hr. Estimate the power output of turbine.

**Solution**




1. *Inlet data,*

$$\dot{m} = 3600\ \text{kg/hour} = 1\ \text{kg/sec}$$
$$V_1 = 80\ \text{m/s}$$
$$Z_1 = 10\ \text{m}$$
$$h_1 = 3276\ \text{kJ/kg}$$

$$Q_{1-2} = -36\ \text{MJ/hour} = -\frac{36\times 10^3}{3600}\ \text{kJ/sec}$$
$$= -10\ \text{kJ/sec}$$

2. *Outlet conditions,*

$$V_2 = 150\ \text{m/sec}$$
$$Z_2 = 3\ \text{m}$$
$$h_2 = 2465\ \text{kJ/kg}$$

3. *Steady flow energy equation,*

$$Q_{1-2} - W_{1-2} = \dot{m}\left[(h_2-h_1) + \frac{V_2^2 - V_1^2}{2\times 10^3} + \frac{g}{10^3}(Z_2 - Z_1)\right]$$

$$-10 - W_{1-2} = 1\left[(2465-3276) + \frac{(150)^2-(80)^2}{2\times 10^3} + \frac{9.81}{10^3}(3-10)\right]$$

$$-10 - W_{1-2} = -811 + 8.05 - 0.06867$$

$$W_{1-2} = \mathbf{793\ kW} \quad \textbf{Ans.}$$

**Example 2.14** In an isentropic flow through a nozzle, air flows at the rate of 600 kg/hr. At inlet to nozzle, pressure is 2 MPa and temperature is 127°C. The exit pressure is 0.5 MPa. If the initial air velocity is 300 m/s, determine:

(i) Exit velocity of air, and
(ii) Inlet and exit area of nozzle.

**Solution**

1. *Inlet conditions,*

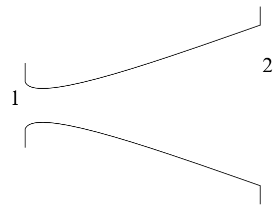
$$p_1 = 2 \text{ MPa} = 2 \times 10^3 \text{ kPa}$$

$$T_1 = 127°\text{C} + 273 = 400 \text{ K}$$

$$\dot{m} = 600 \text{ kg/hr} = \frac{600}{3600} = \frac{1}{6} \text{ kg/s}$$

$$V_1 = 300 \text{ m/s}$$

$$Q_{1-2} = 0$$




2. *Outlet conditions,*

$$p_2 = 0.5 \text{ MPa} = 0.5 \times 10^3 \text{ kPa}$$

The flow through nozzle is isentropic,

$$\therefore \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{\gamma = 1}{\gamma}}$$

For air $\gamma = 1.4$

$$\therefore \quad T_2 = 400\left(\frac{0.5}{2}\right)^{\frac{1.4-1}{1.4}} = 269.18 \text{ K}$$

$$W_{1-2} = 0$$

3. *Steady flow energy equation,*

$$Q_{1-2} - W_{1-2} = \dot{m}\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3}(Z_2 - Z_1)\right]$$

$$h_2 - h_1 = C_p(T_2 - T_1)$$

$$\therefore \quad 0 - 0 = \frac{1}{6}\left[1.005(400 - 269.18) + \frac{V_2^2 - (300)^2}{2 \times 10^3} + 0\right]$$

$$\therefore \quad 0 = 131.474 + \frac{V_2^2 - (300)^2}{2 \times 10^3}$$

$$\therefore \quad V_2 = \sqrt{131.474 \times 2 \times 10^3 + 300^2}$$

$$= \mathbf{594 \text{ m/s}} \quad \textbf{Ans.}$$

$$W_{1-2} = \frac{\gamma}{\gamma - 1}(p_1 V_1 - p_2 V_2)$$

6. Compare the following processes:
   (i) Free expansion process
   (ii) Throttling process
7. Briefly describe the following processes:
   (i) Joule's paddle wheel experiment
   (ii) Joule's-Thomson Porous Plug experiment