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MEASURES OF CENTRAL VALUES

INTRODUCTION OF CENTRAL TENDENCY

Measures of central tendency are numbers computed from the data set that help us locate the 'centre' of a relative frequency distribution.

These are expressed in the form of average, usually central point of a distribution is the point where the concentration of value of items is greatest and the most frequent on the scale of distribution.

Kinds of Averages

There are two types of averages:

1. Mathematical averages
   - (a) Arithmetic average or mean
   - (b) Geometric average
   - (c) Harmonic average
2. Positional average
   - (a) Median
   - (b) Mode

ARITHMETIC MEAN

The mean of a set of quantitative data is equal to the sum of the measurements divided by the number of the measurements contained in the data set.

If $x_1, x_2, x_3, ..., x_n$ is a set of value, then $AM(\bar{x})$ is defined as

$$\bar{x} = \frac{x_1 + x_2 + x_3 + x_4 + ... + x_n}{n}$$

$$= \frac{\sum x}{n}$$

Short Cut Method

Let us arrange the given terms in ascending order, we get

x	$d = x - A$
10	$10 - 23 = -13$
17	$17 - 23 = -6$
19	$19 - 23 = -4$
21	$21 - 23 = -2$
23	$23 - 23 = 0$
24	$24 - 23 = +1$
27	$27 - 23 = +4$
35	$35 - 23 = +12$
39	$39 - 23 = +16$
50	$50 - 23 = +27$
	$\sum di = 35$

Let A be the assumed mean be 23

Then $$\bar{x} = A + \frac{\sum di}{n} = 23 + \frac{35}{10} = 23 + 3.5 = 26.5$$ **Ans.**

Example: The arithmetic mean of the marks obtained by 10 students of class XI in mathematics in a certain examination is 35. The marks obtained are 25, 35, 21, 55, 48, 12, 15, x, 45, 35. Find the value of x.

Solution:

Given $$\bar{x} = \frac{25 + 35 + 21 + 55 + 48 + 12 + 15 + x + 45 + 35}{10}$$

$$\bar{x} = \frac{291 + x}{10}$$

Here $\bar{x} = 35$

Hence, $$35 = \frac{291 + x}{10}$$

$$350 - 291 = x$$

$$x = 59$$ **Ans.**

Some Results of A.M.

The sum of the deviations of the observations $x_1, x_2, x_3, \ldots\ldots x_n$ from the mean $\bar{x}$ is zero.

OR

Prove that $\sum_{i=1}^{n} (x_i - \bar{x}) = 0$

Proof: We know that

$$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

or $$n\bar{x} = \sum_{i=1}^{n} x_i$$

or $$n\bar{x} = x_1 + x_2 + x_3 + \ldots + x_n$$

Then the sum of deviation from the mean

$$\sum_{i=1}^{n}(x_i - \bar{x}) = (x_1 - \bar{x}) + (x_2 - \bar{x}) + (x_3 - \bar{x}) + \ldots + (x_n - \bar{x})$$

$$= (x_1 + x_2 + x_3 + \ldots + x_n) - (\bar{x} + \bar{x} + \bar{x} + \ldots \bar{x})$$

$$= n\bar{x} - n\bar{x} = 0$$ **Proved.**

2. If the mean of a set of n observations $x_1 x_2 x_3 \ldots x_n$ be $\bar{x}$ and if each observation of the data is changed by an amount a. Then its mean $\bar{x}$ is also changed by the same amount a.

Proof: Let new observation be

$$y_1 = x_1 + a,\ y_2 = x_2 + a,\ y_3 = x_3 + a, \ldots\ y_n = x_n + a$$

Then $$\bar{y} = \frac{(x_1 + a + x_2 + a + x_3 + a + \ldots + x_n + a)}{n}$$

$$= \frac{(x_1 + x_2 + \ldots + x_n) - (a + a + \ldots + a)}{n}$$

$$= \frac{\sum_{i=1}^{n} x_i + na}{n}$$

$$\bar{y} = \bar{x} + a$$ **Proved.**

Example: If $\bar{x}$ is the mean of $x_1 x_2 x_3 \ldots x_n$ show that mean of $ax_1 ax_3 \ldots ax_n$ is $a\bar{x}$.

Solution: $$\bar{x} = \frac{x_1 + x_2 + x_3 + \ldots + x_n}{n}$$

Let $\bar{y}$ is the mean of $ax_1, ax_2, ax_3, \ldots, ax_n$

i.e. $$\bar{y} = \frac{ax_1 + ax_2 + ax_3 + \ldots + ax_n}{n}$$

$$= a\frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} = \frac{a\sum_{2=1}^{n} x_i}{n}$$

$$\bar{y} = a\bar{x}$$ **Proved.**

Example: Find the mean of the squares of first n natural numbers.

Solution:

$$\text{Arithmetic mean} = \frac{1}{n}[1^2 + 2^2 + 3^2 + \ldots + n^2] = \frac{1}{n}\frac{[n(n+1)(2n+1)]}{6}$$

$$= \frac{(n+1)(2n+1)}{6} \quad \textbf{Ans.}$$

Example: The mean of 100 observations $x_1 x_2 x_3 \ldots x_{100}$ is 12.6. What is the mean if each observation is divided by 3?

Solution: Since each observation is divided by 3, then

$$\bar{y} = \frac{\frac{x_1}{3} + \frac{x_2}{3} + \ldots + \frac{x_{100}}{3}}{100} = \frac{1}{3 \times 100}(x_1 + x_2 + \ldots + x_{100})$$

$$= \left(\frac{x_1 + x_2 + \ldots + x_{100}}{100}\right)\frac{1}{3} = \frac{12.6}{3} = 4.2 \quad \textbf{Ans.}$$

Example: If the mean of 200 observation is 42; find the mean if each observation is increased by 2.

Solution:

$$\bar{y} = \frac{(x_1 + 2) + (x_2 + 2) + \ldots + (x_{200} + 2)}{200}$$

$$= \frac{x_1 + x_2 + \ldots + x_{200}}{200} + \frac{2 + 2 + \ldots + 2}{200}$$

$$\bar{y} = \bar{x} + \frac{2 \times 200}{200} = 42 + 2 = 44 \quad \textbf{Ans.}$$

Example: A class in a school has 60 students. In the mathematics examination, it was found that the average marks of the class was 48. Later it was found that the marks of one student which was actually 64 was read as 46. Find the correct mean.

Solution: Here $n = 60 \qquad \bar{x} = 48,$

Then $\sum x_i = n\bar{x}$

$= 60 \times 48 = 2880$

But, one entry is with error, 64 was read as 46.

Hence, correct $\sum x_i$ = incorrect $\sum x_i$ + correct item value – wrong item value

$= 2880 + 64 - 46 = 2898$

Thus, correct mean is $\bar{x} = \frac{2898}{60} = 48.3$ **Ans.**

Example: The arithmetic mean of 24 items is 48. If one more item is added to this series, the arithmetic mean becomes 48.2. Find the value of the 25th item.

Solution: We know that

$$\sum x_i = n\bar{x}$$

$$\sum x_i = 24 \times 48 = 1152$$

The new series of 25 items having arithmetic mean as 48.2. Hence,

$$\sum x_i = 25 \times 48.2 = 1205$$

Therefore, $x_{25} = 1205 - 1152$

$= 53$ **Ans.**

Example: Compute the arithmetic mean for the distribution of wages in a factory.

Wages (Rs.)	59	71	61	62	64	65
No. of workers	6	7	9	12	18	8

Solution:

x	f	fx
59	6	354
71	7	457
61	9	549
62	12	744
64	18	1152
65	8	520
	$\sum f = 60$	$\sum fx = 3816$

Thus, $$AM\ (\bar{x}) = \frac{\sum fx}{\sum f} = \frac{3816}{60} = 63.6$$

Example: The postal expenses on the letters despatched from an office on a day resulted in the following frequency distribution.

Postage (in paisa)	20	30	40	50	60	70
No. of letters	45	30	60	40	25	35

Find the mean postage per letter by short-cut and direct method.

Solution:

x	f	$d = x - A$	fd	fx
20	45	20 – 40 = –20	–900	900
30	30	30 – 40 = –10	–300	900
40	60	40 – 40 = 0	0	2400
50	40	50 – 40 = 10	400	2000
60	25	60 – 40 = 20	500	1500
70	35	70 – 40 = 30	1050	2450
	$\sum f = 235$		$\sum fd = 750$	$\sum fx = 10150$

Let the assumed mean be $A = 40$

(i) Direct Method

$$AM(\bar{x}) = \frac{\sum fx}{n} = \frac{10150}{235}$$

$$= 43.1914$$

(ii) Short-cut method

$$AM(\bar{x}) = A + \frac{\sum fd}{\sum f}$$

$$= 40 + \frac{750}{235}$$

$$= 40 + 3.1914$$

$$= 43.1914$$ **Ans.**

Example: Obtain mean wages from the following data

Wages (in Rs.)	1000	1500	2000	2050	2500
No. of workers	20	25	22	19	3

by direct method and short-cut method.

Solution:

Wages (z)	f	$d = x - A$	fx	fd
1000	20	–1000	20000	–20000
1500	25	–500	37500	–12500
2000	22	0	44000	0
2050	19	50	38950	950
2500	3	500	7500	1500
	$\sum f = 89$		$\sum fx = 147950$	$\sum fd = -30050$

ARITHMETIC MEAN OF CONTINUOUS OR GROUPED FREQUENCY DISTRIBUTION

In this frequency distribution, the procedure of computing arithmetic mean is the same as we have discussed earlier. The only difference is that in continuous frequency distributions the frequency within each class are assumed so be distributed uniformly over its range and each class is then represented by its mid-point x_i.

Example: The measurements (in cm) of the diameter of the head of 107 screws gave the following frequency distribution table.

Diameter (in cm)	30-35	35-40	40-50	50-60	60-70
Frequency	17	19	23	21	27

Use direct method and short-cut method to calculate the mean head diameter per screw.

Solution:

Diameter	*Mid value* x	*Frequency* f	$d = x - A$	fd	fx
30–35	32.5	17	– 12.5	– 212.5	552.5
35–40	37.5	19	– 7.5	– 142.5	712.5
40–50	45	23	0	0	103.5
50–60	55	21	10	–210	1155
60–70	65	27	20	540	1755
		$\sum f = 107$		$\sum fd = 395$	$\sum fx = 5210$

Let the assumed mean be $A = 45$.

By direct method

$$A.M.(\bar{x}) = \frac{\sum fx}{\sum f}$$

$$= \frac{5210}{107} = 48.6915$$

By short-cut method

$$A.M.(\bar{x}) = A + \frac{\sum fd}{\sum f}$$

$$= 45 + \frac{395}{107}$$

$$= 45 + 3.6915$$

$$= 48.6915 \quad \textbf{Ans.}$$

Example: Calculate the arithmetic mean of the following using short-cut and direct method.

Class	20–30	30–40	40–50	50–60	60–70
Frequency	8	10	12	20	11

Solution:

Diameter	*Mid value x*	*Frequency f*	*d = x – A*	*fd*	*fx*
20–30	25	8	–20	–160	200
30–40	35	10	–10	–100	350
40–50	45	12	0	0	540
50–60	55	20	10	200	1100
60–70	65	11	20	220	715
		$\Sigma f = 61$		$\Sigma fd = 160$	$\Sigma fx = 2905$

Let the assumed mean be $A = 45$

By direct method

$$A.M.(\bar{x}) = \frac{\Sigma fx}{\Sigma f} = \frac{2905}{61} = 47.622$$

By short-cut method

$$\bar{x} = A + \frac{\Sigma fd}{\Sigma f} = 45 + \frac{160}{61}$$

$$= 45 + 2.622 = 47.\ 622 \quad \textbf{Ans.}$$

Example: In a flour mill, the distribution of earning of 200 workers is as follows:

Monthly wages (in Rs.)	80-90	90-100	100-110	110-120	120-130
No. of workers	30	20	40	20	90

Find the average earning of the workers.

Solution:

Wages (in Rs.)	*No. of worker f*	*mid class x*	*d = x – A*	*fx*	*fd*
80–90	30	85	–20	2550	–600
90–100	20	95	–10	1900	–200
100–110	40	105	0	4200	0
110–120	20	115	10	2300	200
120–130	90	125	20	11250	1800
	$\Sigma f = 200$			$\Sigma fx = 22200$	$\Sigma fd = 1200$

Let assumed mean be $A = 105$

By direct method

$$\bar{x} = \frac{\sum fx}{\sum f} = \frac{22200}{200} = 111$$

By short-cut method

$$\bar{x} = A + \frac{\sum fd}{\sum f} = 105 + \frac{1200}{200}$$

$$= 105 + 6 = 111 \quad \textbf{Ans.}$$

Hence, average earning of the workers = 111 Rs. per month.

Example: The following frequency distribution table shows the number of teaching post sanctioned in higher secondary school in 1997.

Teaching posts sanctioned	*No. of schools*
6–11	899
12–17	1149
18–23	1553
24–29	1464
30–35	1307
36–41	1003
42–47	833
48–53	642
54–59	940

Find the average number of teaching posts sanctioned per higher secondary school.

Solution:

Teaching posts	x	f	$d = x - A$	fd	fx
6–11	8.5	899	–24	–21576	7641.5
12–17	14.5	1149	–18	–20682	16660.5
18–23	20.5	1553	–12	–18636	31836.5
24–29	26.5	1464	–6	–8784	38796
30–35	32.5	1307	0	0	42477.5
36–41	38.5	1003	6	6018	38615.5
42–47	44.5	833	12	9996	37068.5
48–53	50.5	642	18	11556	32421
54–59	56.5	440	24	10560	24860
		$\sum fx = 9290$		$\sum fd = -31548$	$\sum fx = 270377$

Let the assumed mean be $A = 32.5$

By direct method

$$\text{A.M. } (\bar{x}) = \frac{\Sigma fx}{\Sigma f}$$

$$= \frac{270377}{9290} = 29.104$$

By short-cut method

$$\text{A.M. } (\bar{x}) = A + \frac{\Sigma fd}{\Sigma f}$$

$$= 32.5 - \frac{31548}{9290}$$

$$= 32.5 - 3.395 = 29.104$$ **Ans.**

Hence, the average number of teaching posts sanctioned per higher secondary school is 29 only.

EXERCISE

1. Following is the distribution of earning of 200 workers in a flour mill.

Monthly wages (in Rs.)	80-100	100-120	120-140	140-160	160-180
No. of workers	20	30	20	40	90

Find the average earning of the workers by short-cut and direct method.

Ans. 145 per workers.

2. Calculate the arithmetic mean by short-cut and direct method of the following data.

x	*frequency*
6–10	899
11–15	1149
16–20	1553
21–25	1464
26–30	1307
31–35	1003
36–40	833
41–45	642
46–50	440

Ans. = 25

Example: Obtain the mean wages from the following data

Wages (in Rs.)	900	950	1000	1050	1100	1150	1200
No. of workers	26	22	18	19	15	3	2

Solution:

x	f	u	fu
900	26	–4	–104
950	22	–3	–66
1000	18	–2	–36
1050	19	–1	–19
1100	15	0	0
1150	3	1	3
1200	2	2	4
	$\sum f = 105$		$\sum fu = -218$

Let the assumed mean be $A = 110$

$$h = 50 \qquad u = \frac{x - A}{h}$$

Thus,
$$x = A + h\frac{\sum fu}{\sum f}$$
$$= 1100 - 50 \times \frac{218}{105}$$
$$= 1100 - 103.8095$$
$$= 996.1905$$

So the mean wages of the workers = 996.1905

Example: Calculate the arithmetic mean of the following table by step deviation method

x:	2000	2100	2200	2300	2400	2500	2600
f:	5	15	32	42	15	12	4

Solution:

x	f	u	fu
2000	5	–3	–15
2100	15	–2	–30
2200	32	–1	–32
2300	42	0	0
2400	15	1	15
2500	12	2	24
2600	4	3	12
	$\sum f = 125$		$\sum fu = -26$

Let the assumed mean be $A = 2300$

$$u = \frac{x - A}{h}, \quad h = 100$$

Thus, by step deviation method

$$\bar{x} = A + h\frac{\sum fu}{\sum f} = 2300 + 100 \times \frac{(-26)}{125}$$

$$= 2300 - \frac{2600}{125} = 2300 - 20.8$$

$$= 2279.20 \quad \textbf{Ans.}$$

EXERCISE

1. Use step deviation method to calculate arithmetic mean from the following data.

Classes	20–25	25–30	30–35	35–40	40–45	45–50	50–55
Frequency	8	10	12	20	11	4	5

Ans. 35.93

2. Find the mean wage from the following data

Wage (in Rs)	800	820	840	860	880	900
No. of workers	7	14	19	25	20	10

Ans. 854.105

3. Calculate the mean for the distribution

x	73	72	71	70	69	68	67	66	65
y	2	4	6	10	11	7	5	4	1

Ans. 69.18

Arithmetic Mean When Open-Ended Classes are Given

In the case of open-ended frequency distribution, the arithmetic mean cannot be calculated without making some assumptions about the open-ended classes.

Method: In that case, first we will have first to change the given cumulative frequency series to an absolute frequency series.

Initial classes	*Cost of living index (In Rs.)*	*No. of weeks f*	*Mid value x*	*u*	*fu*
Below 150	140–150	5	145	–2	–10
150–160	150–160	10	155	–1	–10
160–170	160–170	20	A = 165	0	0
170–180	170–180	9	175	1	9
180–190	180–190	6	185	2	12
190 and above	190–200	2	195	3	6
		$\Sigma f = 52$			$\Sigma fu = 7$

$$h = 10$$

Then
$$\bar{x} = A + h\frac{\Sigma fu}{\Sigma f}$$

$$= 165 + 10 \times \frac{7}{52} = 166.3$$

Example: Following is the distribution of marks obtained by 60 students in physics test

Marks	*No. of students*	*Marks*	*No. of students*
more than 0	60	more than 30	20
more than 10	56	more than 40	10
more than 20	40	more than 50	3

Calculate the arithmetic mean.

Solution: In this example first we change the given cumulative frequency series to an absolute frequency series. All the 60 students have secured more than 0 marks, but 56 students have more than 10 marks i.e. 4 students have secured marks between 0 and 10.

Marks	*No. of students (f)*	*Mid-mark x*	*u*	*fu*
0-10	4	5	–2	–8
10-20	16	15	–1	–16
20-30	20	A = 25	0	0
30-40	10	35	1	10
40-50	7	45	2	14
50-60	3	55	3	9
	$\Sigma f = 60$			$\Sigma fu = 9$

$$h = 10$$

Then
$$\bar{x} = A + \frac{\Sigma fu}{\Sigma f} = 25 + 10 \times \frac{9}{60} = 26.5 \text{ marks.}$$ **Ans.**

Example: Calculate the arithmetic mean from the following data

Mark:	below 10	10–15	15–20	20–25	25–30	30 and above
Frequency	5	8	7	10	6	4

(IU 2004-05)

Solution: Here the lower limit of the first class and upper limit of the last class is not given and class length is 5 (given) so let first and last classes are (5–10) and (30–35).

Initial classes	*New class*	*Mid value x*	*f*	*u*	*fu*
Below 10	5–10	7.5	5	–2	–10
10–15	10–15	12.5	8	–1	–8
15–20	15–20	$A = 17.5$	7	0	0
20–25	20–25	22.5	10	1	10
25–30	25–30	27.5	6	2	12
30 and above	30–35	32.5	4	3	12
			$\Sigma f = 40$		$\Sigma fu = 16$

$$h = 5$$

Then
$$\bar{x} = A + h\frac{\Sigma fu}{\Sigma f} = 17.5 + 5 \times \frac{16}{40}$$
$$= 17.5 + 2 = 19.5 \quad \textbf{Ans.}$$

Example: In a certain commercial organisation, the weekly wages in rupees is given below. The frequency of the class 49–52 is missing. Given that the mean of the frequency distribution is 47.2. Find the missing frequency.

Week's wages in Rs.:	40–43	43–46	46–49	49–52	52–55
No. of workers	31	58	60	?	27

Solution:

Class	*Mid value x*	*f*	*u*	*f u*
40–43	41.5	31	–2	–62
43–46	44.5	60	–1	–58
46–49	$A = 47.5$	60	0	0
49–52	50.5	f_1	1	f_1
52–55	53.5	27	2	54
		$\Sigma f = 176 + f_1$		$\Sigma fu = -66 + f_1$

Here
$$h = 3, \ \bar{x} = 47.2 \text{ (given)}$$
$$\bar{x} = A + h \times \frac{\Sigma fu}{\Sigma f}$$

3. Find the mean of the following table

Age	*Number of persons*
Less than 20	16
Less than 30	21
Less than 40	35
Less than 50	52
Less than 60	58
Less than 70	78
Less than 80	94
Less than 100	100

Ans. 49.5 years.

4. The mean of the following data is 20.5. Find the missing frequency.

x	10	15	20	25	30
y	5	7	–	12	6

Ans. 40.

5. Compute the arithmetic mean from the following distribution

Wages in Rs.	0–20	20–40	40–60	60–80	80–100	100–120
No. of persons	5	15	25	35	12	08

[I.U. 2006-07]

6. Compute the arithmetic mean from the following data

Classes	10–19	20–29	30–39	40–49	50–59	60–69
Frequency	05	12	17	20	18	8

[I.U. 2005-06]

MEDIAN

The median is a positional measures of a data set. If the data set is prescribed by a relative frequency distribution, the median is a point which will lie in the mid of the distribution. When the data are arranged in ascending or descending order of magnitude, then the value of the most observed middle term is called median.

Computing Median

Arrange the n measurements from smallest to the largest.

$$\text{Median} = \text{size of} \left(\frac{n+1}{2}\right)^{th} \text{term, when } n \text{ is odd}$$

and

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{term} + \left(\frac{n}{2}+1\right)^{th} \text{term}}{2}, \text{ when } n \text{ is even}$$

i.e. When n is odd, the median is the middle term.

When n is even; the median is the mean of the two middle numbers.

Example: A student of B. Pharma. secured the following marks in seven subjects:

50, 53, 61, 49, 45, 63, 48.

Find the median score.

Solution: Arranging it in the ascending order, we get

45, 48, 49, 50, 53, 61, 63.

$$\text{Now median score} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

$$= \frac{7+1}{2} = 4\text{th term} = 50 \text{ marks} \quad \textbf{Ans.}$$

Example: Calculate median from the following data

S. No.	1	2	3	4	5	6	7	8	9	10
Marks	17	32	35	33	15	21	41	32	11	18

Solution: Arranging the marks in ascending order, we have

11, 15, 17, 18, 21, 22, 32, 33, 35, 41

So, $$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2}+1\right)^{th} \text{ term}}{2}$$

$$= \frac{5^{th} \text{ term} + 6^{th} \text{ term}}{2}$$

$$= \frac{21+32}{2} = 26.5 \text{ Marks.} \quad \textbf{Ans.}$$

Example: The following table represents the marks obtained by a batch of 20 students in certain class test in mathematics and physics. Indicate in which subject the performance of students is better (on the basis of median):

Maths 53, 54, 52, 32, 30, 60, 46, 47, 35, 28, 25, 42, 33, 48, 72, 51, 45, 33, 65, 29.

Phy. 58, 55, 25, 32, 26, 85, 44, 80, 33, 72, 10, 42, 15, 46, 50, 64, 39, 38, 30, 36.

Solution: Arranging in the ascending order.

Maths 25, 28, 29, 30, 32, 33, 33, 35, 42, 45, 46, 47, 48, 51, 52, 53, 54, 60, 65, 72.

Phy. 10, 15, 25, 26, 30, 32, 33, 36, 38, 39, 42, 44, 46, 50, 55, 58, 64, 72, 80, 85.

$n = 20$, (even)

$$\text{Median} = \frac{\frac{n}{2}^{th} \text{ term} + \left(\frac{n}{2}+1\right)^{th} \text{ term}}{2}$$

Solution:

Marks	*Frequency*	*c.f.*
10	8	8
18	16	24
23	12	36
38	12	48
40	4	52
58	1	53
65	10	63
92	18	81
		$n = 81$

Since $n = 81$ is odd, so

$$\text{Median} = \left(\frac{81+1}{2}\right)^{th} \text{ term} = (41)^{\text{st}} \text{ terms}$$

Since all items from 37 to 48 have the marks as 38, therefore, the median is 38 **Ans.**

Example: Find the median of the following

Size of shoes	*Frequency*
5	10
5.5	16
6	28
6.5	15
7	30
7.5	40
8	34

Solution:

Sizes of shoes	*Frequency*	*c.f.*
5	10	10
5.5	16	26
6	28	54
6.5	15	69
7	30	99
7.5	40	139
8	34	173
		$n = 173$

Since $n = 173$ is odd, so

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ term}$$

$$= \frac{173+1}{2} = 87^{th} \text{ term}$$

87 lies in c.f. 99 so that median size of shoe = 7 **Ans.**

EXERCISE

1. Calculate the median of the following items.

   (a) 42, 45, 47, 50, 48, 44, 46, 61, 55, 64, 70, 68, 75

   Ans. = 50

   (b) 50, 48, 46, 70, 45, 42, 40, 36, 35, 30, 25, 29, 31

   Ans. = 40

   (c) 68, 65, 66, 67, 70, 71, 73, 72, 74, 76, 80, 81, 83, 84, 89, 90

   Ans. = 73.5

2. Find the value of the median:

Wages (in Rs.):	100	200	250	280	300	350	400
No. of persons:	16	24	66	30	20	3	3

   Ans. = 250

3. Find the median of the following:

Variable value:	4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0
Frequency:	2	5	15	30	60	4	23	11	4	1

   Ans. = 6.5

4. Find the median of the following:

Variable value:	5	7	9	11	13	15	17	19
Frequency:	1	2	9	12	15	8	7	5

   Ans. = 13

5. Calculate the median for the following:

No. of students:	6	16	7	4	2	8
Marks:	20	25	50	9	80	40

[I.U. 2006-07]

Example:

45–49, 50–54, 55–59, 60–64.

To make it in actual class limit, we subtract and add $d/2$ to the lower and the upper limits respectively of each class where d denotes the difference of the lower limit of a class and the upper limit of the previous class.

Here, $d = 1$

$$\frac{d}{2} = \frac{1}{2} = 0.5$$

Here, actual class limit is

44.5–46.5, 49.5–54.5, 54.5–59.5, 59.5–64.5

Example: From the following data calculate the median marks

Marks	*Frequency*
10 – 19	7
20 – 29	15
30 – 39	18
40 – 49	25
50 – 59	30
60 – 69	20
70 – 79	16
80 – 89	7
90 – 99	2

Solution: Since class-intervals are given in inclusive form so, we change it into actual class,

here $d = 1$ then $\frac{d}{2} = 0.5$

Therefore, we have

Marks	*f*	*c.f.*
9.5 – 19.5	7	7
19.5 – 29.5	15	22
29.5 – 39.5	18	40
39.5 – 49.5	25	65
49.5 – 59.5	30	95
59.5 – 69.5	20	115
69.5 – 79.5	16	131
79.5 – 89.5	7	138
89.5 – 99.5	2	140
	$n = 140$	

Then $\frac{n}{2} = \frac{140}{2} = 70^{th}$ observation

Therefore, median will lie in the class having c.f. as 95 and so

$$l_1 = 49.5 \qquad l_2 = 59.5$$

$$F = 30, \qquad c = 65 \qquad l_2 - l_1 = 10$$

$$\text{Median} = l_1 + \frac{\frac{n}{2} * c}{f}(l_2 - l_1)$$

$$= 49.5 + \frac{70 - 65}{30} \times 10$$

$$= 51.17 \text{ marks.}$$

Example: The following table shows age distribution of persons in a particular region:

Age (years)	No. of persons
Below 10	2
Below 20	5
Below 30	9
Below 40	12
Below 50	14
Below 60	15
Below 70	15.5
70 and above	15.6

Solution: Here frequencies are in cumulative frequencies so that

Age (years)	*Actual class*	*f*	*c.f*
Below 10	0-10	2	2
Below 20	10-20	3	5
Below 30	20-30	4	9
Below 40	30-40	3	12
Below 50	40-50	2	14
Below 60	50-60	1	15
Below 70	60-70	0.5	15.5
70 and above	70-80	0.1	15.6
	$n = 15.6$		

So $\frac{n}{2} = \frac{15.6}{2} = 7.8$ lies in c.f. 9

so, $l_1 = 20, \quad l_2 = 30, \quad f = 4, \quad c = 5$

Then
$$\text{median} = l_1 + \frac{n/2 - c}{f}(l_2 - l_1)$$

$$\text{median} = 20 + \frac{7.8 - 5}{4} \times 10$$
$$= 27 \text{ years.}$$ **Ans.**

Example: Compute median from the following data

Mid values :	115	125	135	145	155	165	175	185	195
Frequency :	6	25	48	72	166	60	38	22	3

Solution:

Class-interval	*f*	*c.f*
110 – 120	6	6
120 – 130	25	31
130 – 140	48	79
140 – 150	72	151
150 – 160	116	267
160 – 170	60	327
170 – 180	38	365
180 – 190	22	387
190 – 200	3	390
	$n = 390$	

$$\frac{n}{2} = \frac{390}{2} = 195 \text{ lies in 267 so that}$$
$$l_1 = 150, \quad l_2 = 160, \quad c = 151, \quad f = 161.$$

Then
$$\text{Median} = l_1 + \frac{n/2 - c}{f}(l_2 - l_1)$$
$$= 150 + \frac{195 - 151}{116} \times 10$$
$$= 153.79$$ **Ans.**

EXERCISE

1. From the following table find the value of median:

Class:	11–15	16–20	21–25	26–30	31–35	36–40	41–45	46–50
Frequency:	7	10	13	26	35	22	11	6

Ans. 31.79 [I.U. 2004-05]

2. Obtain the median for the following data:

Marks below:	10	20	30	40	50	60	70	80
No. of students:	15	35	60	84	96	127	198	223

Ans. 55

3. Find the median of the following data:

Marks (more than):	0	10	20	30	40	50
No. of students:	50	46	40	20	10	3

Ans. 27.5

4. Compute the missing frequencies f_1 and f_2 in the following distribution given that n is 100 and the median is 32.

Marks	0–10	10–20	20–30	30–40	40–50	50–60
	9	f_1	26	30	f_2	10

Hint:

Here $n = 100$

$$9 + f_1 + 26 + 30 + f_2 + 10 = 100$$

$$f_1 + f_2 = 25$$

Median = 32, thus median class 30 – 40

$$l_2 - l_1 = 10,\ f = 30,\ c = 9 + f_1 + 26 = f_1 + 35$$

Thus, $$\text{Median} = l_1 + \frac{n/2 - c}{f}(l_2 - l_1)$$

$$32 = 10 + \frac{50 - (35 + f_1)}{30} \times 10$$

$$f_1 = 9$$

Hence, $$f_2 = 16$$

$$f_1 = 9,\quad f_2 = 16 \quad \textbf{Ans.}$$

PARTITION VALUES OF QUARTILES

It has been already discussed that a median is the value of the middle most item of a series of values arranged on ascending or descending order of magnitude. Thus the median divides the series into two equal parts. Quantities denoted by Q_1, Q_2, and Q_3 and are called first quartile (Q_1), second quartile (Q_2) or median and third quartile, (Q_3) respectively.

How to Calculate Quartiles

(a) When simple frequency distribution is given if there are n observations is $x_1, x_2, x_3 \ldots x_n$ given, arrange it in ascending order

Example: The height (in cm). of 60 students of a certain school are given in the following frequency distribution table.

Heights (cm):	151	152	153	154	155	156	157
No. of Students:	6	4	11	9	16	12	2

Find (i) Median (ii) Lower quartile (iii) Upper quartile.

Solution:

Height	*f*	*c.f*
151	6	6
152	4	10
153	11	21
154	9	30
155	16	46
156	12	58
157	2	60
	$n = 60$	

$n = 60$ which is even. So that

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} \text{observation} + \left(\frac{n}{2}+1\right)^{th} \text{observation}}{2} = \frac{154+155}{2} = 154.5$$

(ii) $Q_1 = \frac{n}{4}th$ observation $= 15th$ observation $= 153$.

(iii) $Q_3 = \frac{3n}{4}th$ observation $= 45th$ observation $= 155$.

EXERCISE

1. Find the lower and upper quartiles i.e. Q_1 and Q_3 for the following data.

Score	Frequency
10	5
15	10
20	25
25	30
30	20
35	15
40	2

Ans. $Q_1 = 20$, $Q_3 = 30$

Calculation of Mode

(1) **For an individual series, raw data**

Example: Find the mode of the following:

4, 5, 3, 7, 8, 9, 10, 11, 12, 5, 13, 5, 14, 15, 16, 5, 17, 5, 19

Solution: Here frequency of 5 is maximum i.e. 5, therefore mode is 5.

(2) **For a discrete series:** For discrete series also, mode can be found by observing the highest frequency.

Example: Find the mode of the following data

x :	7	8	9	10	12
f :	42	2	20	22	17

Solution: Mode is the value of x which has maximum frequency, here maximum frequency is 42. Therefore, 7 is the mode

Mode = 7 **Ans.**

Example: Find mode of the following data

Size of item	*Frequency*
1	5
2	6
3	10
4	2
5	3
6	4
7	6
8	7
9	12
10	20

Solution: Here maximum frequency is 20. So that mode is 10

mode = 10 **Ans.**

(3) **Continuous series**

$$\text{Mode} = L_1 + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times C$$

where L_1 = Lower limit of the modal class

f_0 = Frequency of the modal class

f_1 = Frequency of the class before the modal class

f_2 = Frequency of the class after the modal class

C = Interval of the modal class

Note: If we find mode just by inspection, an error is possible, in such cases, we prepare a grouping table and an analysis table to find the mode, these tables help us in determining the correct value of mode.

Method: In this method two tables are framed. These tables are known as 'Grouping table' and 'Analysis Table'. The grouping table consist of six columns which are constructed by using the following steps.

Step 1: Obtain the discrete frequency distribution.

Step 2: Take the column of frequencies as column I and encircle the maximum frequency in it.

Step 3: In column II, the sum of the frequencies taken two at a time, mark the maximum frequency in it.

Step 4: In column III, leave the first frequency containing the sum of the frequencies taken two at a time, again mark the maximum frequency in it.

Step 5: In column IV, containing the sum of three frequencies at a time, mark the maximum frequency.

Step 6: In column V, leave the first frequency, the sum of the three frequencies at a time.

Step 7: In column VI, leave first two frequencies, the sum of the three frequencies at a time. After preparing the grouping table, we prepare an analysis table by using the following steps.

Step I

Prepare a table in which in the topmost row write all values of the variable and in the left most column, write column numbers from I to VI.

Step II

See the maximum frequency in the first column of the grouping table and obtain the corresponding value of the variable. Now, marks in the first row of the analysis table against the value of the variable having the maximum frequency, continue the same procedure for the remaining five columns.

Step III

Find the maximum number of frequency.

Example: Compute the mode from the following table

Size of Item	*Frequency*
0–5	20
5–10	24
10–15	32
15–20	28

Contd.

20–25	20
25–30	16
30–35	34
35–40	10
40–45	8

Grouping Table

Size of Item	*I^st^ Column*	*II^nd^*	*III^rd^*	*IV^th^*	*V^th^*	*VI^th^*
0–5	20	44		76		
5–10	24		56		84	
10–15	32	60				80
15–20	28		48	64		
20–25	20	36			70	
25–30	16		50			60
30–35	34	44		52		
35–40	10		18			
40–45	8					

Analysis Table

Column	*0–5*	*5–10*	*10–15*	*15–20*	*20–25*	*25–30*	*30–35*	*35–40*	*40–45*
I							1		
II			1	1					
III		1	1						
IV	1	1	1						
V		1	1	1					
VI			1	1	1				
	1	3	5	3	1	1			

From analysis table we see that maximum frequency is 5 and so 10-15 is modal class.

Hence, mode will be calculated by

$$f_0 = 32, \quad L_1 = 10, \quad f_1 = 24, \quad f_2 = 28$$

$$C = 5$$

$$\text{mode} = L_1 + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times C$$

$$= 10 + \frac{32 - 24}{64 - 24 - 28} \times 5$$

$$= 10 + \frac{8}{12} \times 5 = 13.33 \quad \textbf{Ans.}$$

Example: The shirt size worn by a group of 200 persons, who bought the shirt from a store are as follows:

Shirt size:	37	38	39	40	41	42	43	44
No. of person:	15	25	34	41	36	17	15	12

Calculate model shirt size of the group.

Solution: **Grouping Table**

	Frequency					
Size of Item	*Ist*	*IInd*	*IIIrd*	*IVth*	*Vth*	*VIth*
37	15					
38	25	30		79		
39	39	80	64			
40	41				105	116
41	36	53	77	94	68	
42	17		32			44
43	15	27				
44	12					

Analysis Table

Column	*37*	*38*	*39*	*40*	*41*	*42*	*43*	*44*
I				1				
II			I	I				
III				I	I			
IV				I	I	I		
V		I	I	I				
VI			I	I	I			
		1	3	6	3	1		

Maximum frequency is 6 hence

Mode of the group is 40 **Ans.**

Example: Calculate the mode of the following:

x:	0–10	10–20	20–30	30–40	40 –50	50–60	60–70	70–80	80–90	90–100
f:	4	6	5	10	20	22	24	6	2	1

Solution: Making grouping table and analysis table we have

Analysis Table

Column	16.5–20.5	20.5–24.5	24.5–28.5	28.5–32.5	32.5–36.5	36.5–40.5
I				1		
II			1	1		
III		1	1			
IV			1	1	1	
V	1	1	1			
VI		1	1	1		
Total	**1**	**3**	**5**	**4**	**1**	

Maximum frequency = 5

So that

$$l_1 = 24.5, \quad f_0 = 14, \quad f_1 = 14, \quad f_2 = 15, \quad C = 40$$

$$\text{Mode} = L_1 + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times C$$

$$= 24.5$$ **Ans.**

EXERCISE

1. Calculate the modal value from the following frequency distribution.

Marks:	0-9	10-19	20-29	30-39	40-49	50-59	60-69	70-79	80-89	90-99
No. of Students:	6	29	87	181	247	263	133	43	9	2

Ans. 47.55

2. Find the mode for the following:

Class:	0–10	10–20	20–30	30–40	40–50	50–60	60–70
Frequency:	5	10	20	22	24	6	2

Ans. = 35

3.

Class:	10–15	15–20	20–25	25–30	30–35	35–40
Frequency:	30	45	75	35	25	15

Ans. = 22.14

4. Determine mode from the following data:

Mid value:	30	40	50	60	70	80	90
Frequency:	7	12	17	29	31	5	3

Ans. = 63.57