CHAPTER 2

Diffraction

"A Deviation from a Straight line path..."

2.1 INTRODUCTION

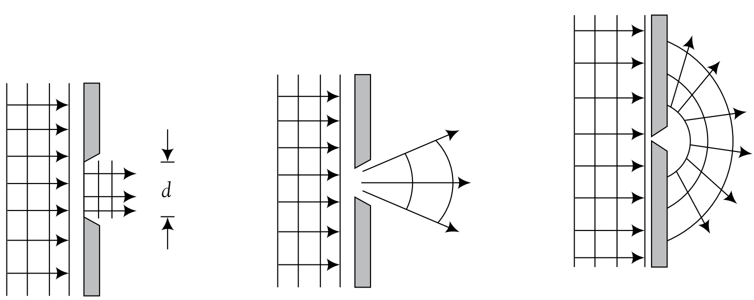
To all outward appearance, light seems to travel in straight line. A very careful observation, however, reveals that light does suffer some deviation from its straight path when it passes close to edges of opaque obstacles or narrow slits. It is found that some light bends into the region of geometric shadow also. This bending of light around corners or the departure of light path from true rectilinear path is called diffraction.

When light passes through an opening and if,

(a) Opening is large compared to the wavelength of light, the waves do not bend around the edges (as shown in Figure 2.1a).

(b) Opening is of the order of the wavelength of light, the bending of light wave round the edges is noticeable (as shown in Figure 2.1b).

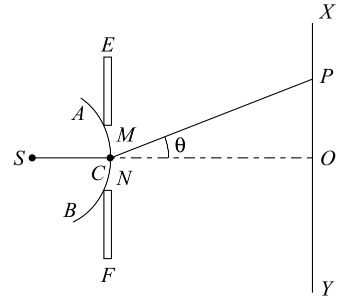
(c) Opening is very small compared to the wavelength of light, the bending takes place to such an extent that the waves spread over the entire surface behind the opening suggesting that the opening acts as a point source (as shown in Figure 2.1c).



(a) $d >> \lambda$ (b) $d = \lambda$ (c) $d < \lambda$

Fig. 2.1 "Diffraction"

Consider a point source S of monochromatic light, MN being small aperture and XY being the screen parallel to the plane of the aperture. From the source S, a spherical wave front ACB is incident on the aperture (Figure 2.4). The resultant effect at a point (P) on the screen will be due to the superposition of the secondary wavelets from the unblocked portion of the wavefront and can be calculated using the following assumptions:



"Fresnel diffraction"

Fig. 2.4

(1) A wavefront can be divided into a large number of zones of small area. The resultant effect at any point (P) will depend on the combined effect of all the secondary waves originating from the various zones of the wavefront.

(2) The effect at a point (P) due to any particular zone will depend on the distance of that point from the zone.

(3) The effect at a point P will also depend upon the obliquity of the point with respect to the zone. Considering an elementary wavefront at C and that we have to find its effect on a point P of the screen XY. Then the obliquity factor is proportional to $(1 + \cos\theta)$, where θ is the angle, which PC makes with OC (Figure 2.4). Thus, the effect is maximum at O, as for O, $\theta = 0$ and $\cos\theta = 1$. As, we move from O towards P, the value of θ increases and hence resultant effect decreases. Similarly, in a direction EF tangential to the wavefront, the resultant effect is one half of that at O, as $\theta = 90°$ and $\cos\theta = 0$. In the direction CS, the resultant effect is zero, as $\theta = 180°$ and $\cos\theta = -1$. This explains the non-existence of the wave in backward direction.

Half Period Zones

Consider a plane wavefront $ABCD$ of monochromatic light of wavelength λ travelling from left to right (fig. 2.5). According to Huygens principle, every point on the wavefront can be regarded as the origin of secondary wavelets. The resultant effect at a point O due to the whole wavefront will be equal to the superposition of the disturbances reaching at O from different points of the wavefront. To find the resultant effect, we divide the whole wavefront into concentric zones as follows. From O drop a perpendicular on $ABCD$, where the foot of the perpendicular 'P' is called the pole of the wave with respect to O (as shown in Figure 2.5). Let OP, i.e. the perpendicular distance of the point O from the incident wavefront, be equal to 'a'. With O as centre draw spheres of radii $(a + \lambda/2)$, $(a + 2\lambda/2)$, $(a + 3\lambda/2)$.... that cut the wavefront in circles of radii r_1, r_2, r_3 ... (as shown in Figure 2.5) with the radius of the n^{th} circle given by,

$$\frac{2\pi}{\lambda}(OM_{n+1} - OM_n) = \pi.$$

As the area of the annular regions is approximately same, for every point Q_n in the annular region between the $(n-1)^{\text{th}}$ and the n^{th} circle, there is a point Q_{n+1} in the annular region between the n^{th} and the $(n+1)^{\text{th}}$ circle, such that $OQ_{n+1} - OQ_n = \lambda/2$. The annular region between the $(n-1)^{\text{th}}$ and the n^{th} circles is called n^{th} half period zone. These are called half period zones because the waves reaching *O* from two consecutive zones differ in path by $\lambda/2$ and in phase by π.

It may be noted that the first half period zone is the region enclosed by the 1st circle, which will be a circle, while all other half period zones are annular rings.

Amplitude at a Point Due to an Unobstructed Wavefront

To find the resultant effect of the whole wavefront at a point *O*, divide the whole wavefront into half period zones. The problem is then reduced to find the resultant of a large number of disturbances originating from the various zones (annular rings) into which the whole wavefront is divided. Let d_1, d_2, d_3 ..., etc. represent the amplitudes at *O* due to secondary wavelets from 1st, 2nd, 3rd ..., etc. half period zones. The amplitude due to the secondary wavelets produced by each zone reaching the point *O* depends upon the following factors;

(i) It is directly proportional to the area of the zone.

(ii) It varies inversely as the distance of zone from the point *O*.

(iii) It varies with the obliquity factor. The amplitude decreases with increasing obliquity.

Now, we know that the areas of the various half period zones are independent of the order of the zones and are nearly equal. As the obliquity increases outward from *P*, we can say that, d_1 is slightly greater than d_2, d_2 is slightly greater than d_3, and so on. Further, due to phase difference of π between the secondary wavelets originating from two consecutive zones, if the amplitudes due to odd number zones are positive, then due to even number of zones the amplitudes will be negative. Thus, the resultant amplitude at *O* at any instant is given by,

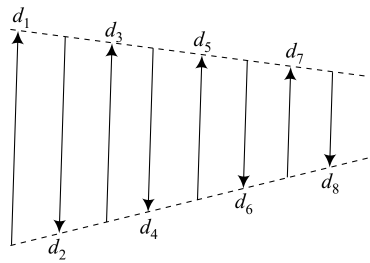


Fig. 2.6 Amplitudes at a point due to half period zones

$$A = d_1 - d_2 + d_3 - d_4 + d_5 - d_6 + ... \tag{2.3}$$

Further as the amplitudes are of gradually decreasing magnitude (Figure 2.6), the amplitude at *O* due to any zone can be approximately taken as the mean of the amplitudes due to the zones preceding and succeeding it, i.e.

$$d_2 = \frac{d_1 + d_3}{2},\ d_4 = \frac{d_3 + d_5}{2} \text{ and so on.} \tag{2.4}$$

unobstructed wavefront. So, at O_2 complete illumination is observed. While for points O_3 and O_4, the corresponding poles are P_3 and P_4. For these points, the Fresnel's theory cannot be used as all the zones are partially exposed to O_3 and O_4.

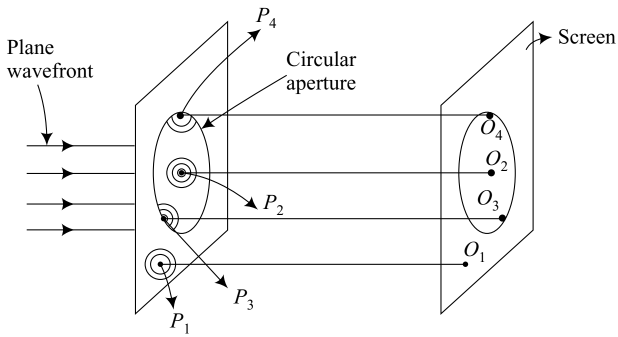


Fig. 2.7 Fresnel's description for rectilinear propagation of light

Thus, the Fresnel's theory leads to darkness in the region of geometrical shadow while complete illumination in the area of the screen that is against the aperture which confirms the rectilinear propagation of light (leaving the area near the edge of the aperture, where the Fresnel's theory fails).

2.5 ZONE PLATE

It is a special diffracting screen designed to obstruct the light from the alternate half-period zones. It is constructed by drawing a series of concentric circles on a sheet of white paper with radii proportional to the square root of the natural numbers. The alternate zones are painted black. A highly reduced photograph of this drawing is then taken on a plane glass plate. The negative so obtained is a zone plate (Figure 2.8).

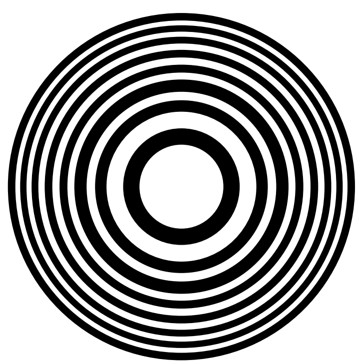


Fig. 2.8 Zone plate

The zone plate behaves as a convex lens. We know that the radius of the *n*th half period zone is given by,

$$r_n = \sqrt{na\lambda} \tag{2.10}$$

So the focal length of the zone plate is given by,

$$f_n = a = \frac{r_n^2}{n\lambda} \tag{2.11}$$

penetrates a little and then there is complete darkness. This pattern on the screen due to diffraction can be explained using Fresnel's theory.

(i) Uniform illumination at points far above *C*: Let the point be *P*, far above the point *C*, where we want to calculate the intensity. As per Fresnel's theory, for calculating the intensity at *P*, drop perpendicular from *P* on *WW′*. Therefore, *O* will be the pole corresponding to *P* (Figure 2.10a). Divide the wavefront into half period zones. The point *P* is completely exposed to the upper half portion *OW* of the wavefront *WW′* and the contribution of the lower half portion *OW′* depends on the number of half period strips (or zones) enclosed between the point *O* and *A*. Since *P* is far above *C*, *OA* will contain almost all the effective strips (or zones) in the lower half portion of the wavefront (Figure 2.10b). If R_1, R_2, R_3 ... represent amplitudes at *P* due to 1st, 2nd, 3rd ... half period strips (or zones of upper or lower portions of the wavefront) respectively, then the total amplitude at *P* will be,

A_P = amplitude due to upper half portion of the wavefront *AP* + amplitude due to lower half portion of the wavefront

or $$A_P = (R_1 - R_2 + R_3 + ...) + (R_1 - R_2 + R_3 + ...)$$

i.e. $$A_P = \frac{R_1}{2} + \frac{R_1}{2} = R_1 \tag{2.13}$$

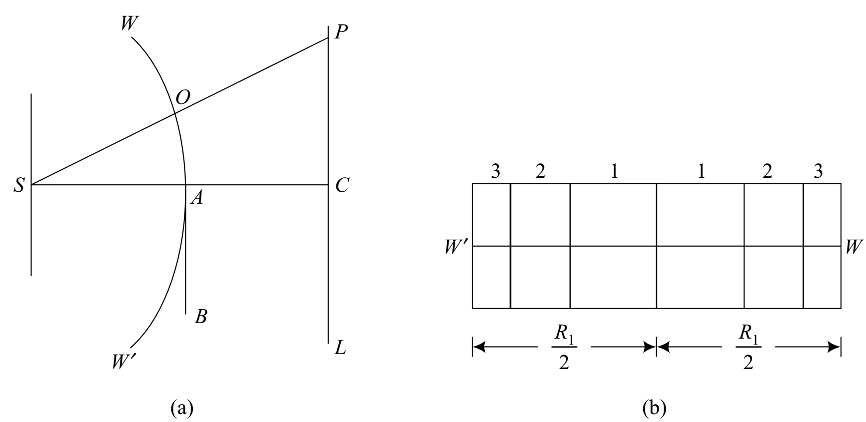


Fig. 2.10

Thus, intensity at a point '*P*' far above '*C*',

$$I \propto R_1^2 \text{ (uniform illumination)} \tag{2.14}$$

(1) Just above *C*: If the position of a point P_1 is such that the corresponding pole on the wavefront is O_1 and the portion $O_1 A$ contains one half period zone or strip (Figure 2.11a), i.e.

$$AP_1 - O_1P_1 = \lambda/2 \tag{2.15}$$

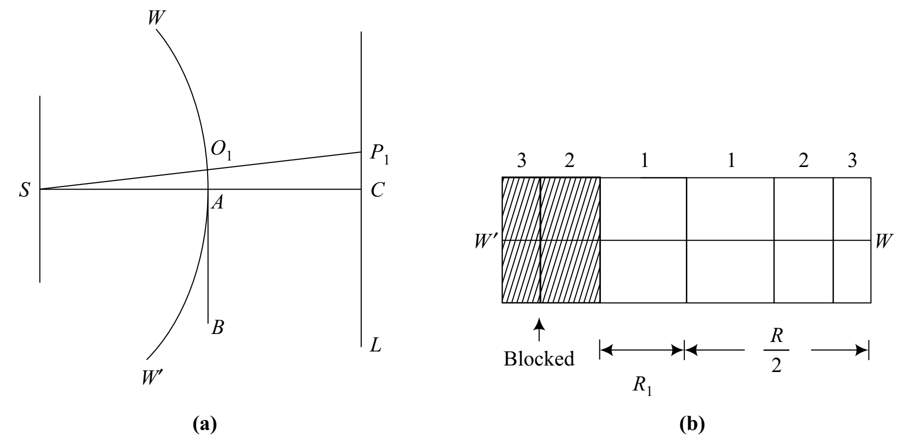


Fig. 2.11

In this case, the upper half of the wavefront (O_1W) reaches to the point P_1 completely while from the lower half of the wavefront (O_1W') only one half period strip is exposed to the point P_1 and others are blocked (Figure 2.11b), hence the total amplitude at P_1 is given by,

A_{P1} = amplitude due to upper half portion of the wavefront + amplitude due to the first half period zone of the lower half portion of the wavefront

i.e., $$A_{P1} = [R_1 - R_2 + R_3 \ldots] + R_1$$

or, $$A_{P1} = \frac{R_1}{2} + R_1 = \frac{3R_1}{2} \tag{2.16}$$

Thus, the intensity at P_1 is given by,

$$I_{P1} \propto \frac{9}{4}R_1^2 \quad \text{(i.e. 2.25 times of the intensity at } P) \tag{2.17}$$

(2) Above *C*: Consider a point P_2 such that the corresponding pole on the wavefront is O_2 and O_2A contains two half period zones or strips (Figure 2.12), i.e.

$$P_2A - P_2O_2 = 2\lambda/2$$

Then, the amplitude at P_2 is given by,

A_{P2} = amplitude due to upper half portion of the wavefront (O_2W) + amplitude due to the first and second half period zones of the lower half portion of the wavefront (O_2W'), i.e.

$$A_{P2} = [R_1 - R_2 + R_3 \ldots] + R_1 - R_2$$

$$\approx \frac{R_1}{2} + 0 = \frac{R_1}{2} \quad \{\text{as } R_1 \approx R_2\} \tag{2.18}$$

$$A_{P3} = [R_1 - R_2 + R_3 \ldots] + R_1 - R_2 + R_3$$

or, $$A_{P3} = \frac{R_1}{2} + 0 + R_3 \qquad \{\text{as } R_1 \approx R_2\}$$

or, $$A_{P3} = \frac{R_1}{2} + R_3 < \frac{3R_1}{2} \qquad \{\text{as } R_3 < R_1\} \tag{2.20}$$

Thus, the intensity at the point P_3, $I_{P3} \propto A_{P3}^2$, which is slightly less than the intensity at the point P_1.

(4) Intensity at *C*: For the point *C* that lies exactly against the edge *A* on the screen, the corresponding pole on the wavefront *WW′* is *A* and the complete upper half portion *AW* of the wavefront is exposed to the point *C*, while lower half portion *AW′* of the wavefront is blocked (Figure 2.14), so the amplitude at *C* is given by,

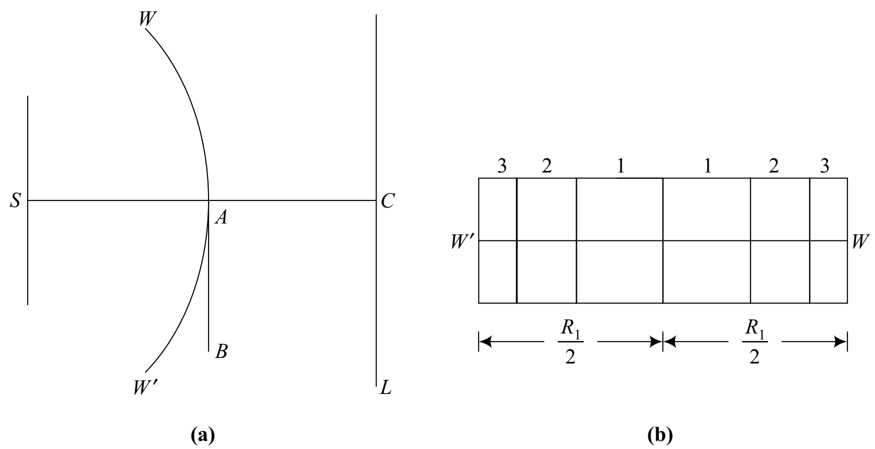


Fig. 2.14

$$A_C = \text{amplitude due to upper half of the wavefront only,}$$

i.e., $$A_C = [R_1 - R_2 + R_3 \ldots]$$

or, $$A_C = \frac{R_1}{2} \tag{2.21}$$

Thus, the intensity at the point *C* is given by,

$$I_C \propto \frac{R_1^2}{4} \text{ (1/4}^{\text{th}} \text{ of the intensity at } P) \tag{2.22}$$

(5) At a point P_1' below *C*: Consider a point P_1' such that the corresponding pole on the wavefront *WW′* is O_1' and $O_1'A$ contains all the half period strips or zones of the upper half portion $O_1'W$ of the wavefront except the first half period zone while the lower half wavefront is completely blocked (Figure 2.15), i.e.,

$$AP_1' - O_1'P_1' = \lambda/2 \tag{2.23}$$

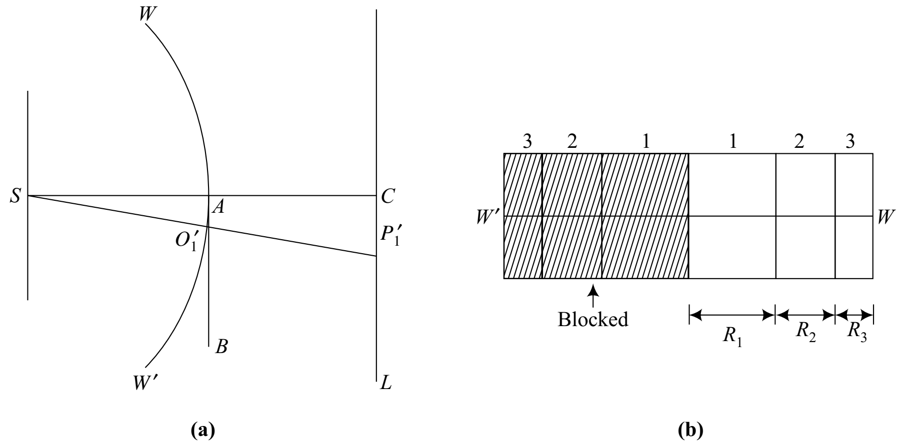


Fig. 2.15

The amplitude at the point P_1' is given by,

$A_{P_1'}$ = amplitude due to upper half portion of the wavefront –amplitude due to the first half period zone of the upper half portion of the wavefront, i.e.

$$A_{P_1'} = [R_1 - R_2 + R_3 - R_4 \ldots] - R_1$$

or, $$A_{P_1'} = [-R_2 + R_3 - R_4 \ldots]$$

or, $$A_{P_1'} = -\frac{R_2}{2} \tag{2.24}$$

Thus, the intensity at the point P_1'

$$I_{P_1'} \propto \frac{R_2^2}{4} \tag{2.25}$$

which is slightly less than the intensity at C (as R_2 is slightly less than R_1).

(6) At a point P_2' below C: Consider a point P_2' such that the corresponding pole on the wavefront is O_2' and $O_2'A$ contains all the strips or zones of the upper half portion $O_2'W$ of the wavefront except the first and second half period zones while the lower half wavefront is completely blocked (Figure 2.16), i.e.,

$$AP_2' - O_2'P_2' = 2\lambda/2 \tag{2.26}$$

The amplitude at P_2' is given by,

$A_{P_2'}$ = amplitude due to upper half portion of the wavefront – amplitude due to the first half period zone of the upper half portion of the wavefront-amplitude due to second half period zone of the upper half wavefront, i.e.

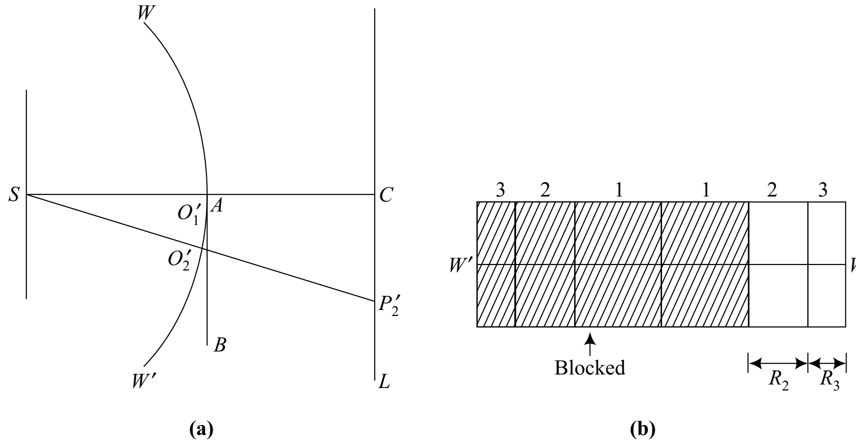


Fig. 2.16

$$A_{P_2'} = [R_3 - R_4 + R_5 \ldots]$$

or,
$$A_{P_2'} = \frac{R_3}{2}$$

Thus, the intensity at P_2'

$$I_{P_2'} \propto \frac{R_3^2}{4} \tag{2.28}$$

which is less than the intensity at the point P_1' (as R_3 is less than R_2). Thus, we find that the intensity below C goes on decreasing and then there is complete darkness (Figure 2.17).

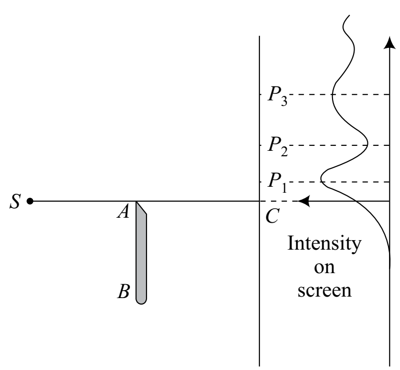


Fig. 2.17 Intensity variation on screen

Position of Maximum & Minimum Intensity on the Screen

From the previous discussion, we can see that the resultant intensity is maximum at a point Q of the screen (Fig. 2.9), if $AQ - OQ = (2n - 1)\lambda/2$ with $n = 1, 2, 3$ ... where O is the pole corresponding to the point Q and resultant intensity is minimum at a point Q, if $AQ - OQ = n\lambda$ with n = 1, `2, 3, ...

Thus, maxima are obtained in the diffraction pattern at the points, which satisfy the condition,

$$AQ - OQ = (2n - 1)\lambda/2 \qquad n = 1, 2, 3, ... \tag{2.29}$$

And minima are obtained in the diffraction pattern at the points which satisfy the condition,

$$AQ - OQ = n\lambda \qquad n = 1, 2, 3, ... \tag{2.30}$$

Consider a point Q at a distance 'x' from the point 'C' of the screen, i.e. $CQ = x$. Then from Figure 2.9 we can write,

$$AQ = (b^2 + x^2)^{1/2} \text{ and } OQ = SQ - SO = \sqrt{(a+b)^2 + x^2} - a \tag{2.31}$$

Thus, for the point Q,

$$AQ - OQ = (b^2 + x^2)^{1/2} - (SQ - SO)$$

$$= (b^2 + x^2)^{1/2} - \left(\sqrt{(a+b)^2 + x^2} - a\right) \tag{2.32}$$

For the points on the screen near C, i.e. for which $x << b$, $(b^2 + x^2)^{1/2} \sim b\{1 + x^2/(2b^2)\}$ and $\sqrt{(a+b)^2 + x^2} \sim (a + b)\ [1 + x^2/\{2(a + b)^2\}]$, so for the point Q,

$$AQ - OQ = b\left[1 + \frac{x^2}{2b^2}\right] - (a+b)\left[1 + \frac{x^2}{2(a+b)^2}\right] + a$$

or, $$AQ - OQ = \frac{x^2}{2}\left(\frac{1}{b} - \frac{1}{a+b}\right) = \frac{x^2}{2}\left(\frac{a+b-b}{b(a+b)}\right) = \frac{x^2}{2}\frac{a}{b(a+b)} \tag{2.33}$$

Therefore, from the condition of maxima (equation 2.29), maxima will be obtained at a point Q if,

$$\frac{x^2}{2}\frac{a}{b(a+b)} = (2n-1)\lambda/2, \qquad n = 1, 2, 3...... \tag{2.34}$$

or, when x is given by,

$$x = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}}, \qquad n = 1, 2, 3...... \tag{2.35}$$

minima at a screen which is at a distance of 4 m from the edge. Given that the wavelength of the light used is 5896 Å.

Solution The positions of n^{th} maxima and n^{th} minima are respectively, given by,

$$(x_n)_{max} = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}} \quad \text{and} \quad (x_n)_{min} = \sqrt{\frac{2n(a+b)b\lambda}{a}}$$

Given that, $\lambda = 5896$ Å $= 5896 \times 10^{-8}$cm, $a = 6$ cm and $b = 4$ m $= 400$ cm.

The position of first maxima on the screen is obtained for $n = 1$ and is given by,

$$(x_1)_{max} = \sqrt{\frac{(2n-1)(a+b)b\lambda}{a}} = \sqrt{\frac{406 \times 400 \times 5896 \times 10^{-8}}{6}} = 1.26 \text{ cm}$$

The position of the first minima at the screen is obtained for $n = 1$ and is given by,

$$(x_1)_{max} = \sqrt{\frac{2n(a+b)b\lambda}{a}} = \sqrt{\frac{2 \times 406 \times 400 \times 5896 \times 10^{-8}}{6}} = 1.786 \text{ cm}$$

2.7 FRAUNHOFER DIFFRACTION DUE TO A SINGLE SLIT

Consider Fraunhofer diffraction of light of angular frequency 'ω' (and wavelength 'λ') due to a slit of width 'b'. A slit can be considered as composed of almost infinite equally spaced points $A_1, A_2, A_3, \ldots A_n$, where $n \to \infty$, the spacing between two points 'Δ' $\to 0$ such that $(n-1)\Delta \to b$. According to Huygen's wave theory, every point on the slit will act as a source of secondary wavelets. Therefore, a slit can be considered as consisting of almost infinite equally spaced point sources, i.e. $A_1, A_2, A_3, \ldots A_n$ of secondary wavelets separated by a distance 'Δ' from each other, where $n \to \infty$, $\Delta \to 0$ such that $(n-1)\Delta \to b$ or $n\Delta \to b$ (Figure 2.18). The secondary wavelets from these point sources superpose with each other to give intensity variation at the screen.

Let the secondary wavelets from the point sources $A_1, A_2, A_3, \ldots A_n$, emitted at an angle θ with the normal to the slit, reach the point P' of the screen and produce the displacements $y_1, y_2, y_3 \ldots y_n$ respectively at P'. According to the principle of superposition, after the superposition of the secondary wavelets, the resultant displacement 'Y' at the point 'P' is given by,

$$Y = y_1 + y_2 + y_3 + \ldots y_n \tag{2.39}$$

The displacement 'y_1' at the point 'P' due to the wavelet from the point source A_1, can be written as,

$$y_1 = a \sin \omega t \tag{2.40}$$

where a is the amplitude at the point 'P' due to the secondary wavelet from A_1.

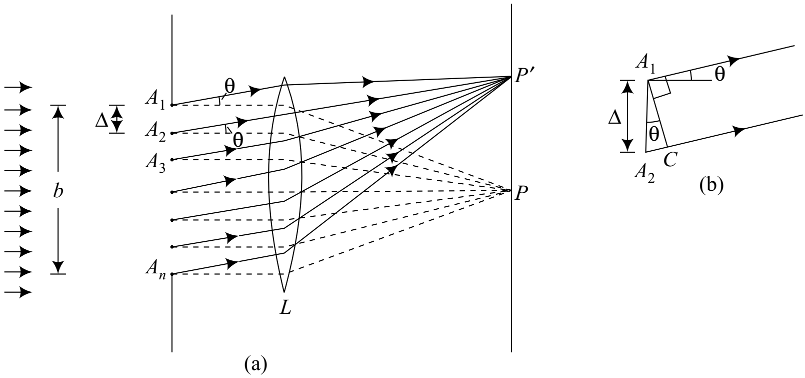


Fig. 2.18 Fraunhaufer diffraction due to a single slit

Now, from Figure 2.18, we can see that in reaching the point P', the secondary wavelet from the point source A_2 traverse an additional distance as compared to the secondary wavelet from A_1. The path difference at the point P' between the secondary wavelets from the sources A_1 and A_2 is given by, $A_2C = \Delta \sin \theta$ (from Figure 2.18b) and the corresponding phase difference 'ϕ' between the two wavelets is given by, $\phi = \frac{2\pi}{\lambda}(\Delta \sin \theta)$. So, the displacement at the point P' due to the secondary wavelet from A_2, can be written as,

$$y_2 = a \sin(\omega t + \phi) \tag{2.41}$$

In the similar way, we can find that,

$$y_3 = a \sin(\omega t + 2\phi)$$

$$y_4 = a \sin(\omega t + 3\phi)$$

....

....

$$y_n = a \sin(\omega t + (n-1)\phi) \tag{2.42}$$

Therefore, due to the superposition of secondary wavelets, the resultant displacement 'Y' at the point P', is given by,

$$Y = \underset{\substack{n\to\infty \\ \Delta\to 0,\, n\Delta\to b}}{\text{Lt}} a\,[\sin \omega t + \sin(\omega t + \phi) + \sin(\omega t + 2\phi) + \ldots + \sin(\omega t + (n-1)\phi)] \tag{2.43}$$

where,

$$\sin \omega t + \sin(\omega t + \phi) + \ldots + \sin(\omega t + (n-1)\phi) = \frac{\sin n\phi/2}{\sin \phi/2} \sin(\omega t + (n-1)\phi/2] \tag{2.44}$$

So, the resultant displacement at *P*, is given by,

$$Y = \lim_{\substack{n\to\infty \\ \Delta\to 0,\, n\Delta\to b}} a \frac{\sin\frac{n\phi}{2}}{\sin\frac{\phi}{2}} \sin(\omega t + (n-1)\phi/2) \tag{2.45}$$

As $n \to \infty$, $\Delta \to 0$, $n\Delta \to b$, we have $\frac{n\phi}{2} = \frac{n}{2}\left[\frac{2\pi}{\lambda}\Delta\sin\theta\right] = \frac{\pi}{\lambda}n\Delta\sin\theta = \frac{\pi}{\lambda}b\sin\theta$ and $\frac{\phi}{2} = \frac{1}{n}\left(\frac{\pi}{\lambda}b\sin\theta\right)$, which will be very small as $n \to \infty$, so we can take $\sin\phi/2 \sim \phi/2 = \frac{1}{n}\left(\frac{\pi}{\lambda}b\sin\theta\right)$. Further as $n \to \infty$ $(n-1)\phi/2 \sim n\phi/2 = \frac{\pi}{\lambda}b\sin\theta$.

Thus, we can write,

$$Y = na\frac{\sin\left(\frac{\pi}{\lambda}b\sin\theta\right)}{\left(\frac{\pi}{\lambda}b\sin\theta\right)} \sin\left(\omega t + \frac{\pi}{\lambda}b\sin\theta\right) \tag{2.46}$$

or,

$$Y = A\frac{\sin\beta}{\beta}\sin(\omega t + \beta) \tag{2.47}$$

where, $\beta = \frac{\pi}{\lambda}b\sin\theta$ and $A = na$.

Therefore, the resultant amplitude '*R*' at the point *P*′ is given by,

$$R = A\frac{\sin\beta}{\beta} \tag{2.48}$$

The resultant intensity '*I*' at a point *P*′ of the screen is proportional to the square of the resultant amplitude at that point, so we can write,

$$I \propto R^2$$

or,

$$I \propto A^2\frac{\sin^2\beta}{\beta^2}$$

or,

$$I = kA^2\frac{\sin^2\beta}{\beta^2} \quad \{k \text{ is a costant}\}$$

or,

$$I = I_0\frac{\sin^2\beta}{\beta^2} \tag{2.49}$$

or, $$\frac{d}{d\beta}\left(\frac{A \sin \beta}{\beta}\right) = 0$$

or, $$\frac{\beta A \cos \beta - A \sin \beta}{\beta^2} = 0$$

or, $$\frac{A[\beta \cos \beta - \sin \beta]}{\beta^2} = 0$$

or, $$\beta \cos \beta = \sin \beta$$

or, $$\beta = \tan \beta \qquad (2.53)$$

This equation can be solved graphically by plotting the curves, $Y = \beta$ and $Y = \tan \beta$. The first of these is a straight line through origin making an angle 45°, while the second is a discontinuous curve having a number of branches. The points of intersection of the two curves give the value of β, satisfying the equation (2.53) (as shown in Figure 2.20).

It can be seen that the two curves intersect each other at $\beta = 0$, $\beta = \pm 1.43\pi$, $\beta = \pm 2.46\pi$, ... etc. The maxima obtained at $\beta = 0$ is also called central maximum.

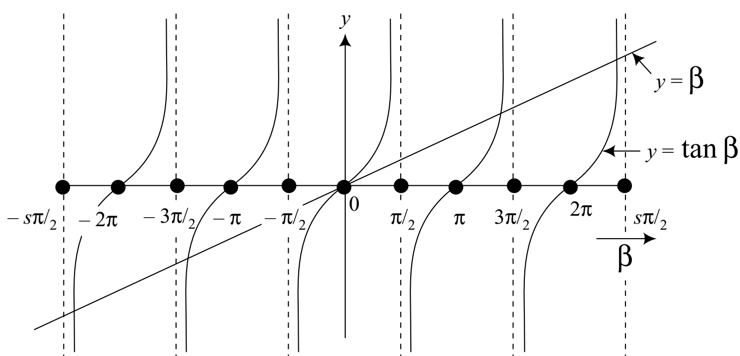


Fig. 2.20 Graphical method for finding the positions of maxima in single slit diffraction pattern

Intensity of Maxima

The intensity of all the maxima can be found using equation (2.49). At the central maximum, i.e. for $\beta = 0$, intensity is given by, $I = I_0$ {as $\sin \beta/\beta = 1$ at $\beta = 0$}. To find the intensity of the other maxima (referred as first, second third ... maxima), we can take $\beta = \pm 1.43\pi \approx \pm 3\pi/2$, $\beta = \pm 2.46\pi \approx \pm 5\pi/2$, ... etc. {from Figure (2.19) also, we can see that the maxima are found almost in the middle of the two minima}. Thus, we can consider that maxima (except central maximum) are obtained when β is approximately an odd multiple of π/2, i.e. when $\beta \approx \pm\left(m + \frac{1}{2}\right)\pi$ with m = 1, 2, 3... which corresponds to the first, second, third,...

maxima obtained on the either sides of the central maxima. Thus, using equation (2.49), the intensity of the m^{th} maxima {i.e. at $\beta \approx \pm\left(m+\frac{1}{2}\right)\pi$} can be written as,

$$I_m = \frac{I_0(\pm 1)^2}{\left\{\left(m+\frac{1}{2}\right)\pi\right\}^2},\ m = 1, 2, 3, \ldots\ \{\text{as } \sin \pm\left(m+\frac{1}{2}\right)\pi\} = \pm 1\}$$

or,
$$I_m = \frac{4I_0}{(2m+1)^2\pi^2} \tag{2.55}$$

If I_1, I_2, I_3... are the intensities of the first, second, third...maxima, then the ratio of the intensities of the maxima can be written as,

$$\mathrm{I}_0 : I_1 : I_2 : I_3 \ldots\ldots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \ldots \tag{2.56}$$

Thus, in the diffraction pattern obtained on the screen, the intensities of the other maxima are small in comparison to the central maximum and decrease rapidly as one goes to higher order maxima (can be seen from Figure 2.19 also).

Example 2.4 Fraunhaufer diffraction pattern of monochromatic light of wavelength 6000 Å is obtained by a slit of width 1.2 μm. Find the half angular width of the central bright maximum in the diffraction pattern.

Solution We know that condition of minima in the single slit diffraction pattern is, $b \sin\theta = \pm m\lambda$, where $m = 1, 2, 3$.... The angular width of central maxima will be equal to the difference between the angular positions of the first order minima obtained on both sides of the central maxima for $m = 1$, thus the half angular width of the central maxima is equal to the angle of diffraction θ for the first minima (i.e. for $m = 1$), which is given by

$$\theta = \sin^{-1}\left(\frac{\lambda}{b}\right) = \sin^{-1}\left(\frac{6000\times 10^{-10}}{1.2\times 10^{-6}}\right) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$$

Example 2.5 In Fraunhoffer diffraction of light due to a narrow single slit of width 0.2 mm, if the first minima lie 5 mm on either side of the central maxima on a screen placed at a distance of 2 m from the lens, find the wavelength of light used. (GGSIPU 2004)

Solution For the Fraunhoffer diffraction by narrow single slit,

The condition of minima is, $b \sin\theta = m\lambda$

The angular position of the first minima is obtained for $m = 1$ in the above equation and is given by,

$$\sin\theta = \frac{\lambda}{b}$$

The angular positions θ of the minima or maxima is related to their linear position x on the screen by the relation, $\sin\theta = \frac{x}{D}$, where D is the distance of the screen from the lens.

Hence, we can write $\frac{x}{D} = \frac{\lambda}{b}$

or, $$\lambda = \frac{bx}{D}$$

Given that $b = 2$ mm = 0.02 cm; $x = 5$ mm = 0.5 cm, $D = 2$ m = 200 cm

Thus, $\lambda = 5 \times 10^{-5}$ cm $= 5 \times 10^{-7}$ m = 5000 Å

Example 2.6 In Fraunhofer diffraction pattern due to single slit of width 0.5 cm, a lens of focal length 40 cm is used to obtain the pattern on the screen. Calculate the distance between the first dark and the next bright fringe on the screen. Given that the wavelength of the light used is 4890 Å.

Solution For the Fraunhoffer diffraction by a narrow single slit, given that,

$b = 0.5$ cm $= 5 \times 10^{-3}$ m; f = focal length of lens = 40 cm = 0.4 m;

$\lambda = 4890$ Å $= 4.89 \times 10^{-7}$ m

Distance between first minima and first order maxima = $(x_2)_{max} - (x_1)_{min}$, where $(x_1)_{min}$ and $(x_2)_{max}$ are the positions of the first dark and the next bright fringe on the screen.

Since condition for minima is, $b \sin\theta = m\lambda$

For first minima $m = 1$, thus $\sin\theta = \frac{\lambda}{b}$, where θ is the angular position of the first minima which is related with the linear position x on the screen as, $\sin\theta = \frac{(x_1)_{min}}{f}$, where f is the focal length of the lens.

Hence, the position of the first minima on the screen, i.e. $(x_1)_{min} = \frac{f\lambda}{b} = 3.912 \times 10^{-5}$ m. Since, condition for maxima is written as,

$$b \sin\theta = \left(m + \frac{1}{2}\right)\lambda,$$

putting $m = 1$, in the above equation, the angular position of the bright fringe next to the first order minima is obtained and is given by,

$$\sin\theta = \frac{3\lambda}{2b}$$

Now, we can write, $\sin\theta = \dfrac{(x_2)_{max}}{f}$

Hence, $(x_2)_{max} = \dfrac{3f\lambda}{2b} = 5.868 \times 10^{-5}$ m

Thus, $(x_2)_{max} - (x_1)_{min} = 1.956 \times 10^{-5}$ m

2.8 FRAUNHOFER DIFFRACTION DUE TO *N*-SLITS (DIFFRACTION GRATING)

Now, we consider Fraunhofer diffraction from N equal width slits that are parallel and equally spaced. Diffraction grating is an example of such an arrangement of slits. Firstly we will consider the case when light is incident normally on such arrangement of slits.

(i) Normal Incidence: Consider, Fraunhofer diffraction of light of angular frequency 'ω' (and wavelength 'λ') falling at normal incidence on N equally spaced parallel slits (1, 2, 3...N), each of width b and spaced at distance 'a' from each other (as shown in Figure 2.21).

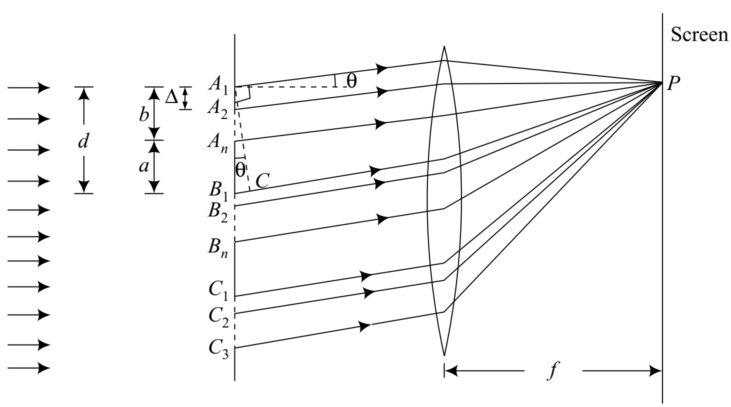


Fig. 2.21 Fraunhofer diffraction due to *N*-slits

As, in the case of single slit, each slit can be considered as composed of almost infinite equally spaced points, $A_1, A_2, A_3, \ldots A_n$, where $n \to \infty$, the spacing between two points 'Δ' $\to 0$ such that $(n-1)\Delta \to b$. According to Huygen's wave theory, each point on a slit acts as a source of secondary wavelets. Thus, the secondary wavelets from all N slits reach the screen and superpose to produce diffraction pattern.

Let, the secondary wavelets from each slit, emitted at an angle θ with the normal to the slit, reach the point 'P' of the screen. Let Y_1 be the displacement at P due to the secondary wavelets from slit 1, Y_2 be the displacement at P due to the

$$\sin(\omega t+\beta)+\sin(\omega t+\beta+\Phi)...+\sin\{\omega t+\beta+(N-1)\Phi\}$$

$$=\frac{\sin\frac{N\phi}{2}}{\sin\frac{\phi}{2}}\sin\left(\omega t+\beta+(N-1)\frac{\phi}{2}\right) \quad (2.61)$$

We can write,

$$Y=A\frac{\sin\beta}{\beta}\frac{\sin\frac{N\Phi}{2}}{\sin\frac{\Phi}{2}}\sin\left(\omega t+\beta+(N-1)\frac{\Phi}{2}\right) \quad (2.62)$$

or, $$Y=A\frac{\sin\beta}{\beta}\frac{\sin N\gamma}{\sin\gamma}\sin\{\omega t+\beta+(N-1)\gamma\} \quad (2.63)$$

where, $$\gamma=\frac{\pi}{\lambda}(a+b)\sin\theta=\Phi/2$$

Therefore, the resultant amplitude '*R*' at the point '*P*' is given by,

$$R=A\frac{\sin\beta}{\beta}\frac{\sin N\gamma}{\sin\gamma} \quad (2.64)$$

The resultant intensity '*I*' at a point '*P*' of the screen is proportional to the square of the resultant amplitude at that point, so we can write,

$$I\propto R^2$$

or, $$I\propto A^2\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

or, $$I=kA^2\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma}$$

or, $$I=\mathrm{I}_0\frac{\sin^2\beta}{\beta^2}\frac{\sin^2 N\gamma}{\sin^2\gamma} \quad (2.65)$$

where, $I_0\frac{\sin^2\beta}{\beta^2}$ represents the intensity pattern due to diffraction at single slit and $\frac{\sin^2 N\gamma}{\sin^2\gamma}$ arises because of interference between *N* diffracted waves from the *N* slits. Note that, for *N* = 1 as expected, equation (2.65) reduces to equation for single slit diffraction.

Positions of Maxima

Let, $Z = \dfrac{\sin N\gamma}{\sin\gamma}$. From equation (2.65) we can see that the resultant intensity 'I' will be maximum, if Z is maximum. Z is maximum, when

$$\frac{dZ}{d\gamma} = 0$$

or,
$$\frac{N\cos N\gamma\sin\gamma - \cos\gamma\sin N\gamma}{\sin^2\gamma} = 0$$

or,
$$N\cos N\gamma\sin\gamma = \cos\gamma\sin N\gamma$$

or,
$$N\tan\gamma = \tan N\gamma \tag{2.66}$$

Thus, the resultant intensity is maximum when the above equation is satisfied. So the roots of the above equation give the positions of maxima.

Principal maxima: Some of the roots of the equation (2.66) are

$$\gamma = \pm n\pi \text{ with } n = 0, 1, 2, 3.... \tag{2.67}$$

We can see that at these values of γ, the value of Z is given by,

$$\underset{\gamma\to\pm n\pi}{\text{Lt}} \frac{\sin N\gamma}{\sin\gamma} = \underset{\gamma\to\pm n\pi}{\text{Lt}} \frac{\frac{d}{d\gamma}(\sin N\gamma)}{\frac{d}{d\gamma}(\sin\gamma)} = \underset{\gamma\to\pm n\pi}{\text{Lt}} \frac{N\cos N\gamma}{\cos\gamma} = \pm N \tag{2.68}$$

{For $\gamma = \pm n\pi$, $n = 1, 2, 3\ldots$, $\sin N\gamma/\sin\gamma = 0/0$, i.e. indeterminate. Thus, the limiting value of z as $\gamma \to \pm n\pi$ can be found using L'Hospitals rule where the limit is found after differentiating the numerator and the denominator.}

So, from equation (2.65), the intensity of maxima corresponding to $\gamma = \pm n\pi$ (n = 0, 1, 2, 3....) is given by,

$$I = N^2 I_0 \frac{\sin^2\beta}{\beta^2} \tag{2.69}$$

Thus, the maxima given by equation (2.67) are very intense and so, are also called principal maxima and equation (2.69) gives the intensities of principal maxima. The positions of these maxima correspond to

$$\gamma = \frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi, \quad \text{where } n = 0, 1, 2, 3, ... \tag{2.70}$$

or,
$$(a+b)\sin\theta = \pm n\lambda, \quad \text{where } n = 0, 1, 2, 3, ... \tag{2.71}$$

where n is called the order of the principal maxima. This equation is also called the condition of principal maxima. The positive and negative sign indicate that there are two principal maxima of the same order lying on either side of the zero order maxima. It can be seen from equation (2.71) that, for higher orders principal maxima, the value of sin θ is larger, for which $\frac{\sin^2\beta}{\beta^2} = \frac{\sin^2\left(\frac{\pi}{\lambda} b \sin\theta\right)}{\frac{\pi}{\lambda} b \sin\theta}$ will be smaller. Thus, the intensity of the principal maxima decreases with increase of order.

Secondary Maxima

The maxima corresponding to the roots of the equation (2.66), other than $\gamma = \pm n\pi$ (n = 0, 1, 2, 3....), are called secondary maxima. The intensity of the secondary maxima can be obtained from equations (2.65) and (2.66). As, for the secondary maxima, $N \tan\gamma = \tan N\gamma$ (where $\gamma \neq \pm n\pi$), using the Figure (2.22), we have

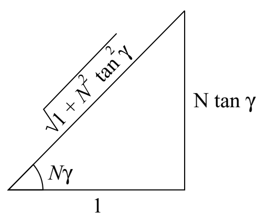


Fig. 2.22

$$\sin N\gamma = \frac{N \tan\gamma}{\sqrt{(1 + N^2 \tan^2\gamma)}} \tag{2.72}$$

Therefore, we can write,

$$\frac{\sin^2 N\gamma}{\sin^2\gamma} = \frac{N^2 \tan^2\gamma}{(1 + N^2 \tan^2\gamma)\sin^2\gamma} = \frac{N^2}{(1 + N^2 \tan^2\gamma)\cos^2\gamma}$$

$$= \frac{N^2}{\cos^2\gamma + N^2 \sin^2\gamma} = \frac{N^2}{1 + (N^2 - 1)\sin^2\gamma} \tag{2.73}$$

Thus, from equation (2.65), the intensities of the secondary maxima are given by,

$$I = I_0 \frac{\sin^2\beta}{\beta}\left\{\frac{N^2}{1 + (N^2 - 1)\sin^2\gamma}\right\} \tag{2.74}$$

And from equations (2.74) and (2.69), the ratio of intensities of secondary maxima to the intensity of principal maxima is given by, If $a = b$, then $n = 2m$, $m = 1, 2, 3....$, So, $n = 2, 4, 6....$ of the principal maxima will be missing in the diffraction pattern obtained with N-slits, if $a = b$.

Example 2.7 Deduce the missing orders for a double slit Fraunhoffer diffraction pattern, if the slit width is 0.16 mm and they are 0.8 mm apart.

Solution The missing orders in the diffraction pattern formed due to N-slits is given by,

$$n = m(a + b)/b \quad \text{where } m = 1, 2, 3....$$

Given that $a = 0.8$ mm and $b = 0.16$ mm, so the missing orders of principal maxima are,

$$n = m(0.96)/0.16 = 6\text{ m} \quad \text{where } m = 1, 2, 3....$$

i.e., $n = 6, 12, 18....$

orders of principal maxima will be missing in the diffraction pattern.

Width of Principal Maxima

The angular separation between the first two minima lying on either side of a principal maximum gives the angular width of that principal maximum.

Let, in the diffraction pattern of light of wavelength λ from N-slits, the nth principal maximum be obtained at an angle 'θ_n', then from the condition of maxima, we can write

$$(a + b)\sin\theta_n = n\lambda \tag{2.83}$$

If $\theta_n + d\theta_{1n}$ and $\theta_n - d\theta_{2n}$ represent the angles of diffraction corresponding to the first minimum on either sides of the n^{th} principal maximum, given by equation (2.78), then $d\theta_{1n} + d\theta_{2n}$ is the angular width of the nth principal maximum. For a large value of 'N', $d\theta_{1n} \approx d\theta_{2n} = d\theta_n$ (Figure 2.23) and the angular width of n^{th} principal maximum can be written as $2d\theta_n$.

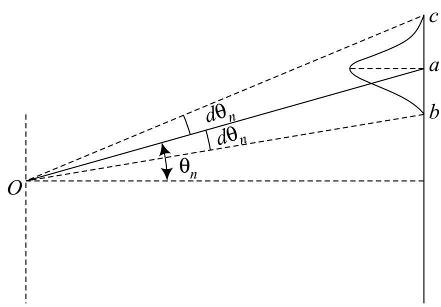


Fig. 2.23 Width of maxima

We know that the condition of minima is

$$(a + b)\sin\theta = \frac{m}{n}\lambda \quad \text{where } m \neq N, 2N, 3N \tag{2.84}$$

Note that the first minimum on either side of n^{th} order principal maxima will be obtained for $m = (nN \pm 1)$. Thus, for the first minimum on either sides of the n^{th} principal maximum we can write,

$$(a + b) \sin [\theta_n \pm d\theta_n] = \frac{(nN \pm 1)}{n}\lambda$$

or,
$$(a + b) \sin [\theta_n \pm d\theta_n] = n\lambda \pm \frac{\lambda}{N} \tag{2.85}$$

Dividing equation (2.85) by equation (2.84),

$$\frac{(a+b)\sin(\theta_n \pm d\theta_n)}{(a+b)\sin\theta_n} = \frac{n\lambda \pm \dfrac{\lambda}{N}}{n\lambda}$$

or,
$$\frac{\sin(\theta_n \pm d\theta_n)}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

or,
$$\frac{\sin\theta_n \cos d\theta_n \pm \cos\theta_n \sin d\theta_n}{\sin\theta_n} = 1 \pm \frac{1}{nN} \tag{2.86}$$

For small value of $d\theta_n$, $\cos d\theta_n \sim 1$ and $\sin d\theta_n \sim d\theta_n$. Thus,

$$\frac{\sin\theta_n \pm \cos\theta_n . d\theta_n}{\sin\theta_n} = 1 \pm \frac{1}{nN}$$

or,
$$1 \pm \cot\theta_n d\theta_n = 1 \pm \frac{1}{nN}$$

or,
$$d\theta_n = \frac{1}{nN\cot\theta_n} \tag{2.87}$$

Therefore, we can write the angular width of the nth principal maximum as,

$$2d\theta_n = \frac{2}{nN\cot\theta_n} \tag{2.88}$$

From equation (2.83), we can write $n = (a + b)\sin\theta_n/\lambda$, so the angular width of n^{th} principal maximum can also be written as,

$$2d\theta_n = \frac{2\lambda}{N(a+b)\cos\theta_n} \tag{2.89}$$

Note that greater the number of slits, smaller will be the angular width of the principal maximum, i.e. the sharper will be the principal maximum.

(ii) Oblique Incidence: In all the above discussions, we have considered the case when light is indent normally on the N-slits. However, for a more general case, when light is incident at an angle of incidence 'i' on the grating, i.e. for oblique incidence, the condition of principal maxima is,

$$(a + b)\{\sin \theta + \sin i\} = \pm n\lambda, \quad \text{where } n = 0, 1, 2, 3.... \tag{2.90}$$

where θ is the angle of diffraction of the nth principal maxima.

2.9 DIFFRACTION GRATING

We have discussed the diffraction pattern produced by a system of parallel, equally spaced slits of equal widths. An arrangement which consists of a large number of equidistant slits is known as a diffraction grating. From equation (2.89), we know that to obtain sharper (small angular width) principal maxima, the number of slits must be very large. Thus, in a good quality diffraction grating, large number of slits, typically 15000 per inch are grooved. Such closely spaced slits are obtained by ruling grooves with a diamond point on a transparent sheet. These grooves act as opaque spaces while the transparent portions act as slits. After the ruling of each groove, the diamond point is lifted by machine and the transparent sheet is forwarded for the ruling of the next groove. Since, the distance between two consecutive grooves must be very small and all the grooves must be equally spaced, the sheet is forwarded with the rotation of a screw (having constant pitch[#]), which drives the carriage carrying the sheet.

Commercial gratings are the replica of an actual diffraction grating. These are produced by taking cast of the actual diffraction grating on a film of cellulose acetate. This is done by pouring an appropriate strength solution of the cellulose acetate on the actual grating and drying it to form a thin film, which is then detached from the actual grating. This film retains the impression of the rulings of the original grating. For making transmission grating, it is fixed between two glass plates and for making reflection grating it is fixed on a silvered surface.

Application: Determination of wavelength using diffraction grating

Diffraction grating can be used to determine the wavelength of a light source. In the diffraction pattern of light obtained with a diffraction grating, the positions of the principal maxima depend on the wavelength of the light according to the relation,

$$(a + b) \sin \theta = n\lambda \tag{2.91}$$

where θ is the angle of diffraction for the nth principal maximum in the diffraction pattern of the light of wavelength λ obtained from a diffraction grating and $(a + b)$ is the grating element. As the number of rulings per unit length of a diffraction grating is known, we can write the grating element as,

[#]To groove the slits at equal distance the screw must forward the sheet by equal distance in each rotation, i.e. the pitch of the screw must be constant. It was only in 1882, when such a nearly perfect screw was achieved by Rowland with which he obtained a grating having 14438 lines per inch.

as $$\sin\theta \le 1,\ (n\lambda/(a+b) - \sin i) \le 1$$

or, $$n \le (a+b)(1+\sin i)/\lambda$$

Thus, the maximum value of 'n' that can be observed with the grating is,

$$n_{\max} = (a+b)(1+\sin i)/\lambda$$

Given that $i = 30°$, grating element $(a + b) = 1/6000$ cm, thus

$$n_{\max} = \frac{(1+\sin 30°)}{6000 \times 5460 \times 10^{-8}} = \frac{3/2}{6000 \times 5460 \times 10^{-8}} = 3.0525 \times 3/2 = 4.5787$$

Thus, the maximum order of spectrum that can be observed with the grating is 4.

2.10 DISPERSIVE POWER OF GRATING

It is defined as the ratio of the difference in the angle of diffraction of any two spectral lines to the difference between the wavelengths of the two spectral lines. If θ and $\theta + d\theta$ are the angles of diffraction corresponding to the maxima due to the wavelengths λ and $\lambda + d\lambda$ respectively, then $d\theta/d\lambda$ gives the dispersive power of the grating.

Now, we know that the angle of diffraction of the n^{th} order principal maximum for a wavelength λ, is given by the equation,

$$(a+b)\sin\theta = n\lambda \tag{2.93}$$

Differentiating, this equation with respect to λ, we get

$$(a+b)\cos\theta \frac{d\theta}{d\lambda} = n$$

or, $$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta} \tag{2.94}$$

This is quite clear that the dispersive power of grating is

(i) directly proportional to the order of the spectrum, n. Thus, the angular spacing between any two spectral lines is more in higher orders.

(ii) directly proportional $1/(a + b)$, i.e. the number of lines per cm in the grating. Thus, the dispersive power of a grating having a large number of lines per unit length will be higher.

(iii) inversely proportional to $\cos\theta$. Thus, larger is the value of θ higher is the dispersive power. As θ increases with increase in the wavelength λ, the dispersive power for large wavelengths is higher.

Example 2.11 A monochromatic light of wavelength 5000 Å is incident normally on a diffraction grating having 2500 lines/cm. Calculate the dispersive power of the grating in the second order spectrum.

Solution The dispersive power of grating is given by,

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$

where given that the grating element $(a + b) = 1/2500$ cm and the wavelength $\lambda = 5000$ Å $= 5000 \times 10^{-8}$ cm. Now, from the condition of principal maxima $\sin\theta = n\lambda/(a + b)$, so $\cos\theta = \sqrt{1 - \{n\lambda/(a+b)\}^2}$.

Thus, $$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\{n\lambda/(a+b)\}^2}}$$

So for second order spectrum, i.e. for $n = 2$, the dispersive power is given by,

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\{n\lambda/(a+b)\}^2}}$$

$$= \frac{2\times 2500}{\sqrt{1-\{2\times 5000\times 10^{-8}\times 2500)^2}} \text{ radians/cm}$$

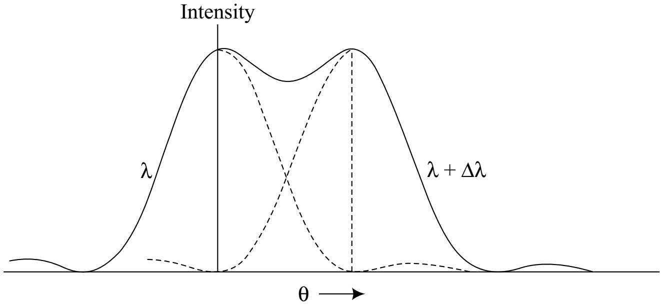
or, $$\frac{d\theta}{d\lambda} = \frac{2\times 2500}{0.9682} \text{ radians/cm} = 5164.2 \text{ radians/cm}$$

2.11 RESOLVING POWER

The ability of any optical instrument to provide distinctly separate image of two objects, located very close to each other, is called its resolving power. It may be precisely defined as the reciprocal of the smallest angle subtended at the objective of instrument by two point objects, which can be distinguished as separate. Resolution of optical instruments is dealt with in two broad ways—Geometric and Spectral resolutions. Geometric resolution, which is defined above is used where the geometric positions between two nearby objects are resolved e.g. telescopes and microscopes if the wavelengths of light in a given source are to be resolved spectral resolution is used e.g. spectroscopes. The spectral resolving power of an optical instrument is defined as its ability to show two spectral lines of very close wavelengths as separate. The spectral resolving power of an optical instrument is given by, $\lambda/d\lambda$.

Two spectral lines from a source are resolvable when their distance is such that the principal maxima in the diffraction pattern of one falls on the first minima in the diffraction pattern of other (Figure 2.24). This criterion of resolution is known as **Rayleigh's Criterion** for resolution.

Note that high dispersive power refers to wide separation of the spectral lines whereas high resolving power refers to the ability of the instrument to resolve two closely spaced spectral lines.



The Rayleigh criterion for the resolution of two spectral lines

Fig. 2.24

2.12 RESOLVING POWER OF OPTICAL INSTRUMENTS

(i) Diffraction Grating: Consider, two wavelengths λ and $\lambda + d\lambda$. According to Rayleigh's criterion, they can be resolved, if the maxima due to one (i.e. $\lambda + d\lambda$) lies on the first minima due to other (i.e. λ) as shown in Figure 2.25.

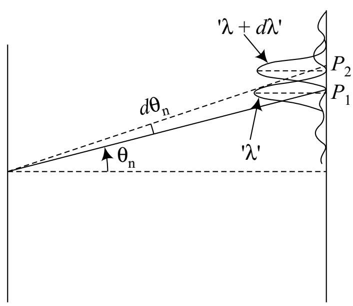


Fig. 2.25 Rayleigh criterion for resolution of two wavelengths in the diffraction pattern formed by diffraction grating

The direction of the n^{th} principal maxima for a wavelength λ is given by

$$(a + b) \sin \theta_n = n\lambda \tag{2.95}$$

The direction of the n^{th} principal maxima for a wavelength $\lambda + d\lambda$ is given by,

$$(a + b) \sin (\theta_n + d\theta_n) = n(\lambda + d\lambda) \tag{2.96}$$

These two lines will appear just resolved, if the angle of diffraction $(\theta_n + d\theta_n)$ also corresponds to the direction of the first minimum after the n^{th} principal maximum at θ_n (corresponding to the wavelength λ).

grating that can just resolve the second order spectrum of sodium lines will not be able to resolve them in first order spectrum.

Example 2.13 Light from sodium lamp is incident normally on a grating having 5000 lines on 1 cm. Find the angles of diffraction for the principal maxima of the two sodium lines (wavelengths 5890 Å and 5896 Å) in the first order spectrum. Are the two lines resolved in the first order spectrum?

Solution For the diffraction through grating, given that–

The grating element $(a+b) = \dfrac{1}{5000}$ cm $= \dfrac{1}{5\times 10^5}$ m, $\lambda_1 = 5890\text{Å} = 5.89\times 10^{-8}$ m and $\lambda_2 = 5896$ Å $= 5.896\times 10^{-8}$ m

Since, the condition for principal maxima is, $(a + b)\sin\theta = n\lambda$. The angles of diffraction (or the angular positions) θ_1 and θ_2 of the first order principal maxima due to the wavelengths λ_1 and λ_2, respectively, are obtained for $n = 1$ and are given by, $(a + b)\sin\theta_1 = \lambda_1$ and $(a + b)\sin\theta_2 = \lambda_2$.

Thus, $\theta_1 = \sin^{-1}\{\lambda_1/(a+b)\} = \sin^{-1}(5890\times 10^{-10}\times 5\times 10^5) = 17.12$ radians

and $\theta_2 = \sin^{-1}\{\lambda_2/(a+b)\} = \sin^{-1}(5896\times 10^{-10}\times 5\times 10^5)$
$= 17.1455$ radians

Thus, the angular separation between the first order principal maxima due to the two wavelengths is given by, $\theta_2 - \theta_1 = 17.1455 - 17.1275 = 0.018$ radians. They are so closely spaced that it looks as if they cannot be viewed separately but from the resolving power of the grating, we know that the given grating can just resolve the two wavelengths having difference between them $d\lambda$, given by,

$$\frac{\lambda}{d\lambda} = nN$$

or,

$$d\lambda = \frac{\lambda}{nN}$$

For the given grating $N = 5000$, thus the two lines having average wavelength $\lambda = (5890 + 5896)/2$ Å $= 5893$ Å can be just resolved in the first order spectrum, if they differ by,

$$d\lambda = \frac{5893\times 10^{-10}}{5000} = 1.179 \text{ Å}.$$ As the difference between the sodium lines is more than 1.179 Å, they can well be resolved by the given grating.

Example 2.14 What is the smallest difference between two wavelengths near the wavelength 5893 Å that a diffraction grating having 5000 lines on 1 cm can resolve in the second order spectrum?

Solution From the resolving power of the grating, we know that the given grating can just resolve the two wavelengths having difference between them $d\lambda$, given by,

$$\frac{\lambda}{d\lambda} = nN$$

or, $$d\lambda = \frac{\lambda}{nN}$$

For the given grating $N = 5000$, thus the two lines near the wavelength $\lambda = 5893$ Å can be just resolved in the second order ($n = 2$) spectrum, if they differ by, $d\lambda = \dfrac{5893 \times 10^{-10}}{2 \times 5000} = 0.5893$ Å. Thus, the smallest difference between two wavelengths near wavelength 5893 Å that can be resolved by the diffraction grating is 0.5893 Å.

(ii) Prism: Let two wavelengths λ and $\lambda + d\lambda$ from a source S are incident on a prism of base length 't' after passing through a collimating lens 'L_1'. The prism is set at minimum deviation position. After getting deviated by the prism the two wavelengths are focussed by the telescope objective L_2 (as shown in Figure 2.26).

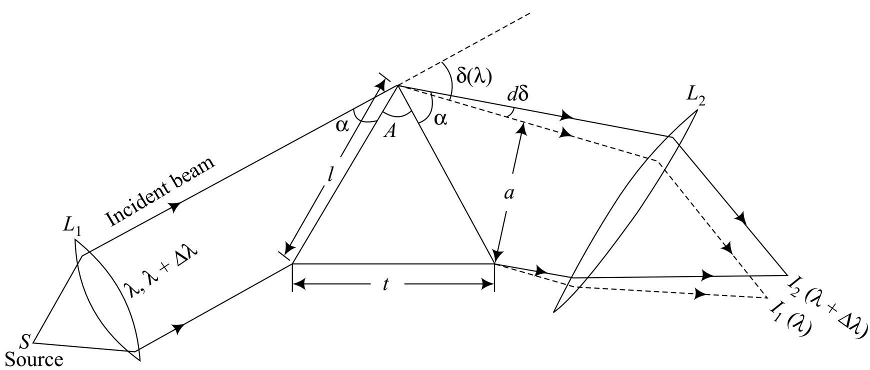


Fig. 2.26 Images corresponding to wavelengths λ and λ + Δλ in prism spectrum

If δ is the deviation produced by the prism in the wavelength λ, we can write,

$$\mu = \frac{\sin\{(A+\delta)/2\}}{\sin\dfrac{A}{2}} \tag{2.100}$$

Where μ is the refractive index of the prism, which depends upon λ. As the refractive index of the prism decreases with increase in the wavelength, the deviation 'δ' produced by the prism, also decreases with increase in wavelength 'λ'. Thus, the wavelength λ and λ + *d*λ are focussed at I_1 and I_2 by the telescope objective after getting deviated by the angles δ and (δ + *d*δ) respectively, by the prism.

From equation (2.100), we can write

$$\frac{d\mu}{d\lambda} = \frac{\cos\left(\frac{A+\delta}{2}\right)}{\sin\frac{A}{2}} \frac{1}{2}\frac{d\delta}{d\lambda}$$

or,
$$\frac{d\delta}{d\lambda} = 2\frac{d\mu}{d\lambda}\frac{\sin\frac{A}{2}}{\cos\left(\frac{A+\delta}{2}\right)} \tag{2.101}$$

Now from Figure 2.26,

$$\alpha + A + \alpha + \delta = \pi \quad \text{or} \quad \alpha = \left[\left(\frac{\pi}{2}\right) - \left(\frac{A+\delta}{2}\right)\right]$$

∴
$$\sin\alpha = \sin\left(\frac{\pi}{2} - \frac{A+\delta}{2}\right) = \cos\left(\frac{A+\delta}{2}\right) \tag{2.102}$$

From Figure 2.26, $\sin\alpha = \frac{a}{l}$ and $\sin\frac{A}{2} = \frac{t}{2l}$ (2.103)

∴
$$\cos\left(\frac{A+\delta}{2}\right) = \frac{a}{l} \tag{2.104}$$

Thus,
$$\frac{d\delta}{d\lambda} = 2\frac{t/(2l)}{a/l}\frac{d\mu}{d\lambda}$$

or,
$$\frac{d\delta}{d\lambda} = \frac{t}{a}\frac{d\mu}{d\lambda} \tag{2.105}$$

The face of the prism limits the incident beam to a rectangular section of width '*a*'. Thus, we can say that the final image I_1 corresponds to the principal maxima for wavelength λ and I_2 corresponds to the principal maxima for wavelength λ + *d*λ after diffraction at the rectangular aperture of width '*a*'. According to Rayleigh's criterion, the two wavelengths λ and λ + *d*λ will be resolvable, if the position of I_2 corresponds to the first minima due to the wavelength λ. The position of I_2 will correspond to the first minima due to the

(iii) The resolving power of grating is much higher than that of a prism.

(iv) In a grating spectra, most of the incident light is distributed in the different order of spectra, whereas it is distributed in a single spectrum in a prism. So the intensities of spectral lines of grating are less than that in a prism.

Short Answer Type Questions

Q1. Differentiate between the phenomenon of interference and diffraction.

Ans. Interference is a result of superposition of two wavefronts originating from a single source, while diffraction is a result of superposition of secondary wavelets from different points of same wavefront. The fringe width in the interference pattern is normally same, while in diffraction pattern, the fringe width goes on decreasing as we move away from the edge of the shadow. In interference all maxima are of same intensity, while in diffraction pattern they are of varying intensity, the intensity decreases as we go from central to higher order maxima.

Q2. Justify why it is easier to observe diffraction of sound waves and antenna waves in daily life as compared to the diffraction of light?

Ans. To observe diffraction of a wave, the dimension of the diffracting body must be of the order of the wavelength of the wave. The wavelength of light is very small and the dimensions of the obstacles or apertures found in daily life is very large in comparison to the wavelength of light, so in daily life diffraction of light is hardly observed. While the wavelength of sound waves and antenna waves is very large in comparison to light and is of the order of the dimensions of the obstacles or apertures found in daily life, thus diffraction of sound and antenna waves can be observed easily in daily life.

Q3. Show that for single slit diffraction pattern to hold well, the width of the slit must be necessarily of the order of one wavelength.

Ans. We know that in single slit diffraction pattern most of intensity of the light is distributed in the central maximum, i.e. between $\sin\theta = -\lambda/b$ and λ/b. Thus, the deviation of light from straight line path, which can be represented by θ, is inversely proportional to the slit width b. If the slit width is very small in comparison to the wavelength λ, θ will be very small (almost negligible) i.e. light continues to move almost undiffracted and if slit width is very small in comparison to the wavelength λ, θ will be very large, that is the incident light spread to a very large angle, i.e. almost uniform illumination is observed on the screen. Thus, for obtaining a good diffraction pattern of light from a single slit, the slit width must be of the order of the wavelength of incident light.

12. Obtain the half angular width of the principal maxima obtained in the diffraction pattern of light from N-slits.
13. How diffraction grating can be used to determine the wavelength of light? If the rulings on the diffraction grating are made closer what will be the effect on the spectra? (GGSIPU 2004)

OBJECTIVE-TYPE QUESTIONS

1. The bending of light around the corners of an obstacle is called
   (i) interference (ii) diffraction
   (iii) polarization (iv) scattering
2. In interference pattern, all the maxima have same intensity, while in diffraction pattern the intensities of maxima
   (i) are same
   (ii) decrease as one go to higher orders
   (iii) increases as one go to higher orders
   (iv) none of these
3. The condition for observing Fraunhofer diffraction from an obstacle is that the light wavefront incident on it must be
   (i) spherical (ii) plane
   (iii) cylindrical (iv) elliptical
4. The maximum order of maxima observed in the Fraunhofer diffraction pattern of light by a diffraction grating is
   (i) directly proportional to the grating element
   (ii) inversely proportional to the grating element
   (iii) independent of grating wavelength
   (iv) directly proportional to the wavelength
5. A diffraction pattern is observed using red light. What happens if the red light is replaced with blue light? (GGSIPU 2004)
   (i) diffraction maxima and minima becomes narrower and the spacing between them decreases
   (ii) diffraction maxima and minima becomes wider and the spacing between them increases
   (iii) no change is seen in the diffraction pattern
   (iv) diffraction pattern is lost.
6. If dispersive power of a grating is high, the spacing between the different maxima in the diffraction pattern obtained with it,
   (i) will be larger
   (ii) will be smaller

7. Calculate the resolving power of a prism which is just able to resolve the sodium lines 5890 Å and 5896 Å. If the base length as well as the dispersive power of the prism is doubled, how close spectral lines can be resolved with the prism? (982, 1.5 Å)
8. What is the resolving power of a diffraction grating which has 15 cm of surface ruled with 6000 lines/cm, for first order of spectrum? (90000)