Framed Structures 2

Objectives:

Differentiate between perfect, imperfect and redundant frames. To compute the member forces in a frame by graphical method. To compute the forces in a truss by method of joints. To compute the forces in a truss by method of sections. To compute the forces in a truss by method of tension coefficients.

2.1 PLANE FRAMES

The plane trusses are here by termed as plane frames. In this chapter, pin jointed plane frames which are statically determinate are considered. A statically determinate frame can be completely analysed by using statics. The number of unknown forces is the same as the number of equations obtained from static equilibrium. The main methods in analysing statically determinate pin jointed plane frames are (i) Graphical solution – Force diagram (ii) Method of resolution at joints (iii) Method of sections and (iv) Tension coefficient method. The first three methods are used in plane frames (trusses) and the fourth method is used for analysing the space frame.

A truss is an assemblage of three or more members which are hinged or pinned. A load applied on the truss is transmitted to all joints so that the members are in pure compression or tension.

Consider a simple truss made up of three members hinged at the ends to form a triangle. A load W is acting at the apex of the triangle and due to symmetry, the reactions are W/2 at each support.

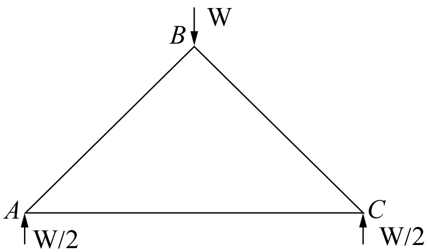
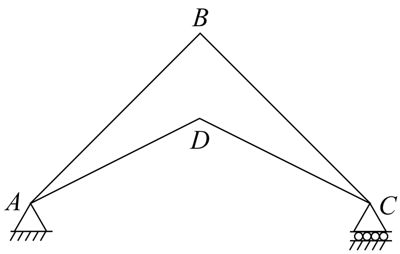


FIG. 2.1

Due to the application of the load, the joint A and C pulls the member out and for equilibrium at joint A there should be an equal and opposite force should move away from joint A. In otherwords, member AC in tension. Due to the downward load W, the joint B is pushed vertically downwards. The forces in the members AB and BC are in compression as the joint B is pushed. A force in the

$$n_m < 2n_j - 3$$
$$4 < (2 \times 4) - 3$$
$$4 < 5$$



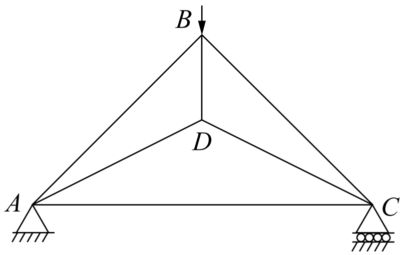
$$\boxed{\begin{array}{l} n_j = 4 \\ n_m = 4 \end{array}}$$

FIG. 2.3

2.2.3 Redundant Frame

A redundant frame is one where the number of member or members are more than $(2j-3)$. In Fig. 2.4, the number of joints are

$$n_m > (2n_j - 3)$$
$$6 > (2 \times 4) - 3$$
$$6 > 5$$



$$\boxed{\begin{array}{l} n_j = 4 \\ n_m = 6 \end{array}}$$

FIG. 2.4

i.e.,' redundant frame is having more member/members necessary to produce stability.

2.3 GRAPHICAL SOLUTION-FORCE DIAGRAMS

Consider the perfect frame in Fig. 2.5. The forces include the applied load and the reactions at *P* and *Q*.

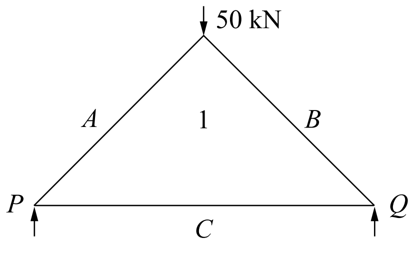


FIG. 2.5

Due to symmetry the reactions are 25 kN at joint *P* and *Q* respectively. In graphical method the loads and reactions are read clockwise. They are represented by capital letters written on either side of the force, commonly known as 'BOW'S Notation'. They are denoted with letters A, B, C and the space inside the member is denoted by numbers. Note that the letters A, B, C are marked in the middle length of the members and not at the joints. The load at the apex 50 kN is denoted as 'load AB'. The reaction at the right support is denoted as 'load BC'. The reaction at the left support 25 kN is denoted as 'load CA'. The member force in the horizontal member is denoted as 'force $1C$'.

The combined force diagram is drawn as follows:

1. Starting from the force AB, the known forces, viz. AB, BC and CA working clockwise round the frame, are set down in order and to scale as ab, bc and ca.
2. Consider the apex joint ab the centre of the clock, and the letters are read clockwise around this centre.
3. Therefore, from the letter 'b' draw a line parallel to $B1$ and from the letter 'a' draw a line parallel to $1A$ and both intersect at the point 1. From the force diagram, the magnitude of the forces are obtained. The member force $A1 =$ Member force $B1 = 5.7 \times 5 = 28.5$ kN. Member force $C1 = 2.8 \times 5 = 14$ kN.
4. Consider the joint at the left hand support reaction. Read clockwise in the frame diagram. Member $A1$ is inclined and in the force diagram a to 1 is downwards and hence mark the arrow correspondingly in the force diagram from 1 to C it is towards right mark this direction at that joint.

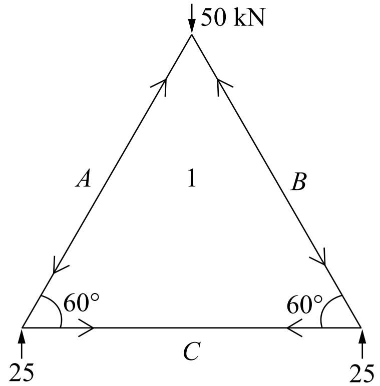


FIG. 2.6 Frame diagram

5. Consider the apex joint. Read clockwise $B1$ is inclined member. In the force diagram, b to 1 is upwards and hence mark the arrow upward for member $B1$ at the apex joint. Member $1A$ is a sloping member. From the force diagram, 1 to a is upwards. Therefore, mark the arrow upwards at the apex joint. (compression).

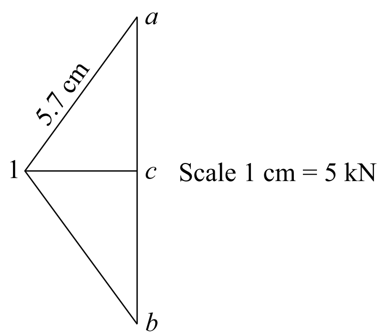


FIG. 2.7 Force diagram

6. Consider the right hand support reaction and again read clockwise. $1B$ is inclined member. In the force diagram 1 to b is downwards. Therefore, mark the arrow downwards at joint.
7. To determine the member force $C1$, from the force diagram it is noted that the force is acting from c towards left to 1. Mark the arrow $C1$ in the same direction in frame diagram.

The final forces are listed below.

Member	Forces in kN	
	Strut	**Tie**
$A1$	28.5	–
$B1$	28.5	–
$C1$	–	14.0

2.3.1 Numerical Problems on Symmetrical Frame and Symmetrical Loading

EXAMPLE 2.1: Determine the forces in the members graphically.

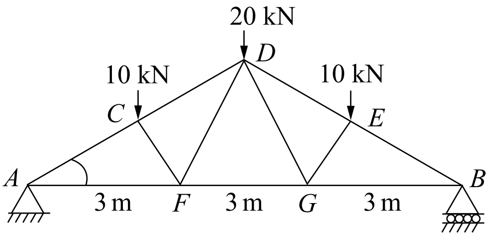


FIG. 2.8

SOLUTION

Due to Symmetry:

$$V_A = V_B = \frac{10 + 20 + 10}{2} = 20 \text{ kN}$$

Using the Bow notations

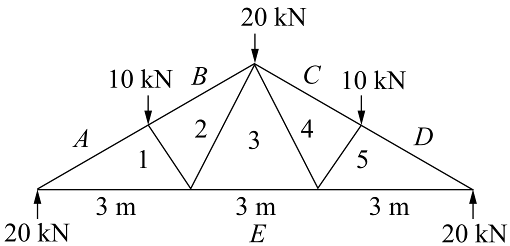


FIG. 2.9

The combined force diagram is drawn as follows:

1. The loads AB, BC, CD, DE and EA are marked to scale.
2. Start with a joint of the left hand reaction. Draw a line through the point 'a' a line parallel to $A1$ and from the point 'e' draw a horizontal line parallel to $1E$.
   They intersect at a point and is marked as 1.
3. Move to the next joint where 10 kN load is acting; Through 'b' draw a line parallel to $B2$ and from 1 draw a line parallel to 12. These two lines intersect at the point 2.
4. After locating point 2 in the force diagram. Consider the joint where the members $1-2, 2-3$, $3-E$ and $E-1$ meet. The point 3 is located on intersection of line $e1$ and a line drawn through point 2 and parallel to $2-3$ of the frame diagram.
5. The point 4 is located by drawing a line through 3 and parallel to $3-4$ in the load diagram which intersects the line drawn from 'C' in the force diagram and parallel to $C-4$.
6. The point 5 is marked from point 4. Draw a line parallel to $4-5$ of the frame diagram from point 4 and this cuts the horizontal line through 'e'.
7. Using the force diagram, the magnitude of the forces and the directions are obtained.
8. It is to be remembered that the arrows indicate not what is being done to the member but what the member is doing at the joint at each end. Hence, if the arrow is acting towards the joint it is compression and if the arrow is acting away from the joint then it is tensile force.

Member	Force in kN	
	Strut	Tie
$A1$ $D5$	36.5	—
$B2$ $C4$	31.5	—
$E1$ $E5$	—	31.0
12 45	8.25	—
23 34	—	8.25

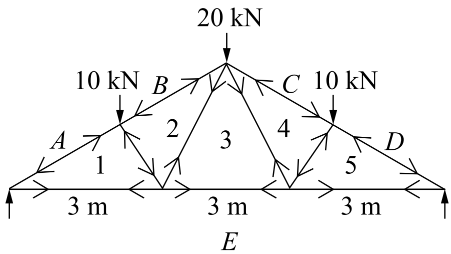


FIG. 2.10 Frame diagram

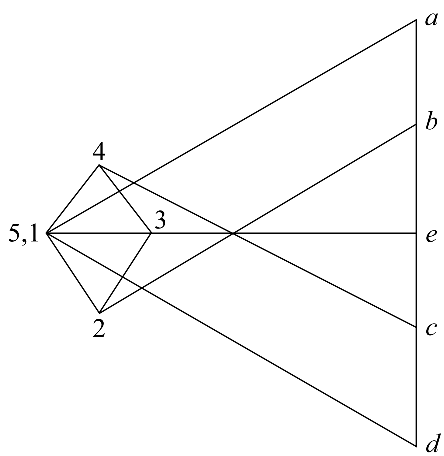


FIG. 2.11 Force diagram

2.3.2 Numerical Example on Frame with Loads Suspended from the Bottom Chord of the Frame in Addition to Loads on the Top Chord

EXAMPLE 2.2: Find the forces in all the members of the truss graphically

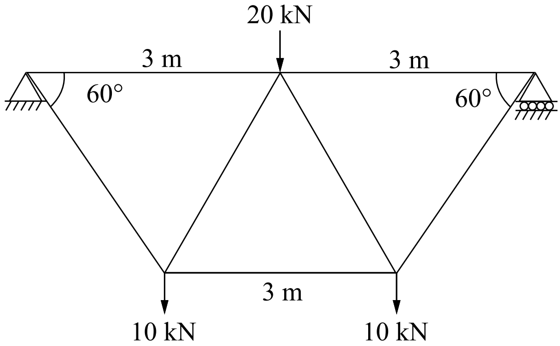


FIG. 2.12

1. The loads AB, BC, CD, DE and EA are marked to scale.
2. Start with the left support joint. Read clockwise and draw. Draw a line from 'a' parallel to $A-1$ in the load diagram. Draw another line e from the load diagram and parallel to the $1E$ of the frame diagram. They intersect at the point 1.
3. The point 2 is located by considering the joint adjacent to the left support. Draw a line from point 1 parallel to $1-2$ of the frame diagram. From the load diagram, draw a horizontal line through d and the line intersect at point 2.
4. From the point 2 draw a line parallel to $2-3$ of the frame diagram and from point 'e' draw a horizontal line and the intersection of above two lines give point 3.
5. Determine the magnitude and nature of the forces from the forces diagram and tabulate.

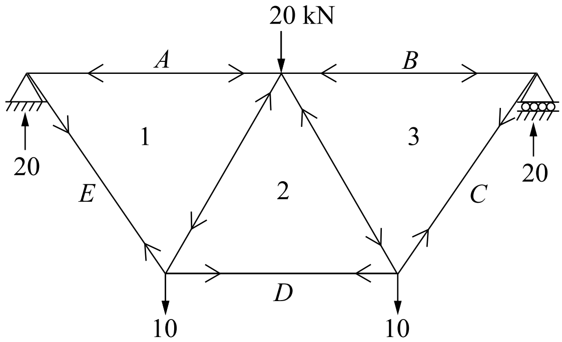


FIG. 2.13 Frame diagram

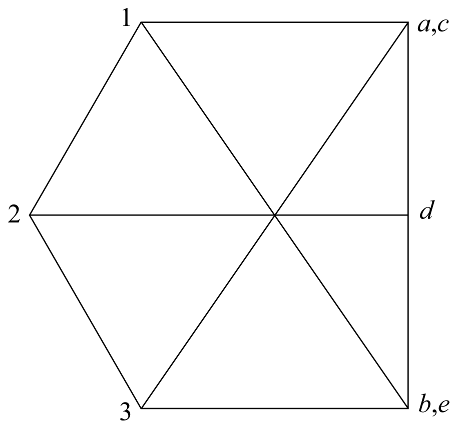


FIG. 2.14 Force diagram

Member	Force in kN	
	Strut	Tie
$A1$	12.5	–
$B3$	12.5	–
12	11.8	–
23	11.8	–
$1E$	–	24.0
$2D$	–	18.5
$3C$	–	24.0

2.3.3 Numerical Example on Cantilever Frames

EXAMPLE 2.3: Use the graphical method and determine the member forces and the reaction at the supports.

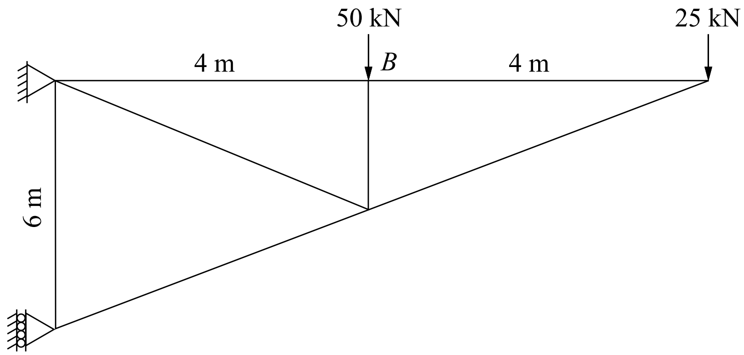


FIG. 2.15

In the force diagram of the cantilever truss, these is no need of reactions before starting of the same.

1. The load line is drawn as in the previous examples. $a-b$, $b-c$, start with a joint at the free end and reading clockwise, draw line from b parallel to $B3$ of the frame diagram and from point C draw line parallel to $C3$ and the intersection of the above lines give point 3.
2. The point 2 is located as follows. From the point 3, draw a line parallel to 23 and from 'a' draw a line parallel to $A2$. The intersection gives the point 2.
3. The point 1 is obtained as follows. A line is drawn from point 3, parallel to $3-C$ and from point 2 draw line parallel to 21 and the intersection of 1.
4. After marking the points 1, 2 and 3 the member forces and their nature are tabulated here.

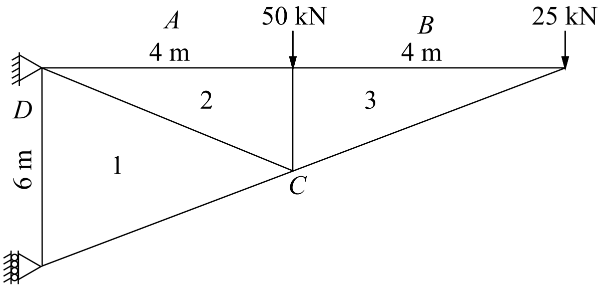


FIG. 2.16

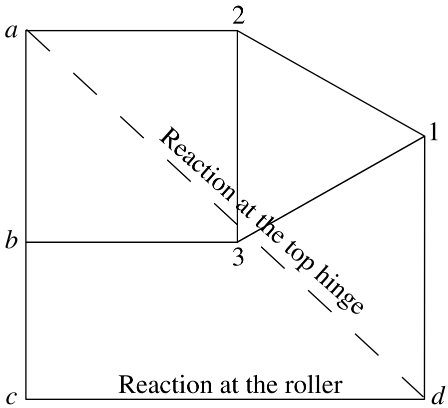


FIG. 2.17

Member	Force in kN	
	Strut	**Tie**
*A*2	–	36
*B*3	–	36
*C*3	43	–
*C*1	86	–
12	–	41.5
1*D*	–	50.0
23	50	–
Reaction at hinge	103	–
Reaction at roller	66	–

2.4 METHOD OF JOINTS

In the method of joints, the member forces are determined using the equilibrium conditions at that particular joint. In this resolution of forces at the joint, the free body diagram at that joint is considered. The procedure is explained as follows:

1. Check the stability and assess its determinacy of the truss.
2. If the truss is of cantilever type, the reactions need not be computed in general. If the truss is stable and determinate where one support is hinge and the other support is on rollers; compute the reactions at the supports.
3. Draw the free body diagram at each joint and analyse the member forces at a joint where only two members meet. Then, consider the adjacent joint where only two unknown forces to be determined. This process is repeated till the analysis of all joints are completed.
4. The results are tabulated along with magnitude of member forces and the nature of forces. The forces are tensile if they are pulling (acting away) the joint. The forces are compressive in nature if they are pushing (acting towards) the joint.

NUMERICAL EXAMPLE

EXAMPLE 2.4: Analyse the truss shown in Fig. 2.18 by method of joints. (May 2010, RVCE)

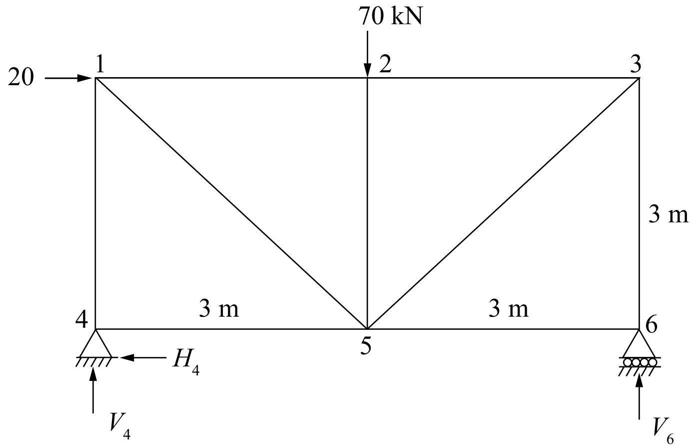


FIG. 2.18

SOLUTION

The reactions at the supports are found out by summing up the forces in horizontal and vertical directions and also by taking moments of applied forces about the hinge support.

$$\sum H = 0; \quad 20 - H_4 = 0$$

$$\boxed{H_4 = 20 \text{ kN}}$$

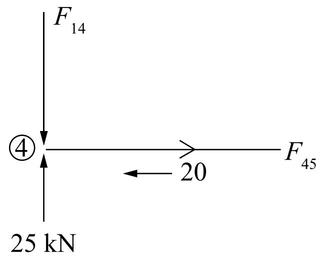
$$\sum V = 0; \quad V_4 + V_6 = 70 \text{ kN}$$

$$\sum M_4 = 0; \quad 20 \times 3 + 70 \times 3 - 6V_6 = 0$$

$$\boxed{V_6 = 45 \text{ kN}}$$

$$\boxed{\therefore \quad V_4 = 25 \text{ kN}}$$

Joint 4



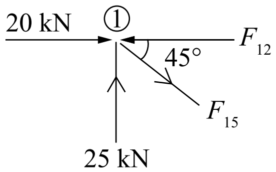
$$\sum H = 0;$$

$$F_{45} - 20 = 0$$
$$F_{45} = 20 \text{ kN}$$

$$\sum V = 0;$$

$$-F_{14} + 25 = 0$$
$$F_{14} = 25 \text{ kN}$$

Joint 1



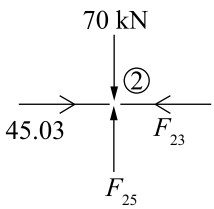
$$\sum V = 0;$$

$$25 - F_{15} \sin 45 = 0$$
$$F_{15} = 35.4 \text{ kN}$$

$$\sum H = 0;$$

$$20 - F_{12} + F_{15} \cos 45 = 0$$
$$20 - F_{12} + 35.4 \cos 45 = 0$$
$$F_{12} = 45.03 \text{ kN}$$

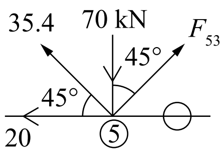
Joint 2



$$45.03 - F_{23} = 0$$
$$F_{23} = 45.03 \text{ kN}$$

$$\sum V = 0; \quad F_{25} = 70.0 \text{ kN}$$

Joint 5



$$\sum V = 0;$$

$$F_{53} \cos 45 + 35.4 \sin 45 - 70 = 0$$
$$F_{53} = 63.61 \text{ kN}$$

Hence the final forces are

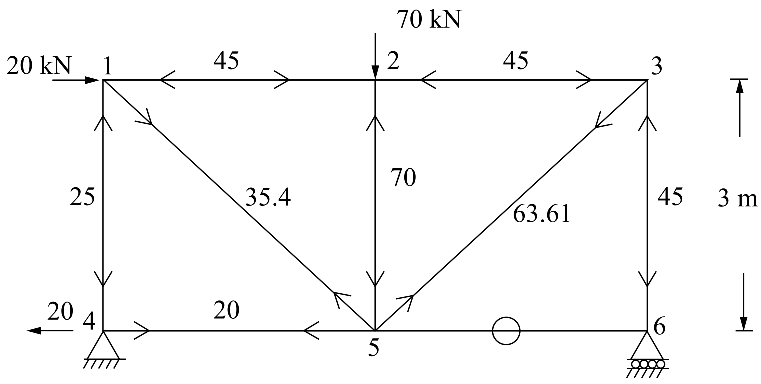


FIG. 2.19

EXAMPLE 2.5: Analyse the truss shown in Fig. 2.20 by method of joints. Indicate the member forces on a neat sketch of the truss. (MSRIT, Jan. 2010)

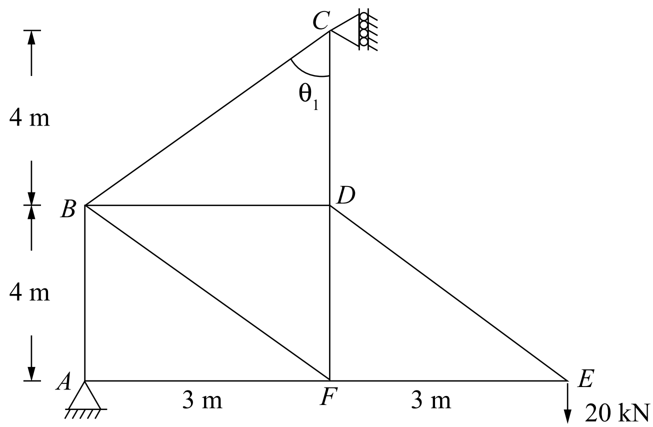


FIG. 2.20

SOLUTION

Taking moment about the hinge at A;

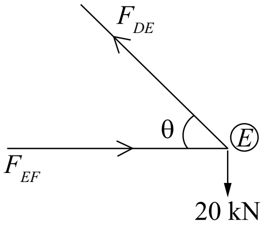
$$\sum M_A = 0; \quad 20 \times 6 - 8H_C = 0$$

$$H_C = 15 \text{ kN}$$

$$\sum H = 0; \quad \therefore \quad H_A = 15 \text{ kN}$$

$$\sum V = 0; \quad V_A = 20 \text{ kN}$$

Joint E

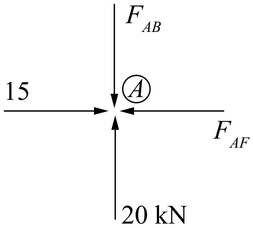


$$\sum V = 0; \qquad F_{DE}\sin\theta = 20$$

$$F_{DE} = \frac{20}{0.8} = 25 \text{ kN}$$

$$F_{EF} = F_{DE}\cos\theta = 25 \times \frac{3}{5} = 15 \text{ kN}$$

Joint A

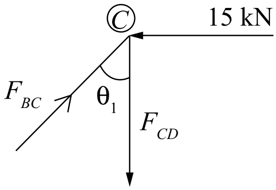


$$\sum H = 0;$$

$$F_{AF} = 15 \text{ kN}$$

$$\sum V = 0; \quad F_{AB} = 20 \text{ kN}$$

Joint C



$$\sum H = 0; \quad F_{BC}\sin\theta_1 = 15$$

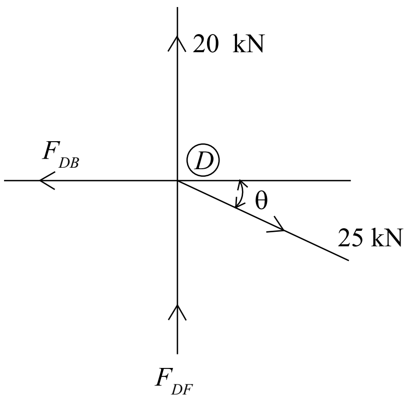
$$F_{BC} = \frac{15}{0.6} = 25 \text{ kN}$$

$$\sum V = 0; \quad -F_{CD} + F_{BC}\cos\theta_1 = 0$$

$$F_{CD} = F_{BC} \times 0.8$$

$$F_{CD} = 25 \times 0.8 = 20 \text{ kN}$$

Joint D



$$\sum H = 0;$$

$$25\cos\theta - F_{DB} = 0$$

$$F_{DB} = 25 \times \frac{3}{5} = 15 \text{ kN}$$

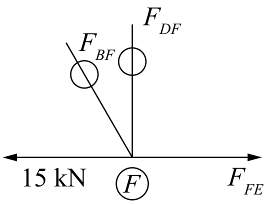
$$\sum V = 0;$$

$$20 + F_{DF} - 25\sin\theta = 0$$

$$20 + F_{DF} - 25 \times 0.8 = 0$$

$$F_{DF} = 0$$

Joint F



$$\sum V = 0;$$
$$F_{DF} = 0$$

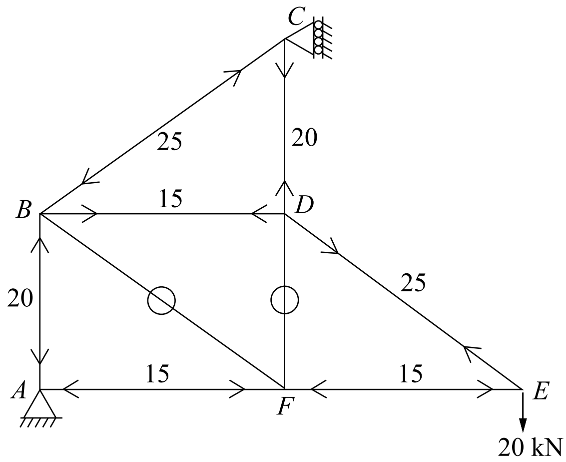


FIG. 2.21

EXAMPLE 2.6: Find the forces in all members of the pin jointed truss shown in Fig. 2.22 by using method of joints. (VTU, June 2009)

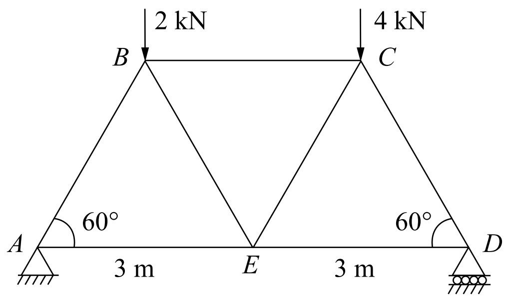


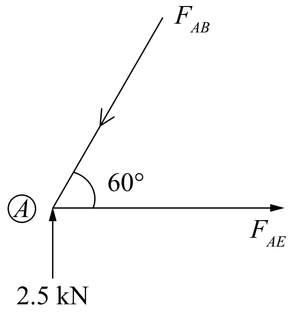
FIG. 2.22

$$\sum V = 0;$$
$$V_A + V_D = 2 + 4 = 6 \text{ kN}$$

$$\sum M_A = 0;$$

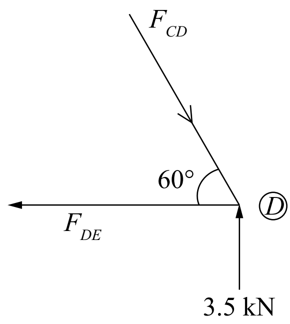
$$2(1.5) + 4(4.5) - 6V_D = 0$$
$$V_D = 3.5 \text{ kN}$$
$$V_A = 2.5 \text{ kN}$$

Consider Joint A



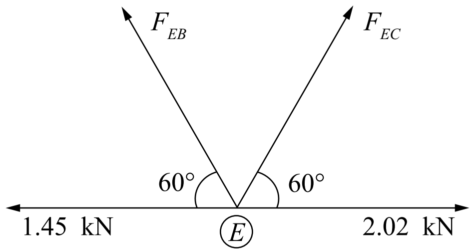
$$\sum V = 0$$
$$F_{AB} \sin 60 = 2.5$$
$$F_{AB} = 2.89 \text{ kN}$$
$$\sum H = 0$$
$$F_{AE} = F_{AB} \cos 60$$
$$= 2.89 \cos 60$$
$$= 1.45 \text{ kN}$$

Joint D



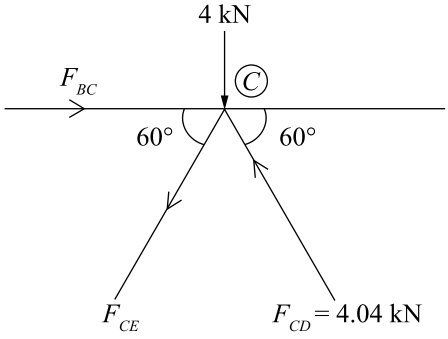
$$\sum V = 0$$
$$F_{CD} \sin 60 = 3.5$$
$$F_{CD} = 4.04 \text{ kN}$$
$$\sum H = 0$$
$$F_{CD} \cos 60 - F_{DE} = 0$$
$$F_{DE} = 4.04 \cos 60 = 2.02 \text{ kN}$$

Joint E



$$\sum V = 0$$
$$F_{EC} \sin 60 + F_{EB} \sin 60 = 0$$
$$F_{EC} = -F_{EB}$$
$$\sum H = 0$$
$$2.02 + F_{EC} \cos 60 - 1.45 - F_{EB} \cos 60 = 0$$
$$2.02 - F_{EB} \cos 60 - 1.45 - F_{EB} \cos 60 = 0$$
$$0.57 = 2F_{EB} \cos 60$$
$$F_{EB} = 0.57;\ F_{EC} = -0.57$$

Joint C



$\sum H = 0;$

$$F_{BC} - F_{CE}\cos 60 - F_{CD}\cos 60 = 0$$
$$F_{BC} - F_{CE}\cos 60 = 4.04\cos 60$$

$\sum V = 0;$

$$-4 - F_{CE}\sin 60 + 4.04\sin 60 = 0$$
$$F_{CE} = -0.57 \text{ kN}$$

Substituting F_{CE} in the above equation

$$F_{BC} = 1.73 \text{ kN}$$

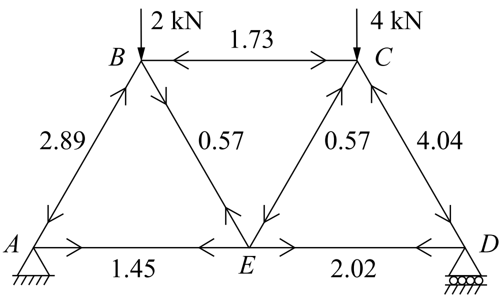


FIG. 2.23

EXAMPLE 2.7: Determine the magnitude and nature of forces in all the number of the pin jointed plane truss shown in Fig. 2.24 by method of joints. (VTU, June 08)

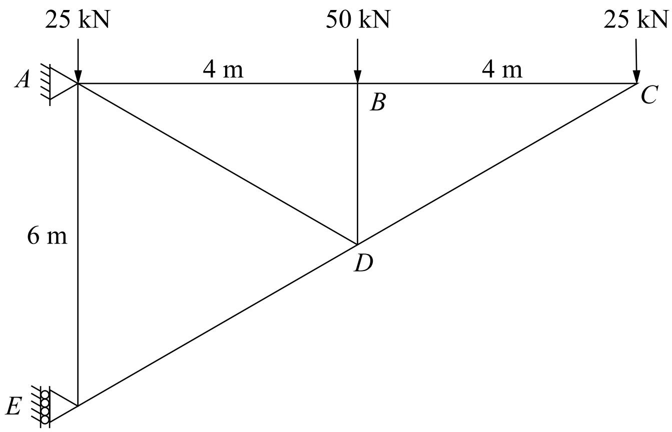


FIG. 2.24

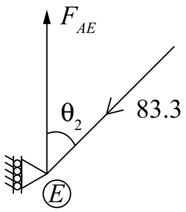
$\sum V = 0;$

$$-50 - 41.67 \sin\theta + F_{AD} \cos\theta_1 + F_{DE} \sin\theta = 0$$
$$-50 - 41.67(0.6) + 0.6F_{AD} + 0.6F_{DE} = 0$$
$$0.6F_{AD} + 0.6F_{DE} = 75$$
$$F_{DE} + F_{AD} = 125 \tag{2.2}$$

Solving Eqns. (1) and (2);

$$F_{DE} = 83.3 \text{ kN}, \ F_{AD} = 41.67 \text{ kN}$$

Joint E



$$\tan\theta_2 = 8/6$$
$$\sin\theta_2 = 0.8$$
$$\cos\theta_2 = 0.6$$

$$\sum V = 0; \ F_{AE} = 83.3 \cos\theta_2 = 50 \text{ kN}$$

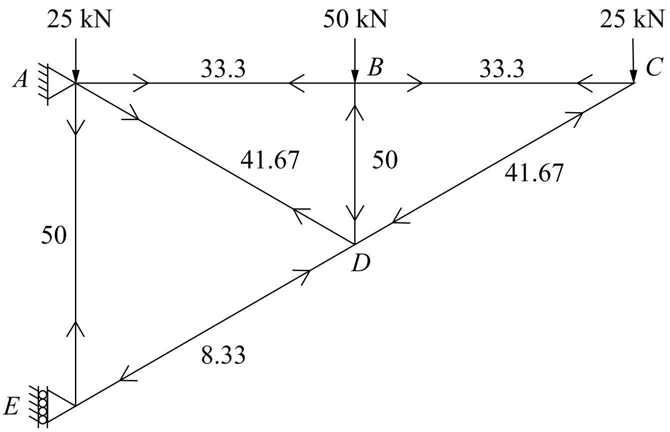


FIG. 2.25

EXAMPLE 2.8: Analyse the truss shown in Fig. 2.26 by method of joints. (VTU, May 2008)

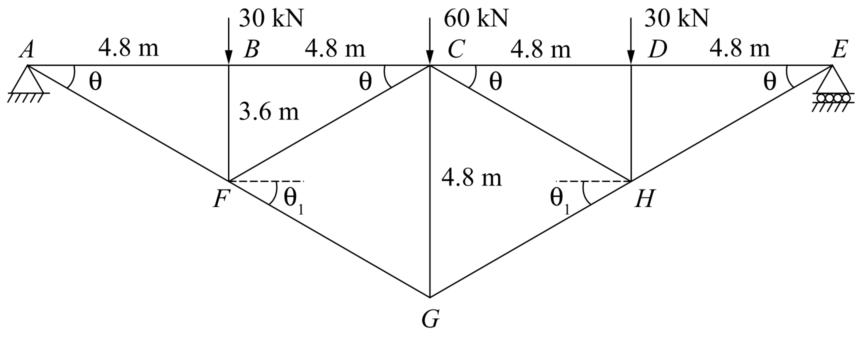
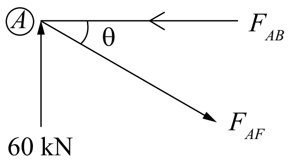


FIG. 2.26

SOLUTION

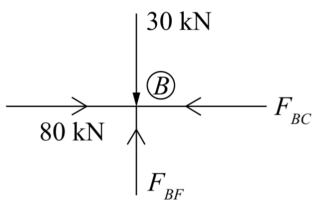
Due to Symmetry: $V_A = V_E = \dfrac{2(30)+60}{2} = 60 \text{ kN}$

Consider Joint A



$$\sum V = 0; \quad F_{AF} \sin\theta = 60$$
$$F_{AF} \times 0.6 = 60$$
$$F_{AF} = 100 \text{ kN}$$
$$\sum H = 0; \quad 100\cos\theta - F_{AB} = 0$$
$$F_{AB} = 100 \times 0.8 = 80 \text{ kN}$$

Joint B



$$\sum V = 0$$
$$F_{BF} = 30 \text{ kN}$$
$$F_{BC} = 80 \text{ kN}$$

EXAMPLE 2.9: Determine the forces in members and tabulate neatly. Use method of joints. (VTU, Dec. 06)

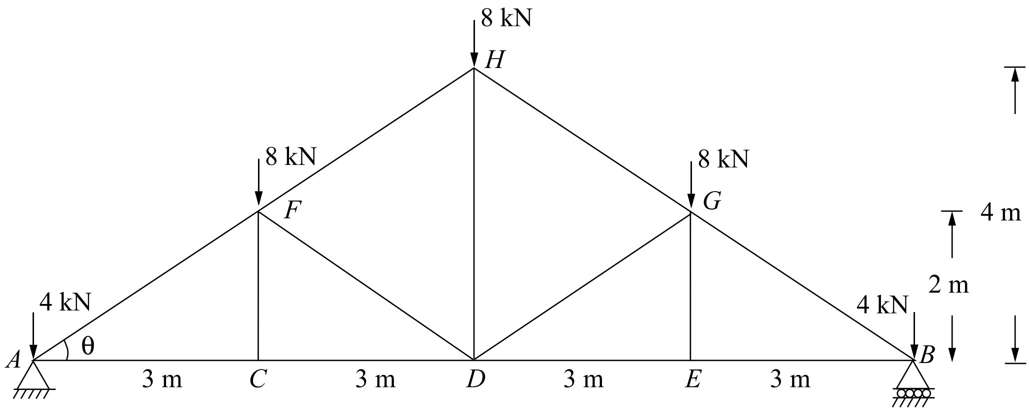
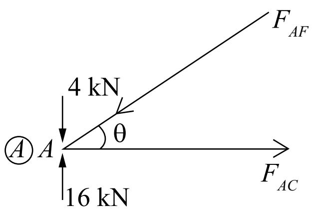


FIG. 2.28

SOLUTION

Due to Symmetry: $V_A = V_B = \dfrac{2(4)+3(8)}{2} = 16$ kN

Joint A



$$\sum V = 0$$

$$-4 - F_{AF}\sin\theta + 16 = 0$$

$$12 - 0.555\ F_{AF} = 0$$

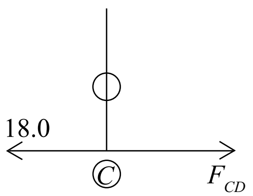
$$F_{AF} = 21.62 \text{ kN}$$

$$\sum H = 0$$

$$F_{AC} - F_{AF}\cos\theta = 0$$

$$F_{AC} = 21.62 \times 0.832 = 18.0 \text{ kN}$$

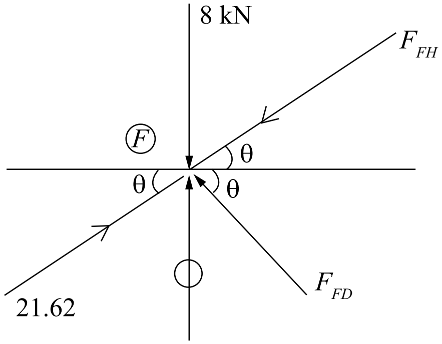
Joint C



$$\sum H = 0;$$

$$F_{CD} = 18.0 \text{ kN}$$

Joint F



$$\sum V = 0$$

$$-8 + 21.62 \sin\theta - F_{FH} \sin\theta + F_{FD} \sin\theta = 0$$

$$-8 + 21.62 \times 0.555 - 0.555\, F_{FH} + 0.555\, F_{FD} = 0$$

$$0.555\, F_{FD} - 0.555\, F_{FH} = -4$$

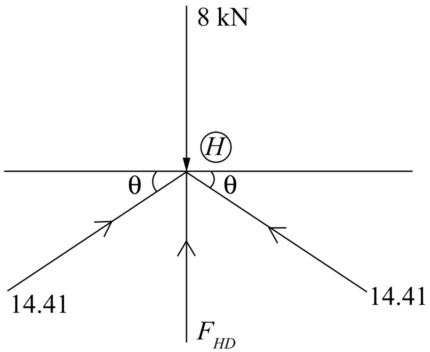
$$\sum H = 0;$$

$$21.62 \cos\theta - F_{FH} \cos\theta - F_{FD} \cos\theta = 0$$

$$F_{FD} + F_{FH} = 21.62$$

$$F_{FD} = 7.21,\ F_{FH} = 14.41 \text{ kN}$$

Joint H



$$\sum V = 0$$

$$-8 + 14.41 \sin\theta + 14.41 \sin\theta + F_{HD} = 0$$

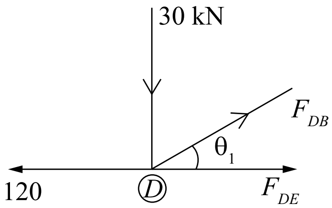
$$F_{HD} = 8 - 16 = -4 \text{ kN}$$

$$\therefore \quad F_{HD} = -4 \text{ kN}$$

This shows that the direction is to be changed, i.e., F_{HD} is tensile

S.No	**Member**	**Force (kN)**	**Nature**
1	*AF*	21.62	Compressive
2	*FH*	14.41	Compressive
3	*HG*	14.41	Compressive
4	*GB*	21.62	Compressive
5	*BE*	18.00	Tensile
6	*ED*	18.00	Tensile
7	*DC*	18.00	Tensile
8	*AC*	18.00	Tensile
9	*CF*	0.00	–
10	*FD*	7.21	Compressive
11	*DH*	14.41	Compressive
12	*DG*	7.21	Compressive
13	*GE*	0.00	–

Joint D



$$\sum V = 0$$

$$F_{DB} \sin\theta_1 - 30 = 0$$

$$F_{DB} = \frac{30}{0.707} = 42.43 \text{ kN}$$

$$\sum H = 0$$

$$-120 + F_{DE} + 42.43 \cos\theta_1 = 0$$

$$F_{DE} = -42.43 \times 0.707 + 120 = -30 \text{ kN} \frac{+120}{90 \text{ kN}}$$

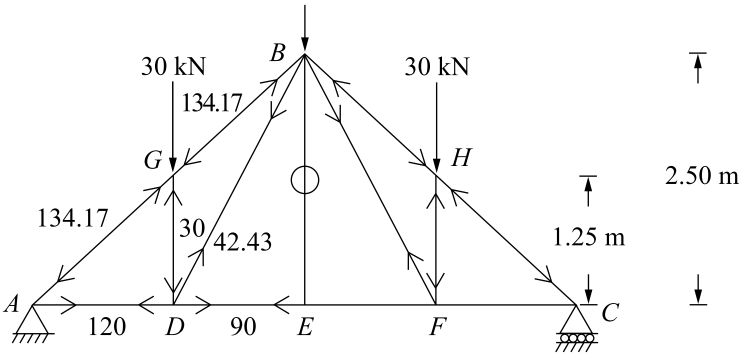


FIG. 2.30

EXAMPLE 2.11: Determine the forces in all members of all Bollman truss by method of joints. (VTU, July 2005)

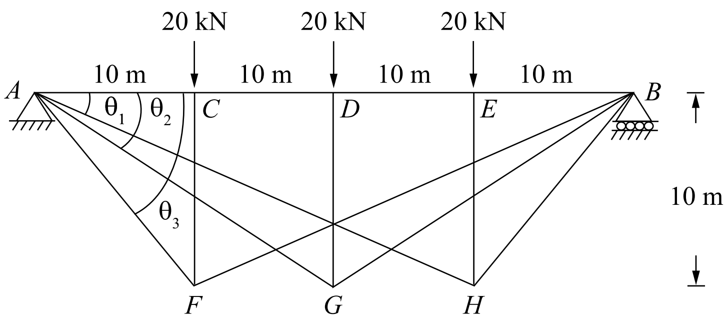


FIG. 2.31

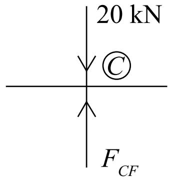
SOLUTION
Due to Symmetry

$$V_A = V_B = \frac{3(20)}{2} = 30 \text{ kN}$$

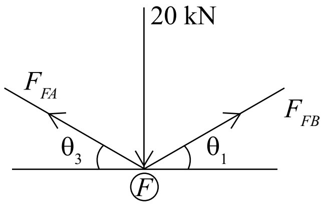
From geometry

$$\tan\theta_1 = \frac{10}{30} \quad \tan\theta_2 = 10/20 \quad \tan\theta_3 = 10/10$$
$$\sin\theta_1 = 0.316 \quad \sin\theta_2 = 0.447 \quad \sin\theta_3 = 0.707$$
$$\cos\theta_1 = 0.949 \quad \cos\theta_2 = 0.895 \quad \cos\theta_3 = 0.707$$

Joint C, D, E
$F_{CF} = 20$ kN, $F_{DG} = 20$ kN, $F_{EH} = 20$ kN



Joint F

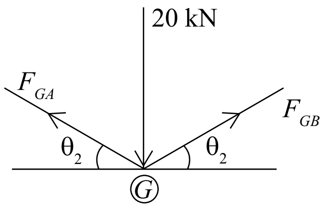


$$\sum H = 0;$$
$$-F_{FA}\cos\theta_3 + F_{FB}\cos\theta_1 = 0$$
$$-0.707F_{FA} + 0.949F_{FB} = 0$$
$$\sum V = 0;$$
$$-20 + F_{FA}\sin\theta_3 + F_{FB}\sin\theta_1 = 0$$
$$0.707F_{FA} + 0.316F_{FB} = 20$$

on solving the above equations

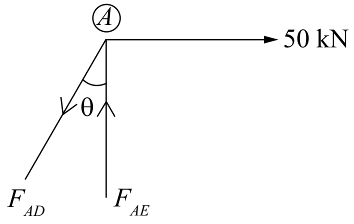
$$F_{FA} = 21.22 \text{ kN}$$
$$F_{FB} = 15.81 \text{ kN}$$

Joint G



$$\sum H = 0;$$
$$-F_{GA}\cos\theta_2 + F_{GB}\cos\theta_2 = 0$$
$$F_{GA} = F_{GB}$$
$$\sum V = 0;$$
$$-20 + F_{GA}\ \sin\theta_2 + F_{GB}\ \sin\theta_2 = 0$$
$$2\ F_{GA}\sin\theta_2 = 20$$
$$2\ F_{GA}(0.447) = 20$$
$$F_{GA} = 22.37 \text{ kN}$$

Joint A



tanθ = 5/6; θ = 39°48′

$$\sum H = 0$$

$$50 - F_{AD}\sin\theta = 0$$

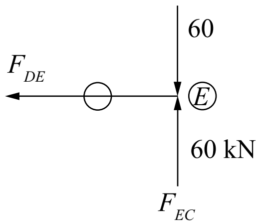
$$F_{AD} = \frac{50}{0.64} = 78.1 \text{ kN}$$

$$\sum V = 0$$

$$F_{AE} - 78.1\cos\theta = 0$$

$$F_{AE} = 60.0 \text{ kN}$$

Joint E



$$\sum V = 0;$$

$$-60 + F_{EC} = 0$$

$$F_{EC} = 60 \text{ kN}$$

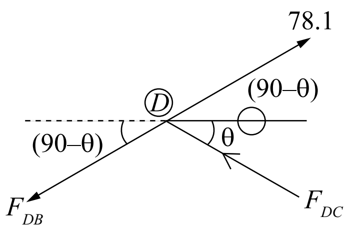
$$\sum H = 0;$$

$$F_{DE} = 0$$

Joint D

$$\sum H = 0$$

$$78.1\cos(90-\theta) - F_{DC}\ \cos\theta - F_{DB}\ \cos(90-\theta) = 0$$



$$78.1\cos 50°11' - F_{DC}(0.768) - 0.64\ F_{DB} = 0$$

$$0.768\ F_{DC} + 0.64\ F_{DB} = 50.01$$

$$\sum V = 0$$

$$78.1\sin\ (90-\theta) + F_{DC}\sin\theta - F_{DB}\sin\ (90-\theta) = 0$$

$$60 + 0.64\ F_{DC} - 0.768\ F_{DB} = 0$$

$$0.64\ F_{DC} - 0.768\ F_{DB} = -60$$

$$F_{DC} = 0,\ F_{DB} = 78.1 \text{ kN}$$

SOLUTION

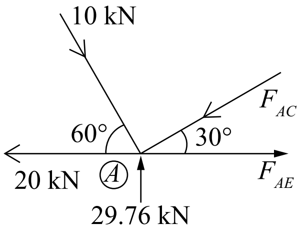
$$AD = \sqrt{AH^2 + DH^2}$$
$$DH = 6\tan 30$$
$$= \sqrt{6^2 + 3.46^2} = 6.93 \text{ m}$$
$$AC = CD = \frac{6.93}{2} = 3.465 \text{ m}$$

The reactions are found as

$$\sum V = 0;$$
$$V_A + V_B = 10\sin 60 + 20\sin 60 + 10\sin 60 + 10$$
$$V_A + V_B = 44.64$$
$$\sum M_A = 0;$$
$$20 \times 3.465 + 10 \times 6.93 + 10(4) - 12V_B = 0$$
$$V_B = 14.88 \text{ kN}$$
$$V_A = 29.76 \text{ kN}$$

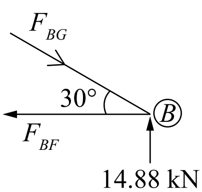
Consider Joint A

$$\sum H = 0$$
$$10\ \cos 60 + 20\ \cos 60 + 10\ \cos 60 - H_A = 0$$
$$H_A = 20 \text{ kN}$$



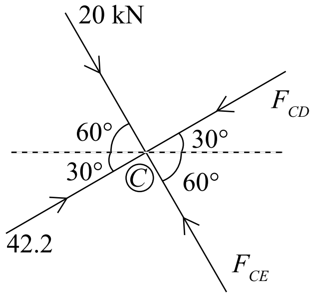
$$\sum H = 0;$$
$$10\cos 60 - 20 + F_{AE} - F_{AC}\cos 30 = 0$$
$$F_{AE} - 0.866\ F_{AC} = 15$$
$$\sum V = 0$$
$$-10\sin 60 + 29.76 - F_{AC}\sin 30 = 0$$
$$F_{AC} = 42.2 \text{ kN}$$
$$\therefore \quad F_{AE} = 51.55 \text{ kN}$$

Joint B



$$\sum V = 0;$$
$$-F_{BG}\sin 30 + 14.88 = 0$$
$$F_{BG} = 29.76 \text{ kN}$$
$$\sum H = 0;$$
$$F_{BG}\cos 30 - F_{BF} = 0$$
$$F_{BF} = 25.77 \text{ kN}$$

Joint C



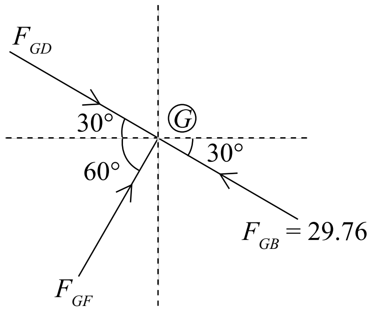
$$\sum H = 0$$

$$20\cos 60 - F_{CD}\cos 30 - F_{CE}\cos 60 + 42.2\cos 30 = 0$$
$$F_{CD}\cos 30 + F_{CE}\cos 60 = 46.55$$
$$0.866\, F_{CD} + 0.5\, F_{CE} = 46.55$$

$$\sum V = 0$$

$$-20\sin 60 - F_{CD}\sin 30 + F_{CE}\sin 60 + 42.2\sin 30 = 0$$
$$-0.5\, F_{CD} + 0.866\, F_{CE} = -3.78$$
$$F_{CD} = 42.20\text{ kN}$$
$$F_{CE} = 20.00\text{ kN}$$

Joint G



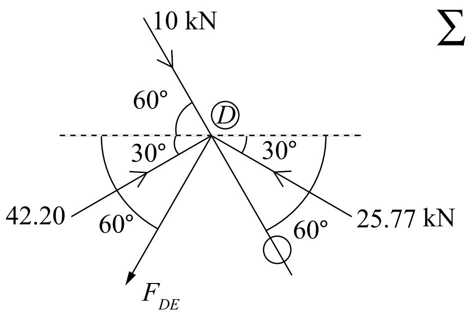
$$\sum H = 0$$

$$F_{GD}\cos 30 - F_{GB}\cos 30 + F_{GF}\cos 60 = 0$$
$$0.866\, F_{GD} - 0.866\, F_{GB} + 0.5\, F_{GF} = 0$$
$$0.866\, F_{GD} + 0.5\, F_{GF} = 0.866 \times 29.76$$
$$0.866\, F_{GD} + 0.5\, F_{GF} = 25.77$$

$$\sum V = 0$$

$$-F_{GD}\sin 30 + F_{GF}\sin 60 + 29.76\sin 30 = 0$$
$$-0.5\, F_{GD} + 0.866\, F_{GF} = -14.88$$
$$F_{GD} = 25.77\text{ kN},\ F_{GF} = 0$$

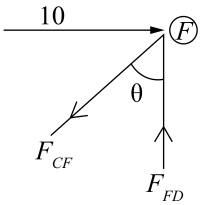
Joint D



$$\sum V = 0;$$

$$-10\sin 60 + 42.20\sin 30 - F_{DE}\sin 60 + 25.77\sin 30 = 0$$
$$F_{DE} = 29.24\text{ kN}$$

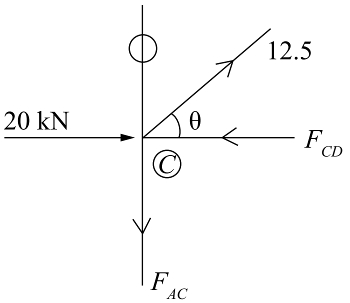
Joint F



$$\tan\theta = 4/3$$
$$\sin\theta = 0.8$$
$$\cos\theta = 0.6$$

$$\sum H = 0$$
$$10 - F_{CF}\ \sin\theta = 0$$
$$F_{CF} = 12.5\text{ kN}$$
$$\sum V = 0$$
$$F_{FD} - F_{CF}\ \cos\theta = 0$$
$$F_{FD} = 7.5\text{ kN}$$

Joint C



$$\tan\theta = 3/4$$
$$\sin\theta = 0.6$$
$$\cos\theta = 0.8$$

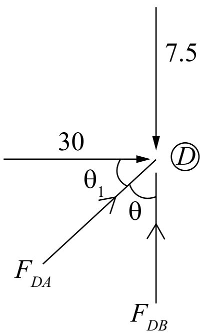
$\sum H = 0;$

$$20 + 12.5\cos\theta - F_{CD} = 0$$
$$20 + 12.5 \times 0.8 = F_{CD}$$
$$F_{CD} = 30\text{ kN}$$

$\sum V = 0;$

$$-F_{AC} + 12.5\sin\theta = 0$$
$$F_{AC} = 12.5 \times 0.6 = 7.5\text{ kN}$$

Joint D



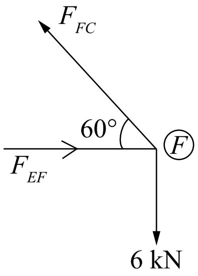
$\sum H = 0;$

$$30 - F_{DA}\ \sin\theta = 0$$
$$F_{DA} = \frac{30}{\sin\theta} = \frac{30}{0.8} = 37.5\text{ kN}$$

$\sum V = 0;$

$$-7.5 + F_{DB} - F_{DA}\ \cos\theta = 0$$
$$-7.5 + F_{DB} - 37.5 \times 0.6 = 0$$
$$F_{DB} = 30.0\text{ kN}$$

Consider Joint F



$$\sum V = 0$$

$$F_{FC} \sin 60 = 6$$

$$F_{FC} = \frac{6}{\sin 60} = 6.93 \text{ kN}$$

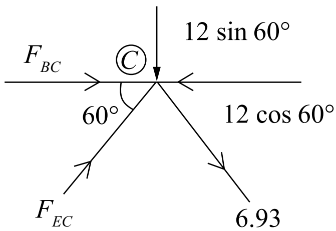
$$\sum H = 0$$

$$F_{EF} - F_{FC} \cos 60 = 0$$

$$F_{EF} = 6.93 \cos 60$$

$$= 3.47 \text{ kN}$$

Joint C



$$\sum V = 0$$

$$-12 \sin 60 - 6.93 \sin 60 + F_{EC} \sin 60 = 0$$

$$F_{EC} = 18.93 \text{ kN}$$

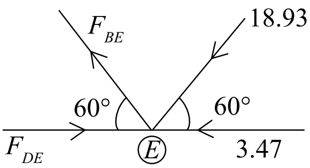
$$\sum H = 0$$

$$F_{BC} + 18.93 \cos 60 + 6.93 \cos 60 - 12 \cos 60 = 0$$

$$F_{BC} = -13.86 \cos 60$$

$$F_{BC} = 6.93 \text{ kN}$$

Joint E



$$\sum V = 0$$

$$F_{BE} \sin 60 - 18.93 \sin 60 = 0$$

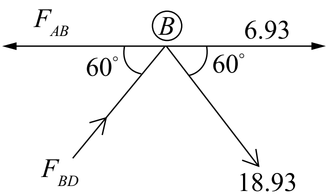
$$F_{BE} = 18.93 \text{ kN}$$

$$\sum H = 0$$

$$-3.47 - 18.93 \cos 60 + F_{DE} - 18.93 \cos 60 = 0$$

$$F_{DE} = 22.4 \text{ kN}$$

Joint B



$$\sum V = 0$$

$$-18.93 \sin 60 + F_{BD} \sin 60 = 0$$

$$F_{BD} = 18.93 \text{ kN}$$

$$\sum H = 0$$

$$6.93 + 18.93 \cos 60 + 18.93 \cos 60 - F_{AB} = 0$$

$$F_{AB} = 25.86 \text{ kN}$$

SOLUTION

The reactions are found using the equilibrium equations.

Summing up all the forces in the vertical direction;

$\sum V = 0;$

$$V_A + V_F = 50 + 40$$

$$V_A + V_F = 90 \tag{1}$$

$\sum M_A = 0;$

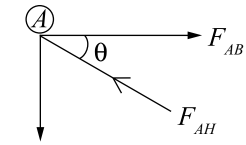
$$50(6) + 40(9) - 6V_F = 0$$

$$V_F = 110 \text{ kN}$$

$$\therefore \quad V_A = 90 - 110 = -20 \text{ kN}$$

This means that the direction of V_A is downwards.

Joint A



Considering the joint A; a vertical downward force of 20 kN is acting down. To balance this there should be an upward force of 20 kN to balance. This is due to F_{AH} since F_{AB} is a horizontal force which do not give a vertical component.

$$\therefore \quad F_{AH} \sin\theta = 20$$

$$F_{AH} = \frac{20}{\sin\theta} = 36 \text{ kN}$$

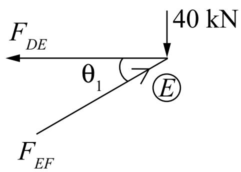
Resolving the forces horizontally at the joint A;

$$F_{AB} = F_{AH} \cos\theta$$

$$= 36 \times 0.832$$

$$= 29.95 \text{ kN}$$

Joint E



$$\tan\theta_1 = 4/3$$

$$\sin\theta_1 = 0.8$$

$$\cos\theta_1 = 0.6$$

Resolving vertically,

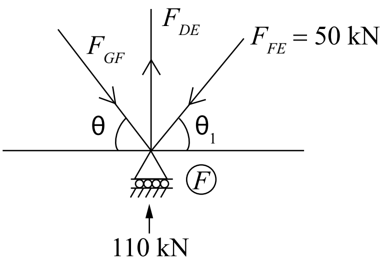
$$F_{EF}\sin\theta_1 = 40$$
$$F_{EF} = \frac{40}{0.8} = 50 \text{ kN}$$

Resolving all the forces in the horizontal direction

$$F_{DE} = F_{EF}\cos\theta_1 = 50 \times 0.6 = 30 \text{ kN}$$

Joint F

The truss is supported on rollers at the joint F. Only one reaction will be acting perpendicular to the base of the roller. Hence,



$\sum H = 0;$

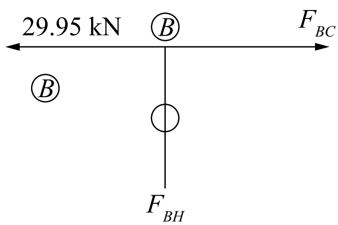
$$F_{GF}\cos\theta - 50\cos\theta_1 = 0$$
$$F_{GF} = 50 \times 0.6/0.832$$
$$F_{GF} = 36.06 \text{ kN}$$

$\sum V = 0;$

$$110 - 36.06\sin\theta - 50\sin\theta_1 + F_{DF} = 0$$
$$F_{DF} = 36.06(0.555) + 50(0.8) - 110$$
$$= -50 \text{ kN}$$

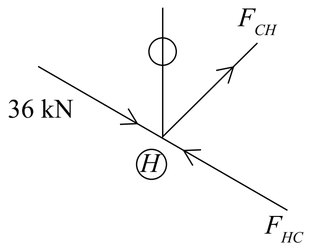
The $-$ve sign indicates that we have to change the nature of the force in F_{DF} and hence F_{DF} is compressive.

Joint B

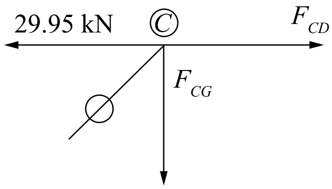


Resolving all the forces meeting at the joint in the vertical direction and as no forces are acting in the vertical direction; $F_{BH} = 0$. Resolving the forces at the joint B along the horizontal direction $F_{BC} = 29.95$ kN

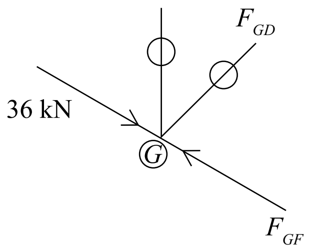
Joint H



A, H and G are on the same line. Hence, resolving the forces meeting at the joint H, $A\,HG$
Resolving all the forces along the line, $F_{HG} = 36$ kN
Joint C



Resolving all the forces in the vertical direction: $F_{CG} = 0$
Resolving all the forces in the horizontal direction: $F_{CD} = 29.95$ kN
Joint G



Resolving all the forces meeting at the joint G, along the line HGF and perpendicular that line;

$$F_{GD} = 0;$$
$$F_{GF} = 36 \text{ kN}$$

1. Imagine the truss to be cut completely through the section $X - X$ passing through the members BC, CG and GF.
2. Assume that the right portion of the truss $X - X$ to be removed. The portion to the left of $X - X$ would then collapse, because three forces in the members BC, GC and GF which were necessary to retain equilibrium had been removed.
3. If three forces F_{BC}, F_{GC} and F_{GF} are now applied to the portion of the frame concerned as shown in Fig. 2.42 then the portion of the frame will be in equilibrium under the action of the reaction 66.7 kN, applied 50 kN load and the forces F_{BC}, F_{GC} and F_{GF}.
4. These three forces are yet knowns in magnitude and direction. The member force F_{GF} is to be determined by knowing that the other two member forces F_{CB} and F_{CG} meet at point C and that they have no moment about that point.

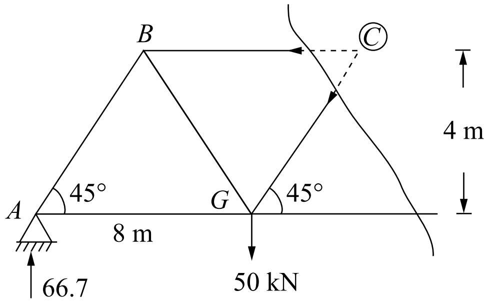


FIG. 2.42

5. Thus taking moments about the point C. The moment of the reaction is a clockwise $(66.7 \times 12 = 800.4)$. The moment of the applied load is anticlockwise (50×4), i.e., 200 kNm For equilibrium, the anticlockwise moment of F_{GF} is $(+4 \times GF)$ and this must be equal to the clockwise moment of $(800.4 - 200) = 600.4$ kNm. Therefore, $4F_{GF} = 600.4$, Hence, $F_{GF} = 150.1$ kN. This force is tension (pulling away from the joint G).
6. To determine the force in F_{BC}, take moments of all the forces about the point G, where other two members of the cut truss F_{GC} and F_{GF} meet.
   (a) The moment of the reaction is (66.7×8), i.e., 533.6 kN, (Clockwise)
   (b) The moment of the forces of 50 kN, F_{GC} and F_{GF} meet at the point G and do not give any moments.
   (c) The moment due to the required member force is $(F_{BC} \times 4)$, i.e., $4F_{BC}$ (anticlockwise)
   (d) $4F_{BC} = 533.6$ and $F_{BC} = 133.4$ kN.
   This force is compressive (Acting towards the joint B).
7. To determine the force in GC member, i.e., F_{GC}; resolve all the forces in the vertical direction of the cut truss considering the left part only. The cut members BC and GF are horizontal and hence they do not give a vertical component; it is easier to consider the vertical equilibrium.

The reaction at A is 66.7 kN acting upwards and hence it is taken as negative. The applied load at G is 50 kN and is acting downwards and therefore it is positive. The vertical component due to F_{GC} is downwards, i.e., $F_{GC} \sin 45$. This is negative.
Summing up all the above forces in the vertical direction and equating upward forces to downward forces.

$$F_{GC} \sin 45 + 50 = 66.7$$
$$\therefore \quad F_{GC} = 23.62 \text{ kN.}$$

This is compressive as the force is acting towards the joint.

8. It must be remembered that the arrows must be considered in respect to the nearest point of that portion of the frame which remains after the cut has been made.

2.5.2 Numerical Problems

EXAMPLE 2.17: Determine the nature and magnitude of forces in members DE, DI and HI of the truss shown in Fig. 2.45 by using method of sections.

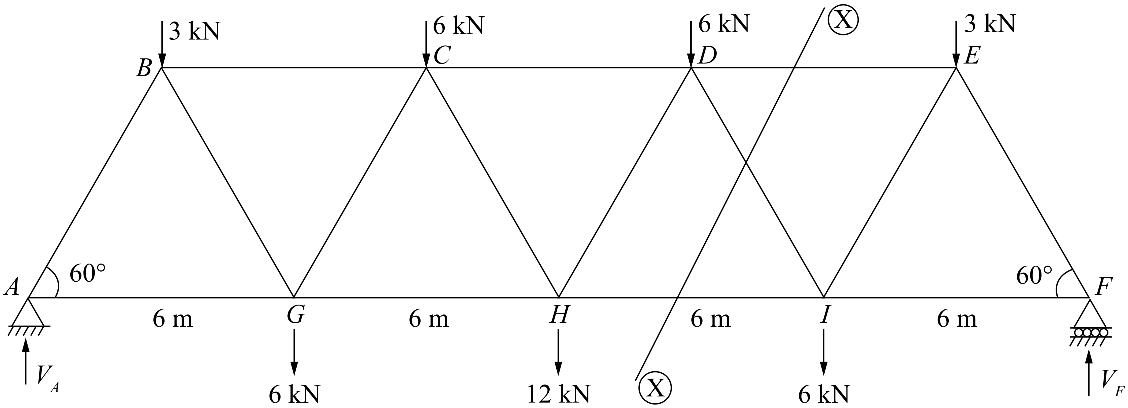


FIG. 2.43

SOLUTION

$$\sum V = 0 \quad V_A + V_F = 42$$
$$\sum M_A = 0 \quad 3(3) + 6(9) + 6(15) + 3(21) + 6(6) + 12(12) + 6(18) - 24V_F = 0$$
$$V_F = 21 \text{ kN}$$
$$\therefore \quad V_A = 21 \text{ kN}$$

EXAMPLE 2.18: Determine the forces in the members *BC*, *FC* and *FG* by method of sections.

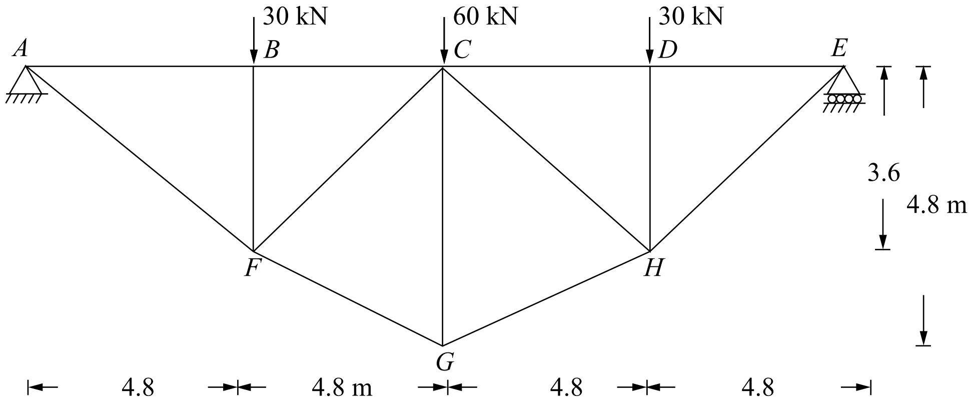


FIG. 2.45

SOLUTION

Due to symmetry, the reactions

$$V_A = V_E = \frac{30 + 60 + 30}{2} = 60 \text{ kN}$$

To determine the forces in *BC, FC* and *FG* cut a section *X-X* as shown in Fig. 2.48
Consider left part of the truss and analyse the equilibrium.

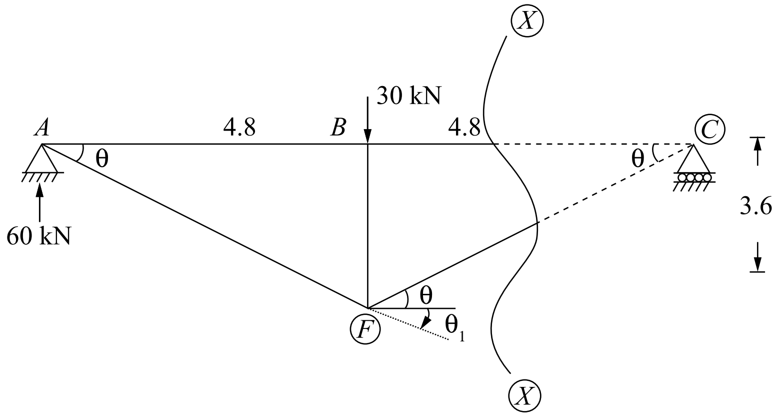


FIG. 2.46

To determine the force in *BC*, take moment about '*F*' where other two members of the cut section *FC* and *FG* meet.

$$60 \times 4.8 = F_{BC} \times 3.6$$

$$F_{BC} = 80 \text{ kN}$$

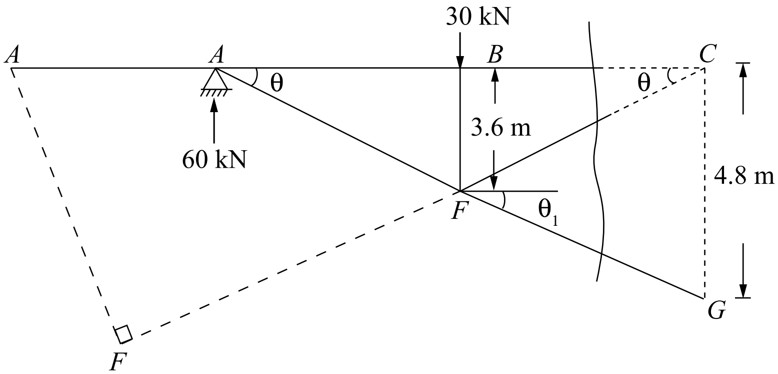


FIG. 2.47

From geometry $\boxed{A'C = 19.2 \text{ m}}$

To determine the force in *FC*, take moment about A' where the other two members of the cut section *BC* and *FG* meet.

Let $A'F'$ be perpendicular to the line CF extended.

$$\sin\theta = \frac{A'F'}{A'C}$$

$$A'F' = 19.2 \times 0.6 = 11.52 \text{ m}$$

$$\sum M'_A = 0$$

$$-60 \times (A'A) + 30\,(A'B) + F_{FC}\,(11.52) = 0$$

$$-60 \times 9.6 + 30\,(19.2 - 4.8) = -11.52\,F_{FC}$$

$$\therefore \quad F_{FG} = 12.5 \text{ kN} \quad \text{(Compressive)}$$

To determine the force in *FG*, take moment about *C* where the members *BC* and *FC* meet.

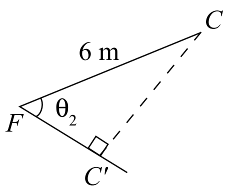


FIG. 2.48

In Fig. 2.45,
$$FC = \sqrt{BC^2 + BF^2}$$

$$= \sqrt{4.8^2 + 3.6^2} = 6 \text{ m}$$

Cut a section through the members CD, CG and FG and consider the right part of the truss.

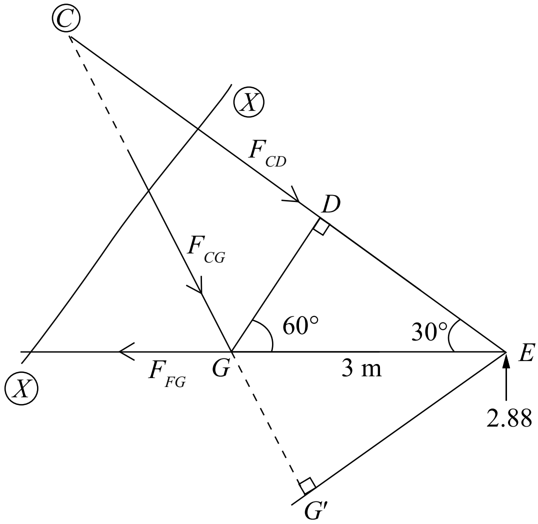


FIG. 2.50

$$\tan 30 = GD/2.6$$
$$GD = 2.6 \tan 30 = 1.5 \text{ m}$$

To determine the force in *CD*, take moment of the forces in the cut truss about *G* where other two members of the cut section, viz. *CG* and *FG* meet.

$$1.5\, F_{CD} = 2.88 \times 3$$
$$F_{CD} = 5.76 \text{ kN}$$

To determine the force in *FG*, take moment about *C* where other two members of the cut portion of the truss, viz. *CD* and *GC* meet.

$$4.5 \tan 30\, (F_{FG}) = 2.88 \times 4.5$$
$$F_{FG} = 4.98 \text{ kN}$$

To determine the force in *GC*, take moment about *E* where the cut members *CD* and *FG* meet.
Let EG' is perpendicular to the line CG extended

$$\therefore \quad F_{CG} \times G'E = 0$$
$$F_{CG} = 0$$

EXAMPLE 2.20: Find the forces in the members ED, EF and FG. Use method of sections. (VTU, Aug. 2004)

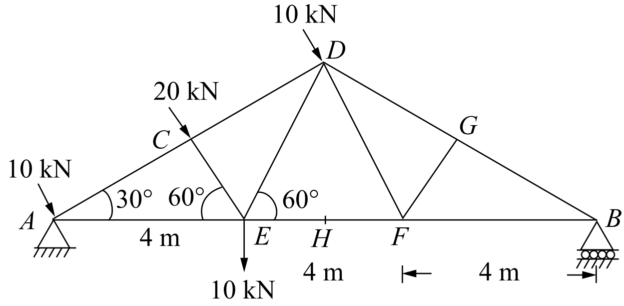


FIG. 2.51

SOLUTION

The length of panel AC, CD are found using geometry.

$$\cos 30 = 6/AD$$

$$AD = 6/\cos 30$$

$$= 6.93 \text{ m}$$

$$\therefore \quad AC = CD = AD/2$$

$$= 3.46 \text{ m}$$

The reactions V_A, H_A and V_B are determined using the equilibrium equations.

$$\sum H = 0;$$

$$10\cos 60 + 20\cos 60 + 10\cos 60 - H_A = 0$$

$$H_A = 20 \text{ kN}$$

$$\sum V = 0;$$

$$V_A + V_B = 10\sin 60 + 20\sin 60 + 10\sin 60 + 10$$

$$V_A + V_B = 44.64 \text{ kN}$$

$$\sum M_A = 0;$$

$$20(3.46) + 10(6.92) + 10(4) - 12V_B = 0$$

$$V_B = 14.87\ KN$$

To determine the forces in the members CD, DE and EF cut a section through these members and for convenience consider the right part of the truss.

$$\sum M_B = 0$$

$$F_{FG} \times GB = 0$$

$$\therefore \quad F_{FG} = 0$$

EXAMPLE 2.21: Determine the forces in the members FE, FD, CD of the truss shown in Fig. 2.54

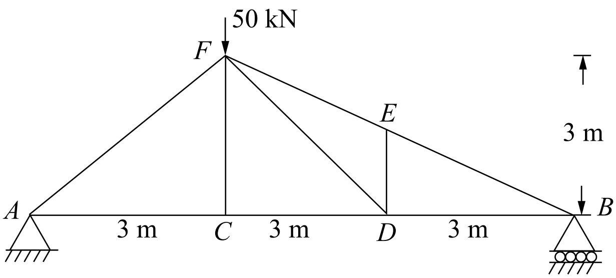


FIG. 2.54

SOLUTION

The reactions are found out by using equilibrium equations.

$$V_A + V_B = 50$$

Taking moment about A;

$$50 \times 3 - 9V_B = 0$$

$$V_B = 16.7 \text{ kN}; \; V_A = 33.3 \text{ kN}$$

To determine the forces in FE, FD and CD, cut a section through these members and consider the right-hand portion of the truss.

$$F_{CD} \times 3 = 16.7 \times 6$$

$$F_{CD} = 33.4 \text{ kN}$$

To determine the force F_{FE}; take moment about D; where other two members of the cut truss, F_{FD} and F_{CD} meet.

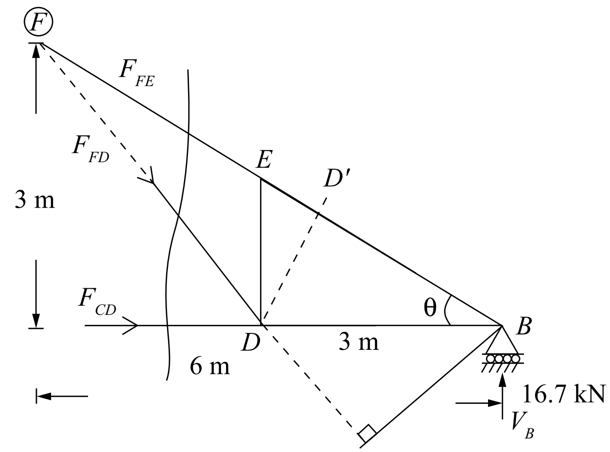


FIG. 2.55

$$\tan\theta = 3/6$$
$$\text{i.e. } \theta = 26^\circ 33' 54''.$$

In $\Delta\, BDD'$

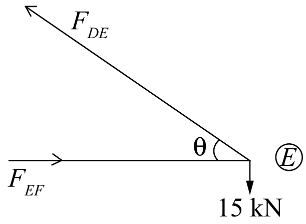
$$\sin\theta = \frac{DD'}{DB}$$
$$\therefore \quad DD' = 3\sin 26^\circ 33' 54''.$$
$$= 1.34 \text{ m}.$$
$$F_{EB} \times 1.34 = 16.7 \times 3$$
$$\boxed{F_{EB} = 37.4 \text{ kN}}$$

To determine the force in *FD*, take moment about *B* where other two members of the cut section, viz. F_{FE} and F_{CD} meet.

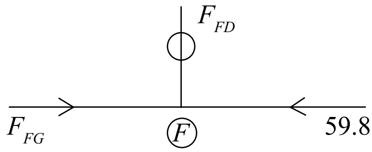
Extend FD downwards to B' such that BB' is perpendicular to the projected line of FD. Considering the $\Delta DBB'$;

$$\sin\theta = \frac{BB'}{3}$$
$$BB' = 3\sin\theta = 3 \times \sin 26^\circ 33' 54''.$$
$$BB' = 1.34 \text{ m}$$

$$\sum M_B = 0; \qquad F_{FD} \times BB' = 0$$
$$F_{FD} = 0$$



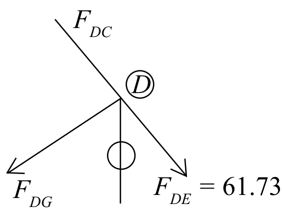
Consider Joint F



At joint F, there is no vertical force acting. Hence, resolving the forces in the vertical direction $F_{FD} = 0$. Resolving the forces in the horizontal direction,

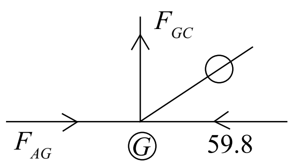
$$\sum H = 0; \quad F_{FG} - 59.8 = 0$$
$$F_{FG} = 59.8 \text{ kN}$$

Consider Joint D



At joint D; resolving the forces perpendicular to the line CDE, $F_{DG} = 0$ as the forces F_{DC} and F_{DE} cannot give components in the perpendicular direction. Resolving the forces along the line of forces. F_{DE} the force F_{DC} is obtained as $F_{DC} = 61.73$ kN.

Joint G



Resolving the forces in the vertical direction at joint G; $F_{GC} = 0$ and resolving the forces in the horizontal direction $F_{AG} = 59.8$ kN.

EXAMPLE 2.23: A truss of 10 m span is loaded as shown in Fig. 2.58 find the forces in the members of the truss by using method of sections.

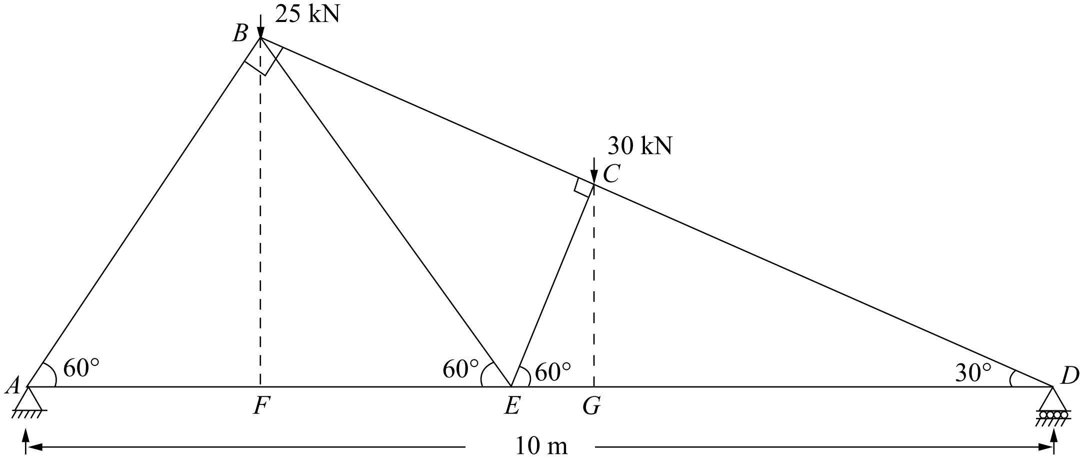


FIG. 2.58

SOLUTION

$$\sum V = 0$$

$$V_A + V_D = 25 + 30$$

$$V_A + V_D = 55 \text{ kN}$$

$$\sum M_A = 0$$

$$25 \times 5 \cos 60 + 30 \times 6.25 - 10 V_D = 0$$

$$V_D = 25 \text{ kN}$$

$$V_A = 30 \text{ kN}$$

$$\sin 30 = \frac{AB}{10}$$

$$AB = 10 \sin 30$$

$$\boxed{AB = 5 \text{ m}}$$

$$\cos 30 = \frac{BD}{10}$$

$$\boxed{BD = 8.66 \text{ m}}$$

$$BC = CD = 4.33 \text{ m}$$

$$\boxed{AF = 5 \cos 60}$$

$$AG = 10 - 4.33 \cos 60$$

$$\boxed{AG = 6.25 \text{ m}}$$

In ΔCGD

$$\sin 30 = \frac{CG}{CD}$$

$$CG = CD \sin 30 = 4.33 \sin 30 = 2.165 \text{ m}$$

In ΔCGE;

$$\sin 60 = \frac{CG}{CE}$$
$$CE = CG/\sin 60 = 2.165/\sin 60$$
$$CE = 2.5 \text{ m}$$

To determine the force in *BC, CE* and *ED* cut a section through these members and consider the right portion of the truss.

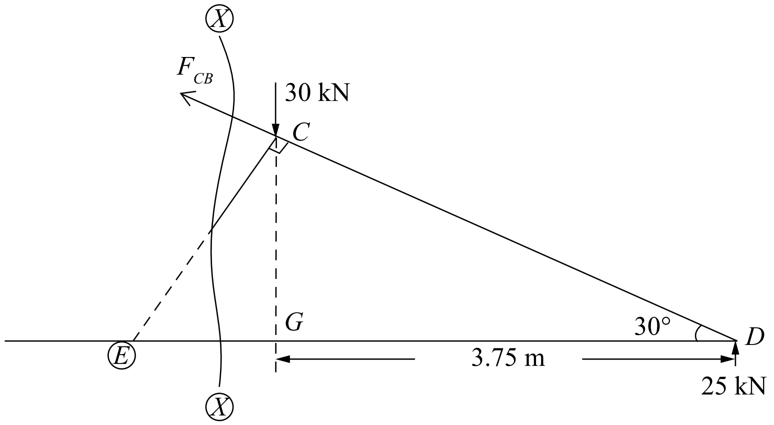


FIG. 2.59

***F_{CB}* is obtained by taking moment about *E* where other two members of the cut truss meet.**

$$30 \times 1.25 - 25 \times 5 - F_{CB} \times 2.5 = 0$$
$$F_{CB} = -35 \text{ kN}$$

-ve sign indicates the force F_{CB} is compressive.
To determine the force in *DE*, take moment of the forces of the cut truss about *C*.

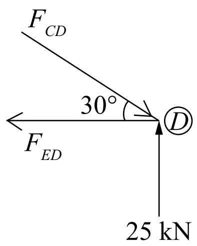
$$F_{DE} \times CG = 25 \times 3.75$$
$$F_{DE} \times 2.165 = 25 \times 3.75$$
$$F_{DE} = 43.3 \text{ kN}$$

To determine the force in F_{CE}, take moment about *D*.

$$30 \times 3.75 = F_{CE} \times 4.33$$
$$F_{CE} = 26 \text{ kN}.$$

To determine the forces in *BE* and *AE*, cut a section through the three members, viz. *BC, BE* and *AE* and consider the left portion of the truss.

Joint D



$$\sum V = 0$$

$$25 = F_{CD}\sin 30$$

$$F_{CD} = 50 \text{ kN}$$

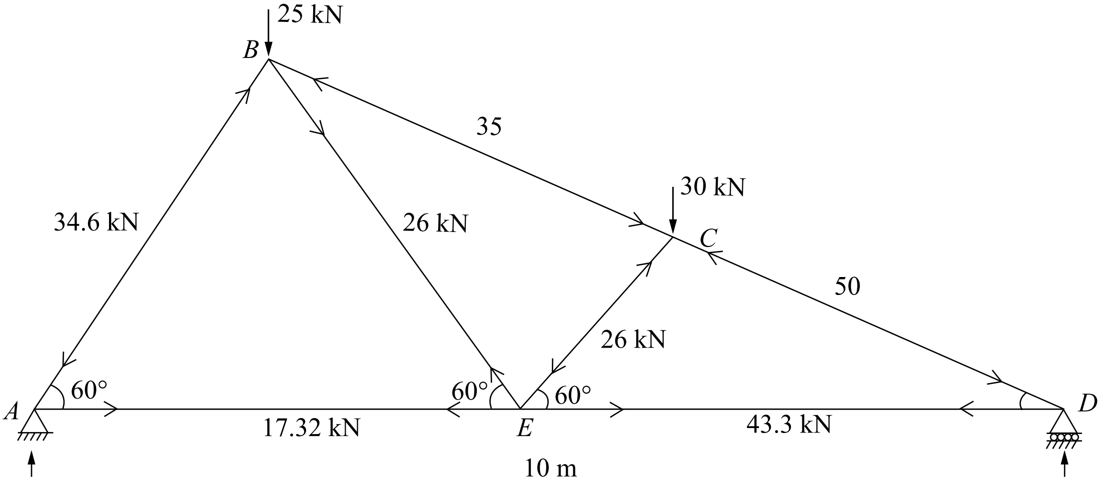


FIG. 2.61

EXAMPLE 2.24: Figure 2.62 shows a pinjointed truss supported by a hinge at *A* and roller at *G*. Determine the force in each of the five members meeting at joint *B*.

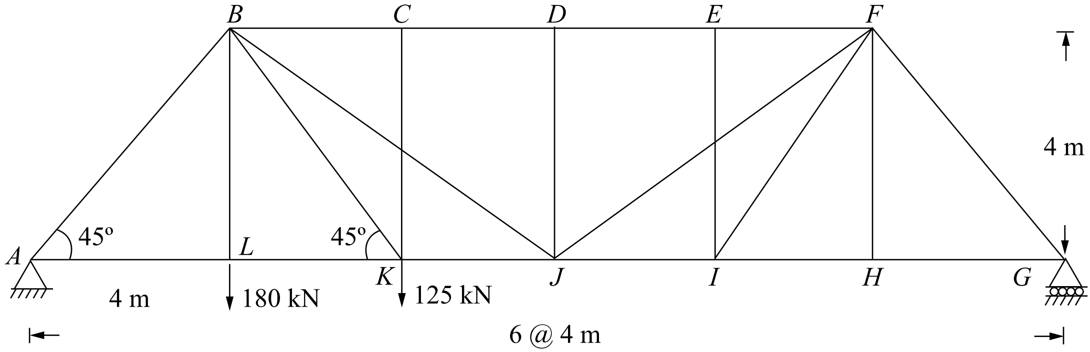


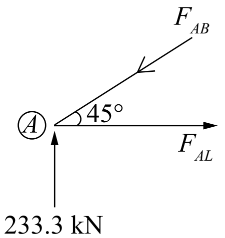
FIG. 2.62

The reactions at A and G are found by using vertical equilibrium equations and moment equilibrium.

$$\begin{aligned}
\sum V = 0; \quad & V_A + V_G = 180 + 125 \\
& V_A + V_G = 305 \\
\sum M_A = 0; \quad & 180 \times 4 + 125 \times 8 - 24V_G = 0 \\
& V_G = 71.7\ \text{kN} \\
& V_A = 233.3\ \text{kN}
\end{aligned}$$

At the joint B, there are five members meeting, viz. BA, BL, BK, BJ and BC. The method of obtaining the forces is by using method of joints as well as method of sections. The forces BA and BL are obtained by using method of joints at A and L. The force in BJ is determined by method of sections. Then the member forces BC and BK are obtained by resolution forces at the joint B.

Joint A



Resolving the forces vertically

$$F_{AB} \sin 45 = 233.3$$

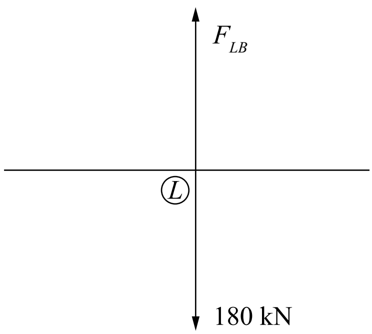
$$\boxed{F_{AB} = 329.9\ \text{kN}}$$

Resolving the forces horizontally

$$F_{AL} = F_{AB} \cos\ 45$$

$$\boxed{F_{AL} = 233.3\ \text{kN}}$$

Joint L



Resolving the forces vertically

$$F_{LB} - 180 = 0$$

$$\boxed{F_{LB} = 180\ \text{kN}}$$

To determine the force in *BJ*, cut a section through *CD, BJ* and *KJ*.

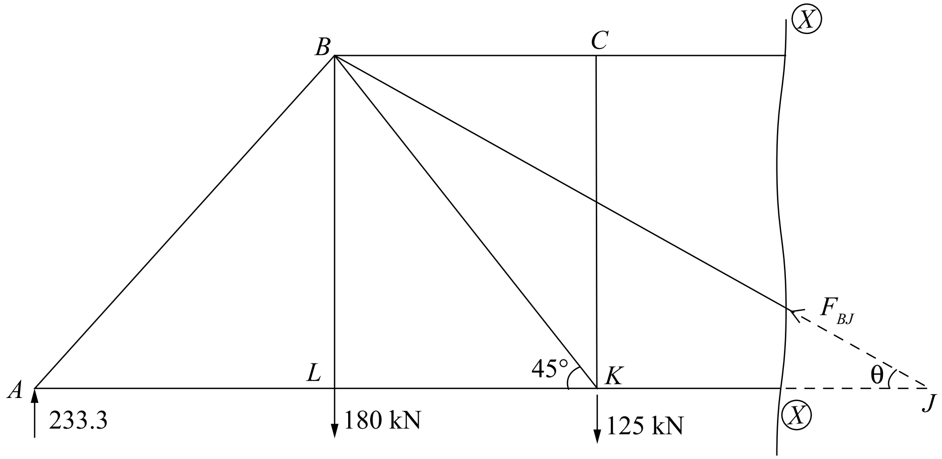


FIG. 2.63

$$\tan\theta = 4/8$$
$$\sin\theta = 0.4472$$

In the cut truss, two cut members, viz. *CD* and *KJ* are horizontal. They do not give any vertical component of the force. Hence to maintain equilibrium in the vertical direction, the force F_{BJ} which is indirect at an angle θ should give a vertical component to balance the effect due to the vertical reaction at *A* and the applied downward loads. Hence resolving vertically.

$$F_{BJ}\sin\theta + 233.3 - 180 - 125 = 0.$$
$$F_{BJ} = 160.33 \text{ kN}$$

Using method of joints at joint *B* (as shown in Fig. 2.64)

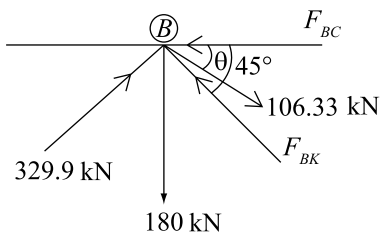


FIG. 2.64

$$\sum H = 0; \quad 329.99\ \cos 45 - F_{BC} + 160.33\ \cos\theta - F_{BK}\ \cos 45 = 0$$
$$F_{BC} + 0.707\ F_{BK} = 376.74$$

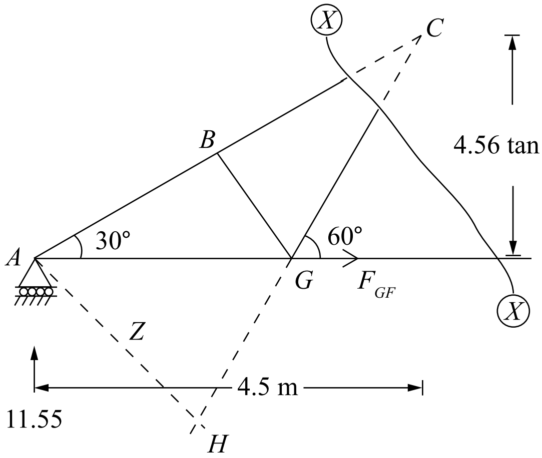


FIG. 2.66

$$\boxed{\therefore \quad F_{GC} = 0}$$

To determine the force in *BC*, take moment about *G*.

$$F_{BC} \times BG = 11.55 \times 3$$
$$F_{BC} \times (3 \sin 30) = 11.55 \times 3$$

$$\boxed{F_{BC} = 23.1 \text{ kN}}$$

2.6 METHOD OF TENSION COEFFICIENTS

The method of tension coefficients was developed by Southwell (1920) and is applicable for plane and space frames. Contemporily this was developed by Muller Breslau independently.

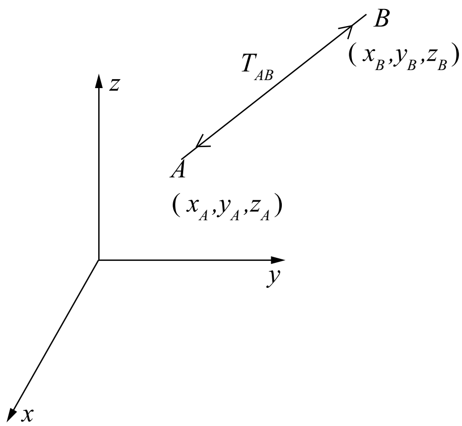
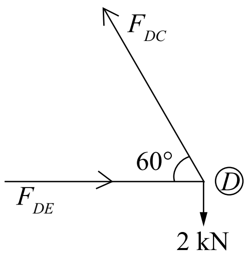


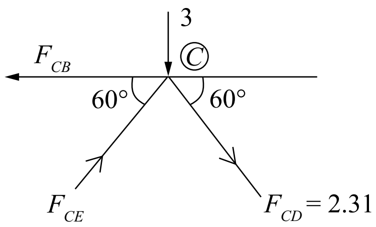
FIG. 2.67

Joint D



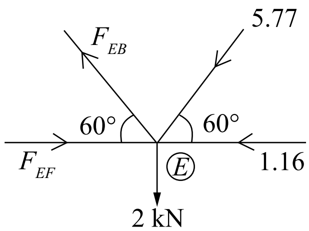
$$\sum V = 0$$
$$F_{DC}\sin 60 - 2 = 0$$
$$F_{DC} = \frac{2}{\sin 60} = 2.31$$
$$\sum H = 0$$
$$F_{DE} - F_{DC}\cos 60 = 0$$
$$F_{DE} = 2.31\cos 60$$
$$F_{DE} = 1.16 \text{ kN}$$

Joint C



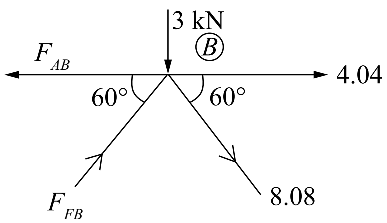
$$\sum V = 0$$
$$-3 - 2.31\sin 60 + F_{CE}\sin 60 = 0$$
$$F_{CE} = 5.77 \text{ kN}$$
$$\sum H = 0$$
$$-F_{CB} + 5.77\cos\ 60 + F_{CD}\cos\ 60 = 0$$
$$F_{CB} = 4.04 \text{ kN}$$

Joint E



$$\sum V = 0$$
$$F_{EB}\sin 60 - 5.77\sin 60 - 2 = 0$$
$$F_{EB} = 8.08 \text{ kN}$$
$$\sum H = 0$$
$$F_{EF} - 8.08\cos 60 - 5.77\cos 60 - 1.16 = 0$$
$$F_{EF} = 8.08 \text{ kN}$$

Joint B



$$\sum V = 0$$
$$-3 - 8.08\sin 60 + F_{FB}\sin 60 = 0$$
$$F_{FB} = 11.54 \text{ kN}$$
$$\sum H = 0$$
$$4.04 + 8.08\cos 60 + 11.54\cos 60 - F_{AB} = 0$$
$$F_{AB} = 13.85 \text{ kN}$$

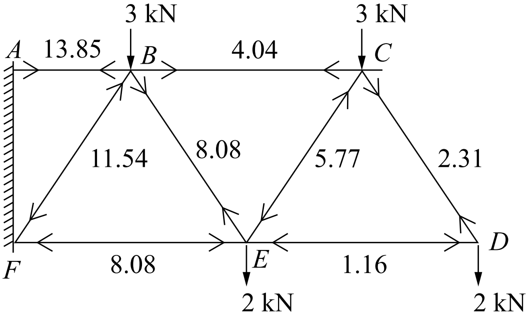


FIG. 2.69

Using tension coefficient method

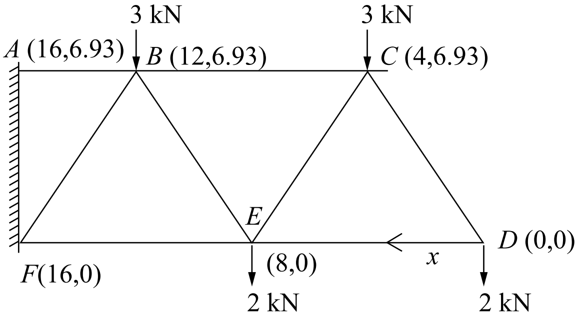


FIG. 2.70

At Joint D

$$\sum H = 0, \qquad t_{DC}(x_C - x_D) + t_{DE}(x_E - x_D) = 0$$
$$t_{DC}(4-0) + t_{DE}(8-0) = 0$$
$$4t_{DC} + 8t_{DE} = 0$$

$$\sum V = 0, \qquad t_{DC}(y_C - y_D) + t_{DE}(y_E - y_D) - 2 = 0$$
$$6.93t_{DC} + 0 = 2$$

$$\boxed{\begin{aligned} \therefore \quad t_{DC} &= 0.2886 \\ t_{DE} &= -0.1443 \end{aligned}}$$

Joint C

$$\sum H = 0; \qquad t_{CD}(x_D - x_C) + t_{CE}(x_E - x_C) + t_{CB}(x_B - x_C) = 0$$
$$0.2886(0-4) + t_{CE}(8-4) + t_{CB}(12-4) = 0$$
$$4\, t_{CE} + 8\, t_{CB} = 1.1544$$

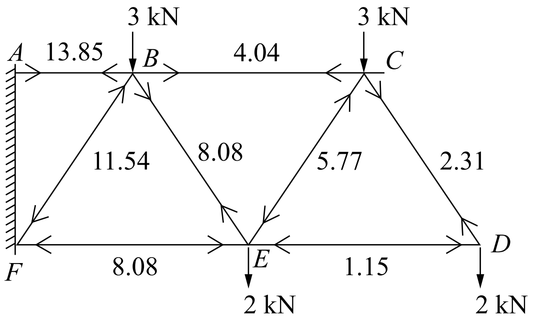


FIG. 2.71

EXAMPLE 2.27: Analyse the space truss shown in Fig. 2.72 using tension coefficient method. (Anna Univ, 2009).

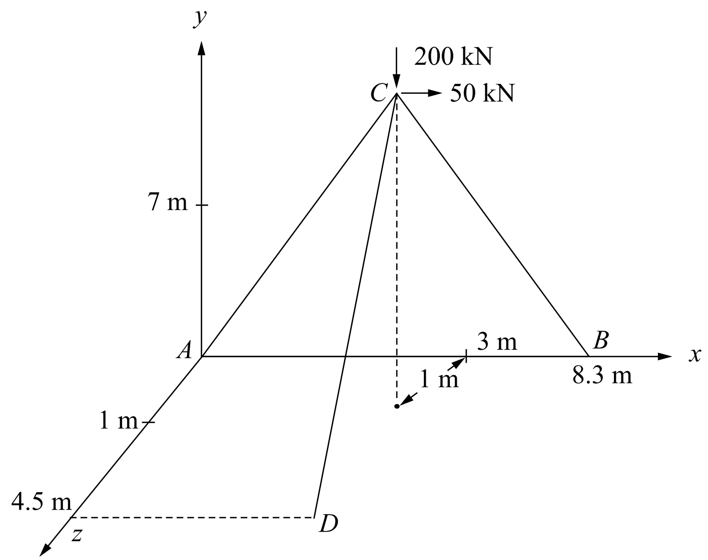


FIG. 2.72

Joint	x	y	z
A	0	0	0
B	8.3	0	0
C	3.0	7	1
D	3	0	4.5

Resolving the forces at joint *C* in *x* direction

$$t_{CA}(x_A - x_C) + t_{CD}(x_D - x_C) + t_{CB}(x_B - x_C) + 50 = 0$$
$$t_{CA}(0-3) + t_{CD}(3-3) + t_{CB}(8.3-3) + 50 = 0$$
$$-3t_{CA} + 5.3t_{CB} = -50 \qquad (1)$$

Resolving the forces at joint *C* in *y* direction

$$t_{CA}(y_A - y_C) + t_{CD}(y_D - y_C) + t_{CB}(y_B - y_C) - 200 = 0$$
$$t_{CA}(0-7) + t_{CD}(0-7) + t_{CB}(0-7) = 200$$
$$-7t_{CA} - 7t_{CB} - 7t_{CD} = 200 \qquad (2)$$

Resolving the forces at the joint *C* in *z* direction

$$t_{CA}(z_A - z_C) + t_{CD}(z_D - z_C) + t_{CB}(z_B - z_C) = 0$$
$$t_{CA}(0-1) + t_{CD}(4.5-1) + t_{CB}(0-1) = 0$$
$$-t_{CA} + 3.5t_{CD} - t_{CB} = 0 \qquad (3)$$

$$\therefore \quad \begin{bmatrix} -3 & 5.3 & 0 \\ -7 & -7.0 & -7.0 \\ -1 & -1.0 & +3.5 \end{bmatrix} \begin{Bmatrix} t_{CA} \\ t_{CB} \\ t_{CD} \end{Bmatrix} = \begin{bmatrix} -50 \\ 200 \\ 0 \end{bmatrix}$$

$$t_{CA} = -8.166 \quad t_{CB} = -14.056 \quad t_{CD} = -6.349$$

$$L_{CA} = \sqrt{(3-0)^2 + (7-0)^2 + (1-0)^2} = 7.68 \text{ m.}$$

$$L_{CB} = \sqrt{(3-8.3)^2 + (7-0)^2 + (1-0)^2} = 4.68 \text{ m.}$$

$$L_{CD} = \sqrt{(3-3)^2 + (7-0)^2 + (1-4.5)^2} = 7.83 \text{ m.}$$

$$T_{CA} = t_{CA}\, L_{CA} = -62.71 \quad \text{(Compression)}$$
$$T_{CB} = t_{CB}\, L_{CB} = -65.78 \quad \text{(Compression)}$$
$$T_{CD} = t_{CD}\, L_{CD} = -49.71 \quad \text{(Compression)}$$

EXAMPLE 2.28: Figure 2.73 shows an elevation and plan of tripod having legs of unequal lengths resting without slipping on a sloping plane, if the pinjointed apex carries a vertical load of 100 kN. Calculate the force in each leg.

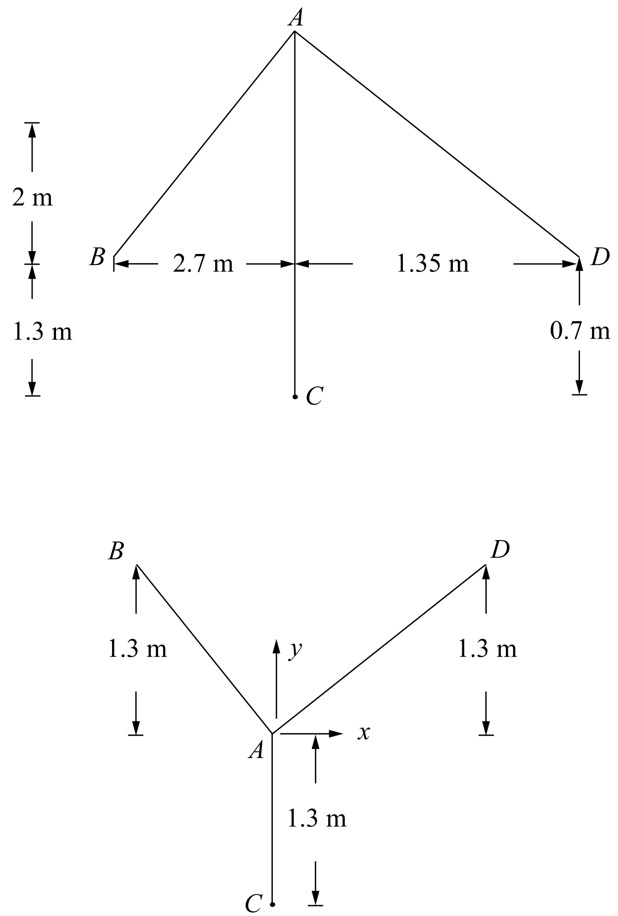


FIG. 2.73

Joint	x	y	z
A	0	0	0
B	−2.7	+1.3	−2
C	0	−1.3	−3.3
D	1.35	+1.3	1.6

Resolving the forces at the joint A is x direction

$$t_{AB}(x_B - x_A) + t_{AC}(x_C - x_A) + t_{AD}(x_D - x_A) = 0$$
$$t_{AB}(-2.7 - 0) + t_{AC}(0 - 0) + t_{AD}(1.35 - 0) = 0$$
$$-2.7t_{AB} + 1.35t_{AD} = 0$$

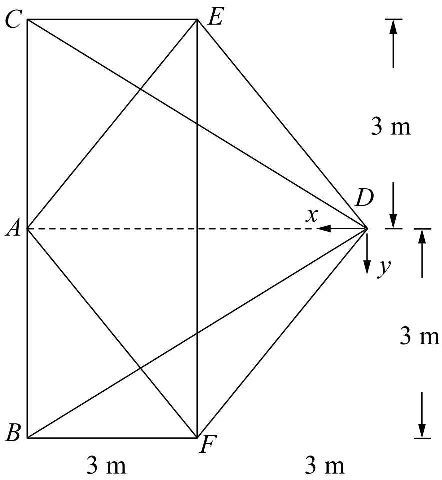


FIG. 2.74

Joint	x	y	z
A	+6	0	+3
B	+6	+3	0
C	+6	−3	0
D	0	0	0
E	+3	−3	+3
F	+3	+3	+3

Resolving all the forces meeting at joint *D* in *z* direction as

$$t_{DE}(z_E - z_D) + t_{DC}(z_C - z_D) + t_{DB}(z_B - z_D) + t_{DF}(z_F - z_D) - 75 = 0$$
$$t_{DE}(3-0) + t_{DC}(0-0) + t_{DB}(0-0) + t_{DF}(3-0) = 75$$
$$3t_{DE} + 3t_{DF} = 75$$
$$t_{DE} + t_{DF} = 25$$

As the structure and loading are symmetrical

$$t_{DE} = t_{DF}$$

and hence

$$t_{DE} = 12.5$$

Resolving all the forces in the directions of *y* axis at the joint *E*

$$\begin{aligned}
t_{EC}(y_C - y_E) + t_{EA}(y_A - y_E) + t_{EF}(y_F - y_E) + t_{ED}(y_D - y_E) &= 0 \\
t_{EC}(-3+3) + t_{EA}(0+3) + t_{EF}(3+3) + t_{ED}(0-3) &= 0 \\
3t_{EA} + 6t_{EF} - 3 \times 12.5 &= 0 \\
3t_{EA} + 6t_{EF} &= 37.5
\end{aligned}$$

Resolving all the forces in the direction of *z*-axis

$$\begin{aligned}
t_{EC}(z_C - z_E) + t_{EA}(z_A - z_E) + t_{EF}(z_F - z_E) + t_{ED}(z_D - z_E) &= 0 \\
t_{EC}(0-3) + t_{EA}(3-3) + t_{EF}(3-3) + t_{ED}(0-3) &= 0 \\
-3t_{EC} - 3t_{ED} &= 0 \\
t_{EC} &= t_{ED}
\end{aligned}$$

Resolving all the forces in the direction of *x*-axis

$$\begin{aligned}
t_{EC}(x_C - x_E) + t_{EA}(x_A - x_E) + t_{EF}(x_F - x_E) + t_{ED}(x_D - x_E) &= 0 \\
t_{EC}(6-3) + t_{EA}(6-3) + t_{EF}(3-3) + t_{ED}(0-3) &= 0 \\
3t_{EC} + 3t_{EA} - 3t_{DE} &= 0 \\
3(-12.5) + 3t_{EA} - 3 \times 12.5 &= 0 \\
3t_{EA} &= 75
\end{aligned}$$

$$\boxed{t_{EA} = 25}$$

Substituting in the equation:

$$3 \times 25 + 6\, t_{EF} = 37.5$$

$$\boxed{t_{EF} = -6.25}$$

$$\begin{aligned}
L_{DE} &= \sqrt{(x_E - x_D)^2 + (y_E - y_D)^2 + (Z_E - Z_D)^2} \\
&= \sqrt{3^2 + 3^2 + 3^2} = 5.2\text{m} \\
L_{EF} &= \sqrt{(x_F - x_E)^2 + (y_F - y_E)^2 + (Z_F - Z_E)^2} \\
&= \sqrt{(3-3)^2 + (3+3)^2 + (3-3)^2} = 6\text{m}
\end{aligned}$$

$$\therefore \quad T_{DE} = t_{DE} L_{DE} = 12.5 \times 5.2 = 65 \text{ kN (Tensile)}$$

$$T_{EF} = t_{EF} L_{EF} = -6.25 \times 6 = -37.5 \text{ kN (Compressive)}$$

EXAMPLE 2.30: A space frame shown in figure is supported at A, B, C and D in a horizontal plane through ball joints. The member EF is horizontal and is at a height of 3 m above the base. The loads at the joints E and F shown in Fig. 2.75 act in a horizontal plane. Find the forces in all of the members of the frame. (Anna Univ, 2004)

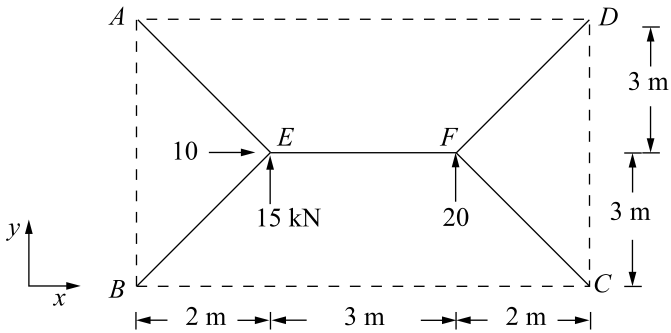


FIG. 2.75

Joint	x	y	z
A	0	6	0
B	0	0	0
C	7	0	0
D	7	6	0
E	2	3	3
F	5	3	3

Resolving all the forces at joint E in x direction

$$t_{EA}(x_A - x_E) + t_{EB}(x_B - x_E) + t_{EF}(x_F - x_E) + 10 = 0$$
$$t_{EA}(0-2) + t_{EB}(0-2) + t_{EF}(5-2) + 10 = 0$$

Resolving all the forces in y direction at joint E

$$t_{EA}(y_A - y_E) + t_{EB}(y_B - y_E) + t_{EF}(y_F - y_E) + 15 = 0$$
$$t_{EA}(6-3) + t_{EB}(0-3) + t_{EF}(3-3) + 15 = 0$$
$$3t_{EA} - 3t_{EB} = -15$$

Resolving all the forces in z direction at joint E

$$t_{EA}(z_A - z_E) + t_{EB}(z_B - z_E) + t_{EF}(z_F - z_E) + 0 = 0$$
$$t_{EA}(0-3) + t_{EB}(0-3) + t_{EF}(3-3) = 0$$
$$-3t_{EA} - 3t_{EB} = 0$$

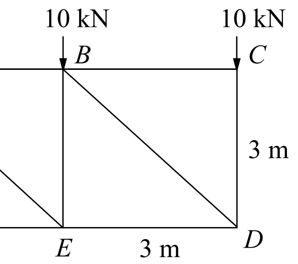
REVIEW QUESTIONS

Remembrance:

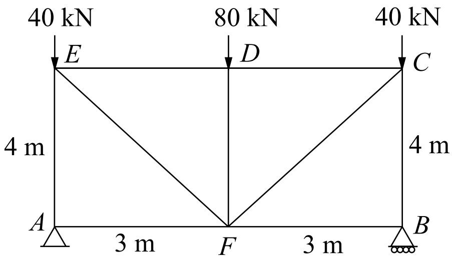
2.1 Define plane and space truss?
2.2 What are the different types of analysis of trusses?
2.3 List the assumptions made in truss analysis?
2.4 Explain the steps involved in method of joints?
2.5 Explain the steps involved in method of sections?
2.6 Explain the method of tension coefficients?
2.7 What type of analysis is used in determining the forces of a space truss?
2.8 For what kind of trusses can be analysed by method of joints and method of sections?
2.9 Define Tension coefficient?
2.10 Which method is preferable to find out the forces in a few members of a truss?
2.11 What is the primary function of a truss?
2.12 What is the minimum numbers of elements to make a simple truss?

Understanding:

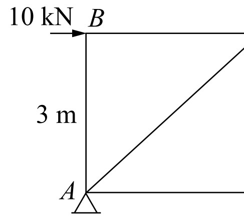
2.1 Distinguish between a simple truss, compound truss and complex truss?
2.2 What kind of stresses developed if the loads are not applied at the joints?
2.3 What are the limitations of method of joints?
2.4 Identify the truss members having zero forces joints?



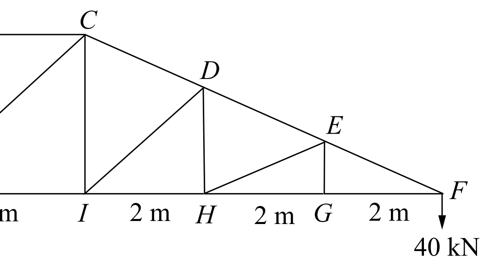
10 kN
10 kN
B
C
3 m
E
3 m
D



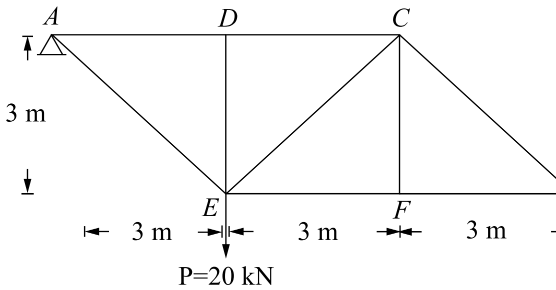
40 kN
80 kN
40 kN
E
D
C
4 m
4 m
A
3 m
F
3 m
B



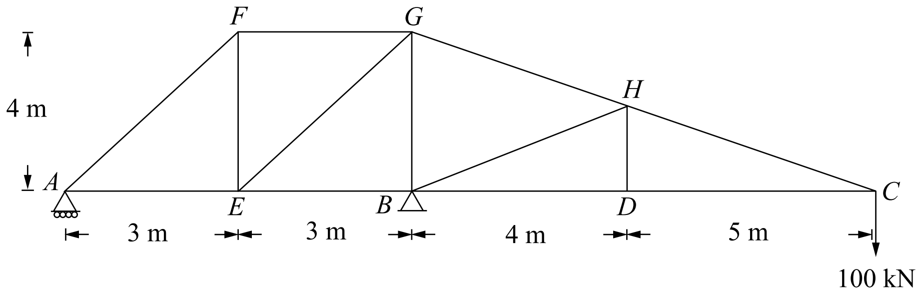
10 kN
B
3 m
A



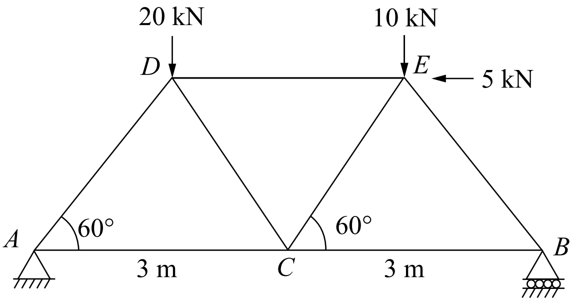
C
D
E
m
I
2 m
H
2 m
G
2 m
F
40 kN



A
D
C
3 m
E
F
3 m
3 m
3 m
P=20 kN

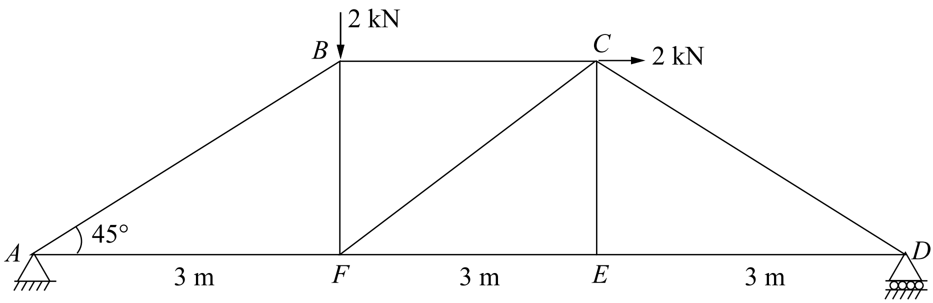


F
G
H
4 m
A
E
B
D
C
3 m
3 m
4 m
5 m
100 kN



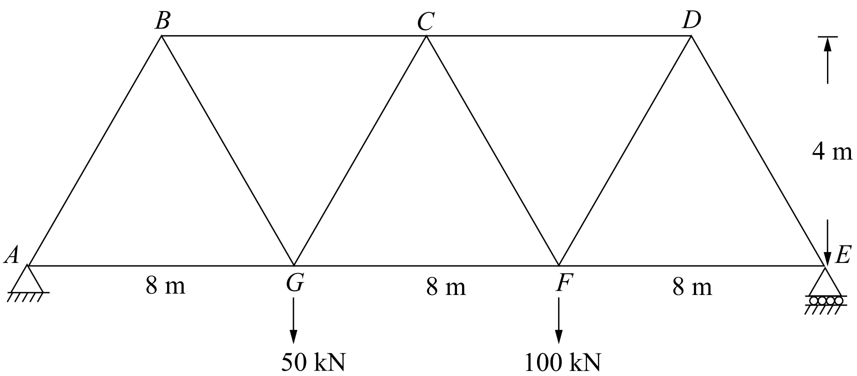
(***Ans*** $AD = -22.75$, $DE = -11.16$, $EB = -11.9$, $AC = 11.38$, $CB = 10.95$, $DC = -0.43$, $EC = -0.43$)

2.4 Determine the forces in all the members of the following truss by method of joints. (VTU, Feb. 2003)



(***Ans*** $AB = -0.95$ kN, $BC = -0.658$, $CD = -1.88$, $DE = +1.33$, $EF = 1.3$ $AF = +2.67$ kN, $BF = -1.33, FC = +1.88$, $CE = 0$)

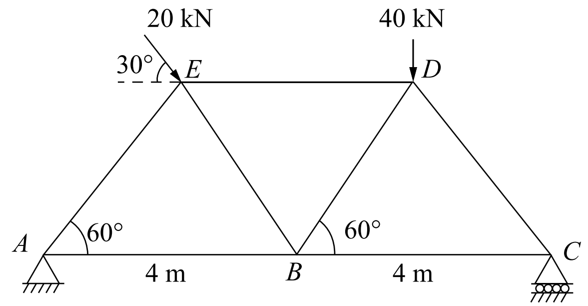
2.5 Determine the forces in all the members by method of joints.



(***Ans*** $AB = -94.3$, $BC = -133.36$, $CD = -166.6$, $DE = -117.8$, $CG = -23.55$, $AG = 66.7$, $GF = 149.93$, $FE = 83.3$, $BG = +94.3$, $CF = 23.55$, $FD = 117.8$ kN)

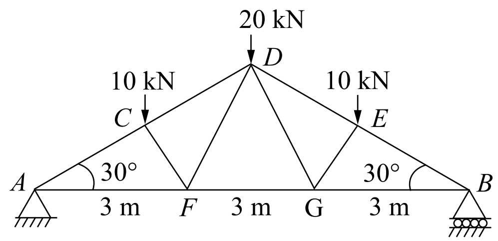
(***Ans*** $AD = +18.75$, $DE = 15.63$, $EF = 15.63$, $CF = -18.75$, $BC = 0$, $AB = 50$, $BD = -15.63$, $BF = 46.88$)

2.9 Determine the magnitude and nature of forces in all the members of the plane truss shown in figure. (Anna Univ. Dec 2008)

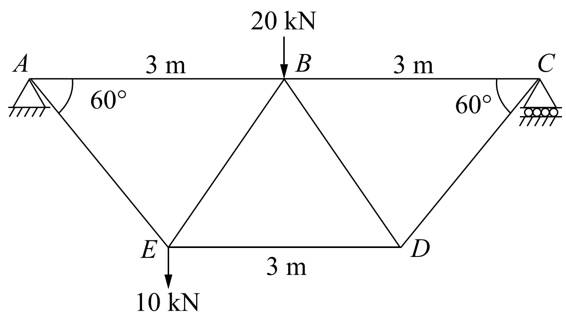


(***Ans*** $AE = -11.55$, $ED = -23.1$, $DC = -46.18$, $AB = 23.1$, $BC = +23.1$, $BE = 0$, $BD = 0$)

2.10 Find the forces in all members of the pinjointed plane truss shown in figure. Use method of joints. (VTU, July 2007)

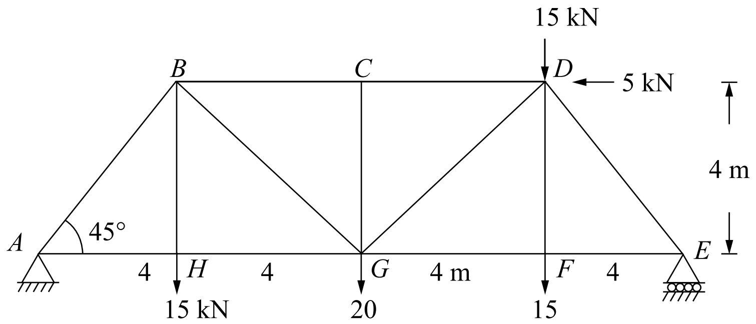


2.11 Find the forces in all members of the truss shown in figure by method of joints. (VTU, Dec 2007)



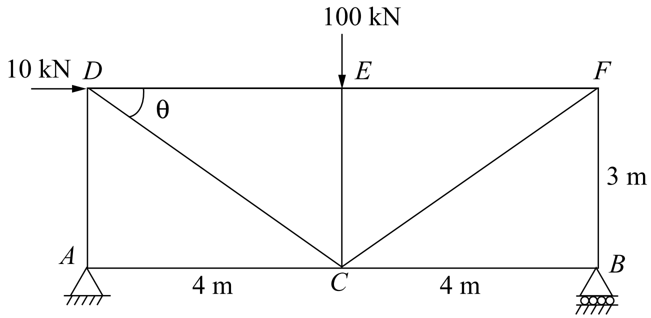
(***Ans*** $AE = 20.21$, $ED = 14.43$, $CD = 14.43$, $AB = -10.11$, $BC = -7.22$, $BE = -8.66$, $BD = -14.43$ kN)

2.12 Determine the forces in the members of the truss by method of joints. Draw a neat sketch showing nature and magnitude of forces. (VTU, 2001)



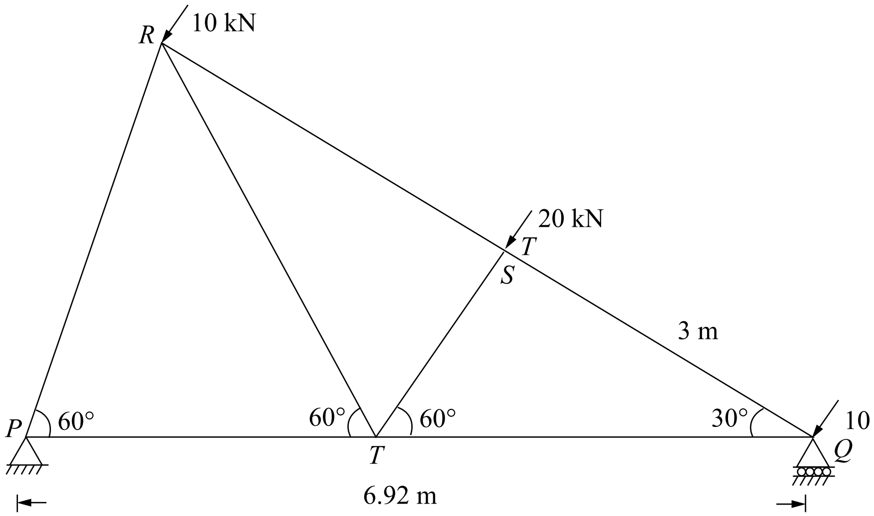
(***Ans*** $AB = -35.36$ kN, $BC = -35$, $DC = -35$, $DE = -35.36$, $AH = -25$, $HG = -25.00$, $GF = -25.00$, $FE = -25.00$, $HB = +15.00$, $CG = 0$, $DF = 15.00$, $BG = 14.14$, $GD = 14.14$)

2.13 Analyse the frame shown in figure by method of joints and tabulate the forces in all the members. (VTU, Feb 2004)



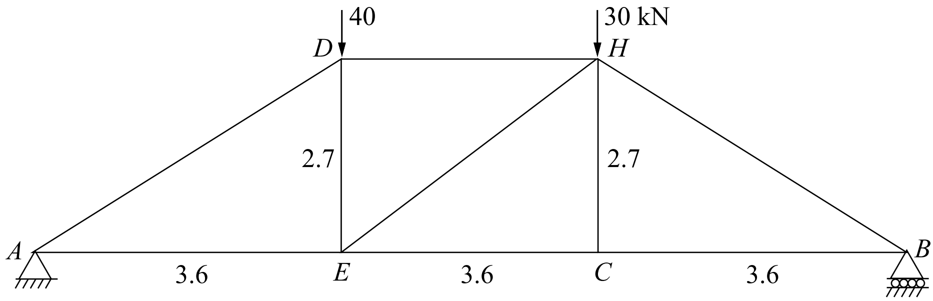
(***Ans*** $AD = -46.25$, $DE = -71.67$, $EF = -71.67$, $BF = -33.75$, $BC = 0$, $CA = 10$, $CD = 77.08$, $CF = 89.58$, $EC = -100$)

2.14 Find the forces in members RS, ST and TQ by method of sections.



(***Ans*** $RS = -1990$ kN, $TQ = -9.96$ kN, $ST = +20$ kN)

2.15 A plane truss is subjected to point loads as shown in figure. Find the forces in the member EH and EC by method of sections. (MSRIT-2009)



(***Ans*** $EH = 5.5$ kN; $EC = 44.4$ kN)

2.16 Figure shows a pin jointed frame which is hinged to the foundation at A and is resting on rollers at B. Determine the magnitude and nature of force in BC.