CHAPTER 2

Control System Representation

2.1 INTRODUCTION

In previous chapter, physical control systems were discussed like servosystems, water level control in a tank, temperature control system, etc. These physical systems were basically the collection of physical objects connected together to fulfill an objective. These physical systems can also be represented mathematically in the following forms:

(a) Transfer function representation
(b) Block diagram representation
(c) Signal flow graph representation

2.2 TRANSFER FUNCTION REPRESENTATION

Transfer function of a linear time invariant (LTI) system is defined as the ratio of Laplace transform of input variable of the system to the Laplace transform of output variable of the system, with the assumption that all the initial conditions are kept at zero.

Consider a block diagram representation of the simplest open loop system:

For Fig. 2.1,

$$T(s) = G(s) = \frac{C(s)}{R(s)}$$

where, $T(s) = G(s)$ = open loop transfer function
$C(s)$ = Laplace transform of output variable
$R(s)$ = Laplace transform of input variable

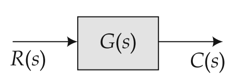


Fig. 2.1

Transfer function gives only the relationship between input and output variables of the system and does not tell anything about the internal state of the system.

poles of the transfer function. So, p_1, p_2, …, p_n are poles of the transfer function. Poles can be found out by solving characteristic equation for variable s. They are represented by × (small cross) on pole-zero diagram.

(iv) **Type of the system:** The number of poles that lie on origin i.e., the number of values of s equal to zero in characteristic equation gives the type of the system.

If either poles or zeros coincide, then they are called multiple or repeated poles or multiple or repeated zeros respectively, otherwise, they are called simple poles or simple zeros.

Example 2.1 For the transfer function given:

$$T(s) = G(s) = \frac{s+5}{s^2(s+6)(s^2+25)}$$

Find characteristics equation, pole-zero diagram, order and type of the system.

Solution:

Step 1: Characteristics equation is:

$$s^2(s+6)(s^2+25)=0 \Rightarrow (s^3+6s^2)(s^2+25)=0 \Rightarrow s^5+6s^4+25s^3+150s^2=0$$

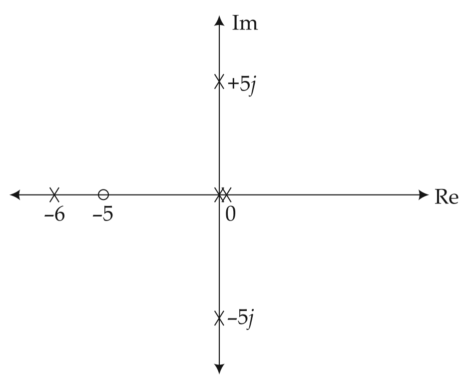


Fig. 2.2

Step 2: Poles of the transfer function are at s = 0, 0, –6, ± 5j.

Step 3: Zero of the transfer function is at s = –5.

Step 4: Pole-zero diagram is shown in Fig. 2.2.

Step 5: Order of the system is 5.

Step 6: Type of the system is 2.

2.2.2 Transfer Function of Electric Circuits

Number of examples using different electric circuits are solved:

Solution:

Step 1: Applying KVL in loop 1,

$$v_i(t) - i_1(t) - 3\int [i_1(t) - i_2(t)]\, dt = 0$$

Taking Laplace transform on both sides,

$$V_i(s) - I_1(s) - 3\,\frac{I_1(s)}{s} + 3\,\frac{I_2(s)}{s} = 0 \tag{2.8}$$

Step 2: Applying KVL in loop 2,

$$-5i_2(t) - \frac{di_2(t)}{dt} - 3\int [i_2(t) - i_1(t)]\, dt = 0$$

Taking Laplace transform on both sides,

$$-5I_2(s) - sI_2(s) - 3\,\frac{I_2(s)}{s} + 3\,\frac{I_1(s)}{s} = 0$$

$$\Rightarrow \quad I_1(s) = I_2(s)\,\frac{s}{3}\left[5 + s + \frac{3}{s}\right] = I_2(s)\left[\frac{s^2}{3} + \frac{5s}{3} + 1\right] \tag{2.9}$$

Step 3: Also, in loop 2,

$$v_o(t) = 1\,\frac{di_2(t)}{dt}$$

Taking Laplace transform on both sides,

$$V_o(s) = sI_2(s) \tag{2.10}$$

Step 4: Putting equation (2.9) in (2.8),

$$V_i(s) = \frac{-3I_2(s)}{s} + I_2(s)\left[\frac{s^2}{3} + \frac{5s}{3} + 1\right]\left(1 + \frac{3}{s}\right)$$

$$= I_2(s)\left[-\frac{3}{s} + \frac{s^2}{3} + \frac{5s}{3} + 1 + s + 5 + \frac{3}{s}\right]$$

$$= I_2(s)\left[\frac{s^2}{3} + \frac{8s}{3} + 6\right] \tag{2.11}$$

Step 5: Dividing equation (2.10) by (2.11),

$$\frac{V_o(s)}{V_i(s)} = \frac{s}{\frac{s^2}{3} + \frac{8s}{3} + 6} = \frac{3s}{s^2 + 8s + 18} \qquad \textbf{(Answer)}$$

Example 2.4 Obtain transfer function of the circuit given in Fig. 2.5.

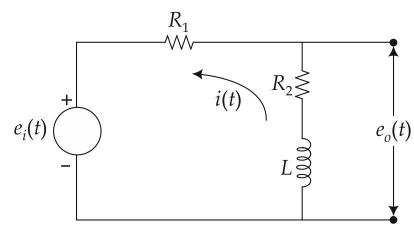


Fig. 2.5

Solution:

Step 1: Applying KVL in loop 1,

$$e_i(t) - R_1\, i(t) - R_2\, i(t) - L\frac{di(t)}{dt} = 0$$

Applying Laplace transform on both sides,

$$E_i(s) - R_1 I(s) - R_2 I(s) - LsI(s) = 0$$

$$\Rightarrow \quad E_i(s) = I(s)\left[R_1 + R_2 + sL\right] \quad (2.12)$$

Step 2: In loop 2,

$$e_o(t) = R_2 i\,(t) + L\frac{di(t)}{dt}$$

Taking Laplace transform on both sides,

$$E_o(s) = R_2\; I(s) + sLI(s)$$

$$\Rightarrow \quad E_o(s) = I(s)\left[R_2 + sL\right] \quad (2.13)$$

Step 3: Dividing equation (2.13) by (2.12),

$$\frac{E_o(s)}{E_i(s)} = \frac{R_2 + sL}{R_1 + R_2 + sL} \quad \textbf{(Answer)}$$

2.2.3 Transfer Function of a Closed Loop System

For a negative feedback control system, shown in Fig. 2.6, the transfer function or the relationship between input and output variables is derived as follows:

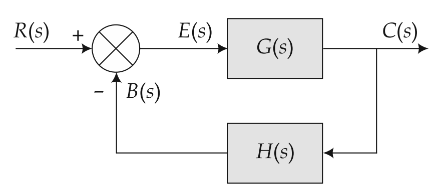


Fig. 2.6

The terms used in Fig. 2.6 are given below:

$R(s)$ = Reference Input Signal
$E(s)$ = Actuating or Error Signal
$C(s)$ = Output Signal
$B(s)$ = Feedback Signal
$G(s)$ = Open Loop Transfer Function or Forward Path Transfer Function = $C(s)/E(s)$
$H(s)$ = Feedback Path Transfer Function = $B(s)/C(s)$
$T(s)$ = Overall Transfer Function of the System or Closed Loop Transfer Function = $C(s)/R(s)$

From Fig. 2.6,
$$E(s) = R(s) - B(s) \tag{2.14}$$
$$= R(s) - C(s)\, H(s)$$
and
$$C(s) = E(s)\, G(s) \tag{2.15}$$
Putting (2.14) in (2.15),
$$C(s) = [R(s) - C(s)\, H(s)]\, G(s)$$
$$\Rightarrow \quad C(s)\, [1 + H(s)\, G(s)] = R(s)\, G(s)$$
$$\Rightarrow \quad \frac{C(s)}{R(s)} = T(s) = \frac{G(s)}{1 + H(s)\, G(s)} \tag{2.16}$$

Equation (2.16) gives the transfer function of a closed loop system.

The transfer function obtained in equation (2.16) is obtained for negative feedback system, also known as **degenerative system.**

If the output of system is fed back with positive sign, then transfer function is changed to
$$\frac{C(s)}{R(s)} = T(s) = \frac{G(s)}{1 - H(s)\, G(s)}$$
and the system is called **regenerative system.**

In general, $T(s) = \dfrac{G(s)}{1 \pm H(s)\, G(s)}$, + sign for negative feedback
– sign for positive feedback

2.3 BLOCK DIAGRAM REPRESENTATION AND REDUCTION

Another way of representing a physical control system mathematically is block diagram representation. As discussed earlier, the input-output behavior of a linear system is described by its transfer function as,
$$T(s) = C(s)/R(s),$$
where $C(s)$ = Laplace transform of output variable
$R(s)$ = Laplace transform of input variable

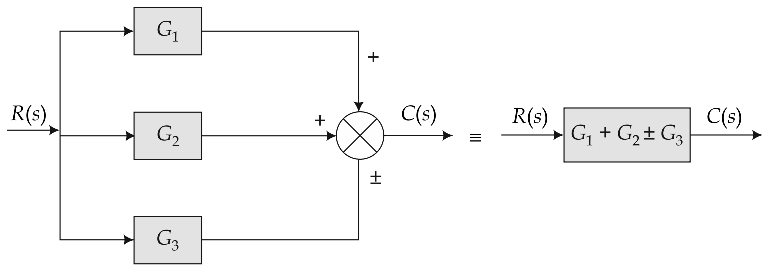


Fig. 2.10

Rule 3: Moving a Take-off Point After the Block

If take-off point is to be moved after the block, then the block with reciprocal of original transfer function appears in the signal of take-off point, as shown in Fig. 2.11.

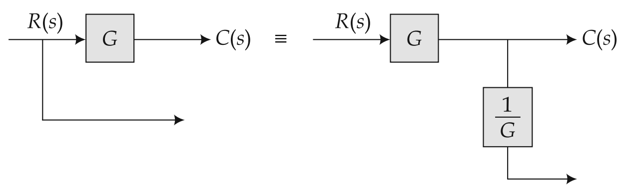


Fig. 2.11

Rule 4: Moving a Take-off Point Before the Block

If take-off point is to be moved before the block, then the block with same transfer function appears in the signal of take-off point, as shown in Fig. 2.12.

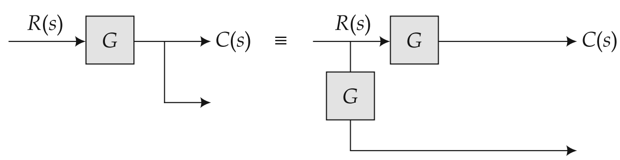


Fig. 2.12

Rule 5: Moving a Summing Point After the Block

If summing point is to be moved after the block, then the block with same transfer function appears in the signal of summing point, as shown in Fig. 2.13.

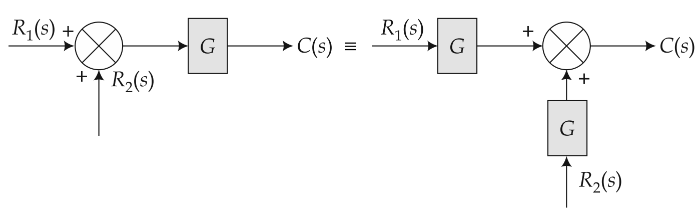


Fig. 2.13

Rule 6: Moving a Summing Point Before the Block

If summing point is to be moved before the block, then the block with reciprocal of original transfer function appears in the signal of summing point, as shown in Fig. 2.14.

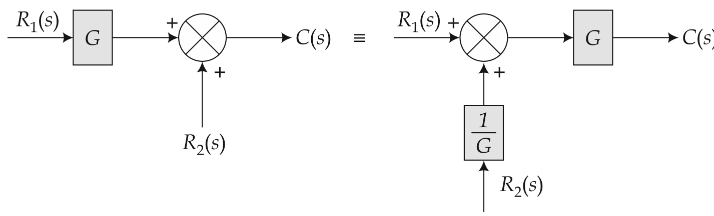


Fig. 2.14

Rule 7: Eliminating a Feedback Loop

The formula derived in section 2.2.3 is used to eliminate the feedback loop, as shown in Fig. 2.15.

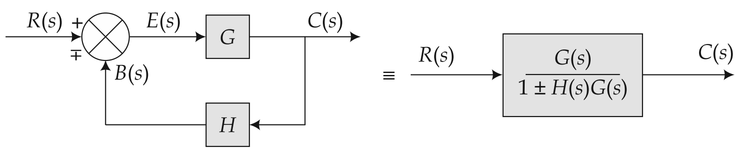
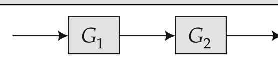
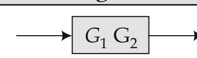
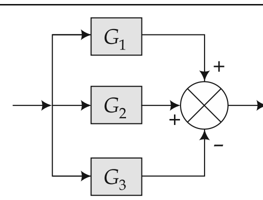
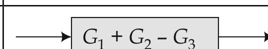
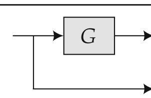
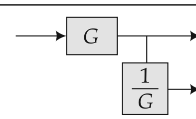
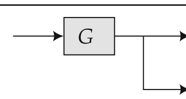
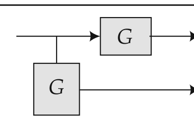


Fig. 2.15

Summary

Rule	*Original Block Diagram*	*Equivalent Block Diagram*
Combining blocks in cascade		
Combining blocks in parallel		
Moving a take-off point after the block		
Moving a take-off point before the block		

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Step 2: Eliminating feedback loop, Fig. 2.17 reduces to Fig. 2.18.

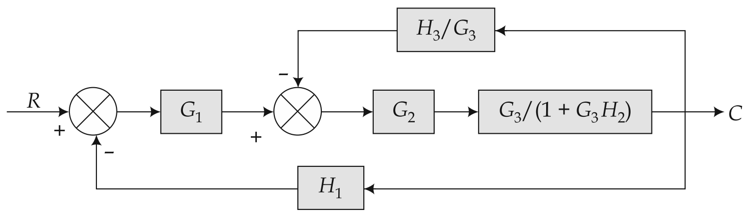


Fig. 2.18

Step 3: Combining blocks in cascade, Fig. 2.18 reduces to Fig. 2.19.

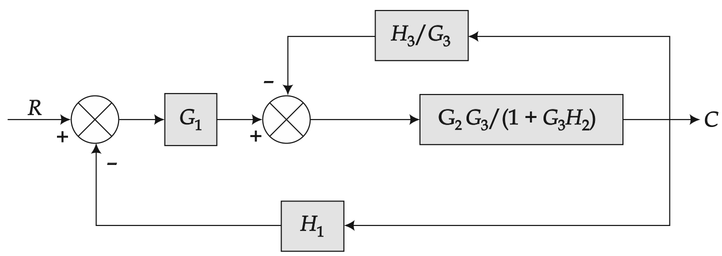


Fig. 2.19

Step 4: Eliminating feedback loop, Fig. 2.19 reduces to Fig. 2.20.

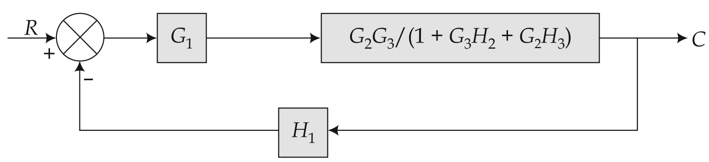


Fig. 2.20

Step 5: Combining blocks in cascade, Fig. 2.20 reduces to Fig. 2.21.

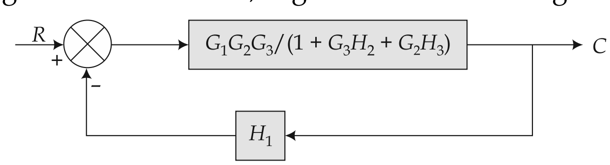


Fig. 2.21

Step 6: Eliminating feedback loop, Fig. 2.21 reduces to Fig. 2.22.

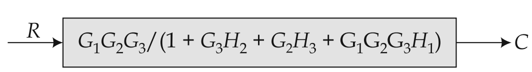


Fig. 2.22

So, transfer function becomes, $T(s) = \dfrac{G_1 G_2 G_3}{1 + G_2 H_3 + G_3 H_2 + G_1 G_2 G_3 H_1}$ **(Answer)**

Example 2.6 Obtain transfer function of the following block diagram:

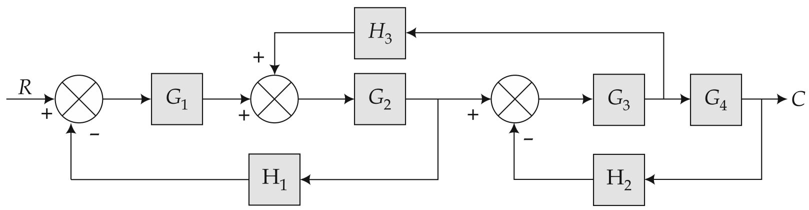


Fig. 2.23

Solution:

Step 1: Moving take-off point between G_3 and G_4 after G_4 and summing point between G_1 and G_2 before G_1, Fig. 2.23 reduces to Fig. 2.24.

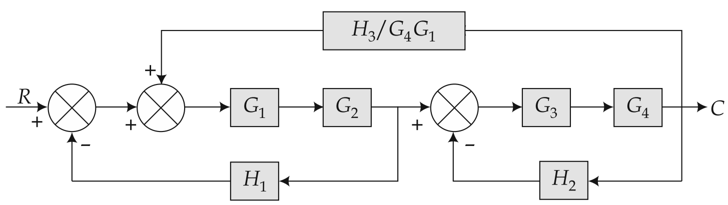


Fig. 2.24

Step 2: Combining first and second summing points, Fig. 2.24 reduces to Fig. 2.25.

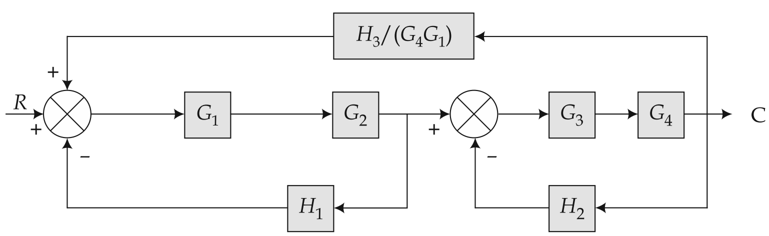


Fig. 2.25

Step 3: Simplifying first and second feedback loops, Fig. 2.25 reduces to Fig. 2.26.

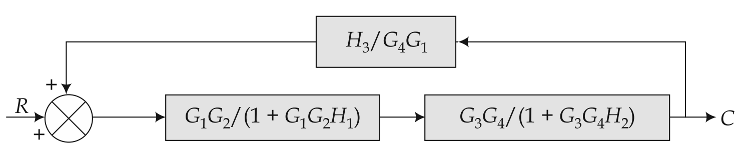


Fig. 2.26

Step 4: Combining the blocks in cascade, Fig. 2.26 reduces to Fig. 2.27.

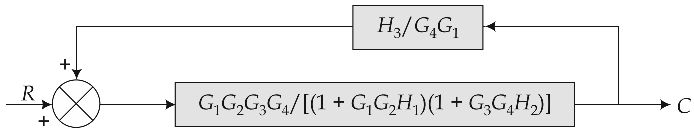


Fig. 2.27

Step 5: Eliminating feedback loop, Fig. 2.27 reduces to Fig. 2.28.

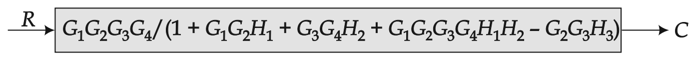


Fig. 2.28

So, transfer function is,

$$T(s) = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 + G_1 G_2 G_3 G_4 H_1 H_2 - G_2 G_3 H_3} \quad \textbf{(Answer)}$$

Example 2.7 Obtain the transfer functions C/R and C/D of the system shown in Fig. 2.29.

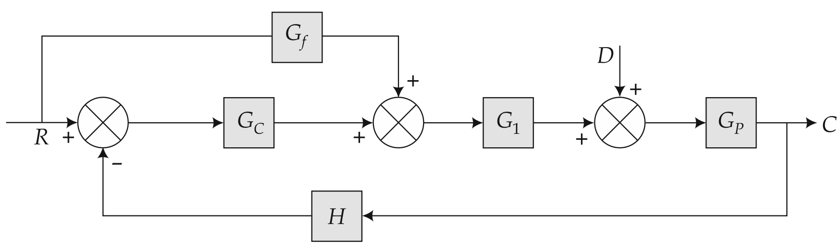


Fig. 2.29

Solution:

Step 1: To find C/R, second input of the system i.e., D is put equal to zero, so that Fig. 2.29 reduces to Fig. 2.30.

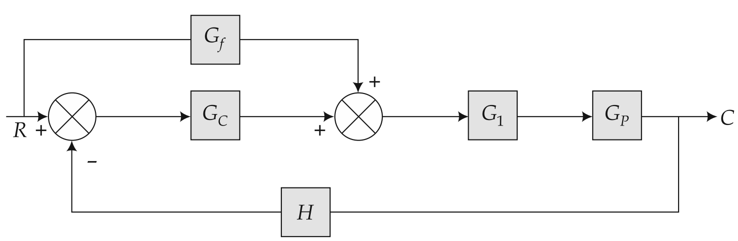


Fig. 2.30

Step 2: Combining G_1 and G_p in cascade and moving summing point between G_C and G_1 before G_C, Fig. 2.30 reduces to Fig. 2.31.

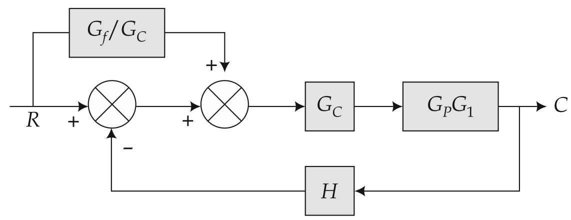


Fig. 2.31

Step 3: Combining two summing points and combining G_C and G_1G_P in cascade, Fig. 2.31 reduces to Fig. 2.32.

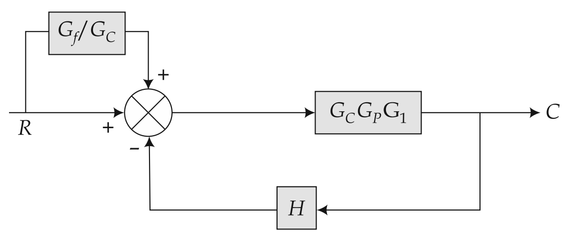


Fig. 2.32

Step 4: Applying parallel blocks and feedback blocks rules, Fig. 2.32 reduces to Fig. 2.33.

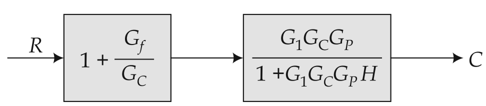


Fig. 2.33

Step 5: Combining blocks in cascade, Fig. 2.33 reduces to Fig. 2.34.

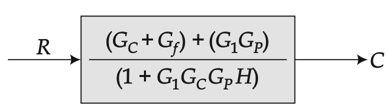


Fig. 2.34

So, $$\frac{C}{R} = \frac{\left(G_C + G_f\right)\left(G_1\,G_P\right)}{1 + G_1\,G_C\,G_P\,H}$$ **(Answer 1)**

Step 6: To find C/D, first input of the system i.e., R is put equal to zero, so that Fig. 2.29 reduces to Fig. 2.35.

Step 2: To find C_1/R_1, put $C_2 = R_2 = 0$, Fig. 2.38 reduces to 2.39.

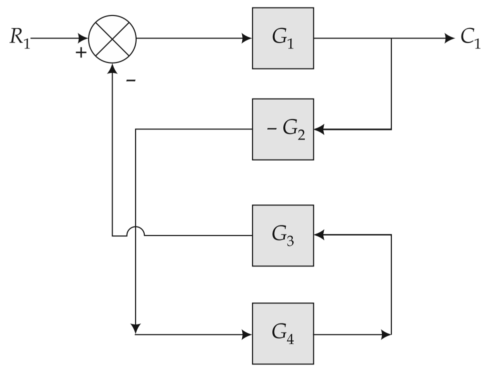


Fig. 2.39

Step 3: G_2, G_3 and G_4 are in cascade, so Fig. 2.39 reduces to Fig. 2.40.

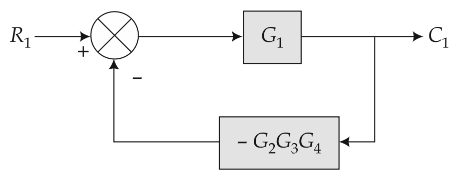


Fig. 2.40

Step 4: Eliminating feedback loop, Fig. 2.40 reduces to Fig. 2.41.

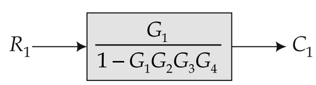


Fig. 2.41

Step 5: To find C_1/R_2, put $C_2 = R_1 = 0$, Fig. 2.38 reduces to 2.42.

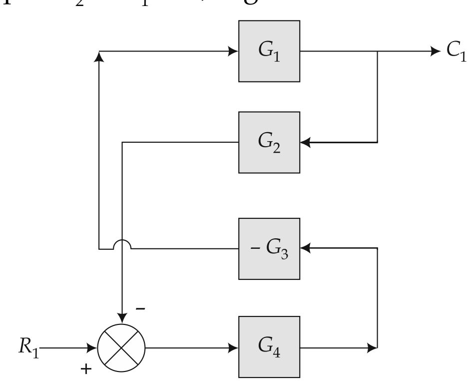


Fig. 2.42

Step 6: G_2, G_3 and G_4 are in cascade, Fig. 2.42 reduces to Fig. 2.43.

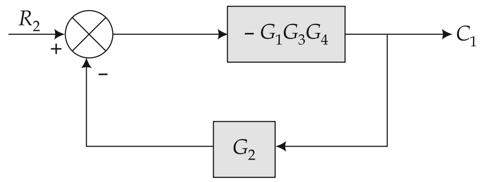


Fig. 2.43

Step 7: Eliminating feedback loop, Fig. 2.43 reduces to Fig. 2.44.

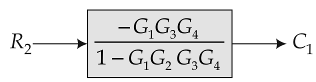


Fig. 2.44

Step 8: To find C_2/R_1, put $C_1 = R_2 = 0$, Fig. 2.38 reduces to 2.45.

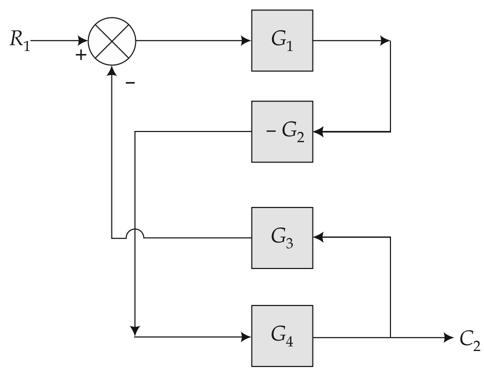


Fig. 2.45

Step 9: G_1, G_2 and G_4 are in cascade, Fig. 2.45 reduces to Fig. 2.46.

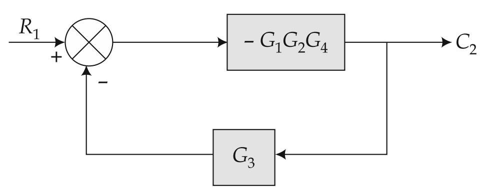


Fig. 2.46

Step 10: Eliminating feedback loop, Fig. 2.46 reduces to Fig. 2.47.

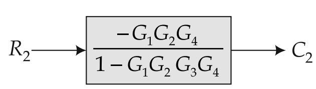


Fig. 2.47

Step 11: To find C_2/R_2, put $C_1 = R_1 = 0$, Fig. 2.38 reduces to Fig. 2.48.

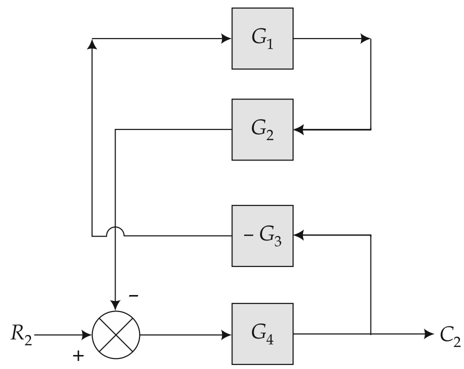


Fig. 2.48

Step 12: G_1, G_2 and G_3 are in cascade, Fig. 2.48 reduces to Fig. 2.49.

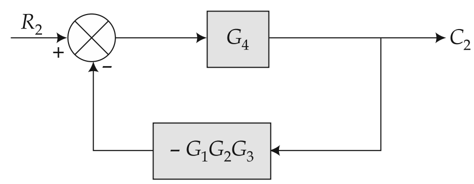


Fig. 2.49

Step 13: Eliminating feedback loop, Fig. 2.49 reduces to Fig. 2.50.

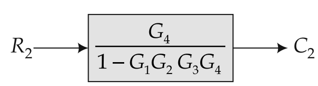


Fig. 2.50

Step 14: The transfer functions are:

$$\frac{C_1}{R_1} = \frac{G_1}{1 - G_1 G_2 G_3 G_4}, \frac{C_1}{R_2} = \frac{-G_1 G_3 G_4}{1 - G_1 G_2 G_3 G_4}$$

$$\frac{C_2}{R_2} = \frac{-G_1 G_2 G_4}{1 - G_1 G_2 G_3 G_4}, \frac{C_2}{R_2} = \frac{G_4}{1 - G_1 G_2 G_3 G_4} \quad \textbf{(Answer)}$$

2.4 SIGNAL FLOW GRAPH REPRESENTATION AND REDUCTION

Block diagram representation is a very advantageous approach to represent a control system. But it has one disadvantage that, for complicated systems, it becomes very difficult and sometimes impossible to reduce the block diagrams. A new and alternate method to represent a control system is signal flow graph (SFG) representation, which was developed by S.J. Mason. A readymade gain formula is available in this representation that relates the system input and output variables and directly gives the transfer function of the system without any transformation.

An SFG representation is a graphical figure that represents a set of linear equations, giving relationship between various variables of the system. It basically consists of two basic elements: node and branch. Node is a small circle representing a variable of the system. Branch is a line connecting two nodes and represents the flow of signal. Branch always contains one arrow, giving the direction of flow of signal.

2.4.1 Construction of SFG from Set of Linear Equations

Consider a set of linear equations:

$$\left.\begin{aligned} x_2 &= h_{12}x_1 + h_{42}x_4 \\ x_3 &= h_{13}x_1 + h_{23}x_2 + h_{33}x_3 \\ x_4 &= h_{24}x_4 + h_{34}x_4 \end{aligned}\right\} \tag{2.17}$$

First step is to represent the various variables i.e., x_1, x_2, x_3 and x_4 by nodes, shown in Fig. 2.51(a). The first equation in (2.17) states that x_2 is sum of two signals, coming from x_1 and x_4 and its SFG is shown in Fig. 2.51(b).

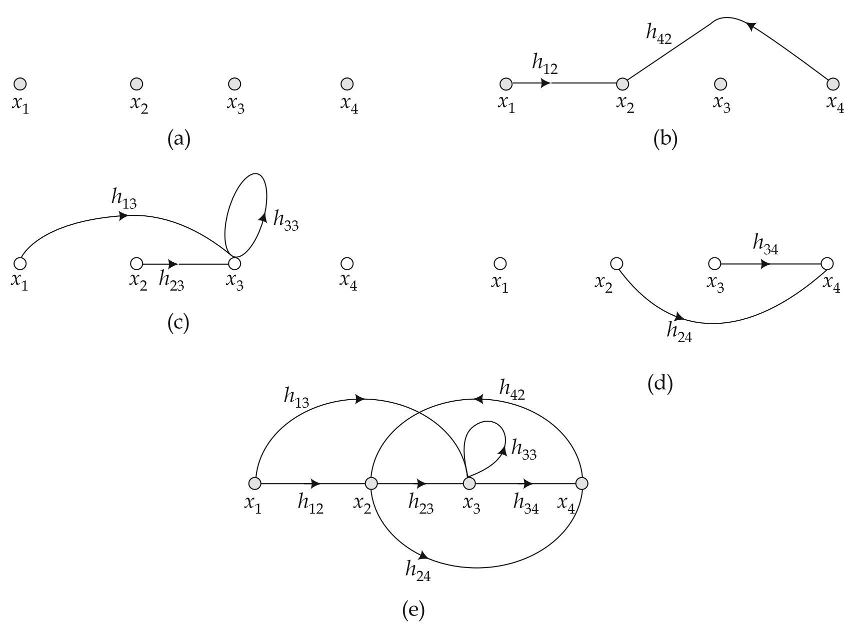


Fig. 2.51

Similarly, SFG for the remaining equations are shown in Figs. 2.51 (c) and (d) and complete SFG is shown in Fig. 2.51(e).

(i) Input node or source node: Node representing the input variable in block diagram or node having only outgoing branches, e.g., node R in Fig. 2.53.

(ii) Output node or sink node: Node representing the output variable in block diagram or node having only incoming branches, e.g., node C in Fig. 2.53.

(iii) Path: It is the interconnection of branches in the direction of arrows, such that no node is followed more than once, e.g., $R - x_1 - x_2 - x_3$, $x_1 - x_2 - x_3$, $R - x_4 - C$ are some of the paths in Fig. 2.53.

(iv) Forward path: It is the type of path starting from input node and terminating on output node, e.g., $R - x_1 - x_2 - x_3 - x_4 - C$, $R - x_1 - x_4 - C$ are two forward paths in Fig. 2.53.

(v) Forward path gain: It is the gain obtained by the product of all branch gains of a forward path, e.g., forward path gain for the forward path $R - x_1 - x_2 - x_3 - x_4 - C$ is $G_1 G_2$ in Fig. 2.53.

(vi) Loop: It is the special type of path, which starts and terminates at the same node. Except starting or terminating node, no node is to be followed more than once, e.g., $x_1 - x_2 - x_3 - x_4 - x_1$, $x_1 - x_4 - x_1$, $x_2 - x_3 - x_2$, are the three loops in Fig. 2.53.

(vii) Loop gain: It is the gain obtained by the product of all branch gains of a loop, e.g., loop gain for the loop $x_2 - x_3 - x_2$ is $- G_2 H_1$ in Fig. 2.53.

(viii) Non-touching loops: Two or more loops are non-touching, if they do not have any common node, e.g., $x_1 - x_4 - x_1$ and $x_2 - x_3 - x_2$ are two non-touching loops in Fig. 2.53.

2.4.4 Mason's Gain Formula

It is the readymade gain formula giving the relationship between input and output variable. It is very useful in finding transfer function of a signal flow graph. It is given by:

$$T(s) = \frac{\sum_i P_i \Delta_i}{\Delta}$$

where

$T(s)$ = overall transfer function of the system.

P_i = forward path gain of i^{th} forward path.

Δ_i = 1 – (sum of loop gains of those loops which are not touching i^{th} forward path).

Δ = Determinant of the graph = 1 – (sum of loop gains of all individual gains) + (sum of loop gain products of all possible combinations of two non-touching loops) – (sum of loop gain products of all possible combinations of three non-touching loops) + …

Consider the signal flow graph of Fig. 2.53. Its overall transfer function can be found out using Mason's gain formula.

(a) There are two forward paths. Forward path gains are:

$$P_1 = G_1 G_2 \quad P_2 = G_3$$

(b) There are three loops. Loop gains are:

$$L_1 = -\ G_1 G_2 H_2 \quad L_2 = -\ G_2 H_1 \quad L_3 = -\ G_3 H_2$$

(c) P_1 touches all the loops. So, $\Delta_1 = 1 - 0 = 1$

P_2 does not touch L_2. So, $\Delta_1 = 1 - L_2 = 1 + G_2 H_1$

(d) There is one possible combination of two non-touching loops i.e., $L_2 L_3$.

So, $\Delta = 1 - (L_1 + L_2 + L_3) + (L_2 L_3) = 1 + G_1 G_2 H_2 + G_2 H_1 + G_3 H_2 + G_2 G_3 H_1 H_2$

Now, the overall transfer function is given by:

$$T(s) = \frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 + G_3 (1 + G_2 H_1)}{1 + G_1 G_2 H_2 + G_2 H_1 + G_3 H_2 + G_2 G_3 H_1 H_2}$$

$$\Rightarrow \qquad T(s) = \frac{G_1 G_2 + G_3 + G_2 G_3 H_1}{1 + G_1 G_2 H_2 + G_2 H_1 + G_3 H_2 + G_2 G_3 H_1 H_2} \qquad \textbf{(Answer)}$$

Example 2.9 Obtain the transfer function of the signal flow given in Fig. 2.54.

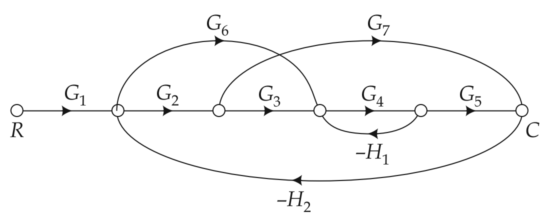


Fig. 2.54

Solution:

Step 1: There are three forward paths. Forward path gains are:

$$P_1 = G_1 G_2 G_3 G_4 G_5; \ P_2 = G_1 G_6 G_4 G_5; \ P_3 = G_1 G_2 G_7$$

Step 2: There are four loops. Loop gains are:

$L_1 = -\ G_4 H_1; \ L_2 = -\ G_6 G_4 G_5 H_2; \ L_3 = -\ G_2 G_3 G_4 G_5 H_2; \ L_4 = -\ G_2 G_7 H_2$

Step 3: First and second forward paths touch all loops:

Thus, $\Delta_1 = 1 - 0 = 1$

and $\Delta_2 = 1 - 0 = 1$

Third forward path does not touch L_1. So, $\Delta_3 = 1 - L_1 = 1 + G_4 H_1$.

Step 4: There is one pair of two non-touching loops i.e., L_1 and L_4

Thus, $\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_4)$

$= 1 + G_4 H_1 + G_6 G_4 G_5 H_2 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_4 H_1 G_2 G_7 H_2$

Step 5: According to Mason's gain formula,

$$\text{Transfer function} = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}$$

$$\Rightarrow \text{Transfer function} = \frac{G_1 G_2 G_3 G_4 G_5 + G_1 G_4 G_5 G_6 + G_1 G_2 G_7 (1 + G_4 H_1)}{1 + G_4 H_1 + G_4 G_5 G_6 H_2 + G_2 G_3 G_4 G_5 H_2 + G_2 G_7 H_2 + G_2 G_4 G_7 H_1 H_2}$$

(Answer)

Example 2.10 Find the transfer function of the signal flow graph shown in Fig. 2.55.

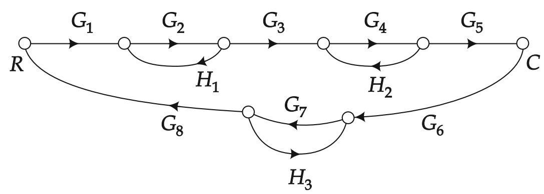


Fig. 2.55

Solution:

Step 1: There is one forward path. Forward path gain is:

$$P_1 = G_1 G_2 G_3 G_4 G_5$$

Step 2: There are four loops. Loop gains are:

$$L_1 = G_2 H_1;\; L_2 = G_4 H_2;\; L_3 = G_7 H_3;\; L_4 = G_1 G_2 G_3 G_4 G_5 G_6 G_7 G_8$$

Step 3: First forward path does not touch L_3.

Thus, $\Delta_1 = 1 - L_3 = 1 - G_7 H_3$

Step 4: There are three pairs of two non-touching loops i.e., $L_1 L_2$, $L_2 L_3$ and $L_3 L_1$ and there is one pair of three non-touching loops i.e., $L_1 L_2 L_3$.

Thus, $\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_2 + L_2 L_3 + L_3 L_1) - (L_1 L_2 L_3)$

$= 1 - G_2 H_1 - G_4 H_2 - G_7 H_3 - G_1 G_2 G_3 G_4 G_5\, G_6 G_7 G_8 + G_2 G_4 H_1 H_2 + G_4 G_7 H_2 H_3 + G_2 G_7 H_1 H_3 - G_2 G_4 G_7 H_1 H_2 H_3$

Step 5: According to Mason's gain formula,

$$\text{Transfer function} = \frac{C(s)}{R(s)} = \frac{P_1\Delta_1}{\Delta}$$

$\Rightarrow$ Transfer function =

$$\frac{G_1 G_2 G_3 G_4 G_5 (1 - G_7 H_3)}{1 - G_2 H_1 - G_4 H_2 - G_7 H_3 - G_1 G_2 G_3 G_4 G_5 G_6 G_7 G_8 + G_2 G_4 H_1 H_2 + G_4 G_7 H_2 H_3 + G_2 G_7 H_1 H_3 - G_2 G_4 G_7 H_1 H_2 H_3}$$

(Answer)

Example 2.11 Obtain the transfer function of the following block diagram. Convert it into signal flow graph and again obtain transfer function using Mason's gain formula. Verify the results obtained.

Step 5: There is one forward path. Forward path gain is:

$$P_1 = G_1 G_2$$

Step 6: There are two loops. Loop gains are:

$$L_1 = G_2 H_2;\ L_2 = -\ G_1 G_2 H_1 H_2$$

Step 7: First forward path touches all loops. So, $\Delta_1 = 1 - 0 = 1$

Step 8: There is no combination of non-touching loops.

So, $$\Delta = 1 - (L_1 + L_2) = 1 - G_2 H_2 + G_1 G_2 H_1 H_2$$

Step 9: According to Mason's gain formula,

$$\text{Transfer function} = \frac{C}{R} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2}{1 - G_2 H_2 + G_1 G_2 H_1 H_2} \qquad \textbf{(Answer 2)}$$

Step 10: Answer 1 and Answer 2 are same. So, results are verified.

Exercise

1. Find characteristic equation, order of the system, type of the system and poles and zeros of the systems whose transfer functions are given by:

   (i) $\dfrac{10}{s(1+0.4\,s)(1+0.1\,s)}$ (ii) $\dfrac{4(s+2)}{s^2(s^2+7s+12)}$

   (iii) $\dfrac{16(1+0.5\,s)}{s^2\,(1+0.125\,s)(1+0.1\,s)}$ (iv) $\dfrac{80(s+5)}{s^3(1+8s)}$

2. A system is given by the block diagram shown in Fig. 2.61.

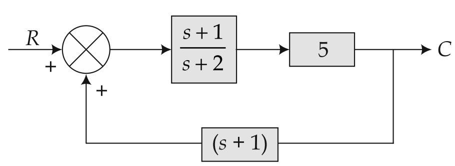


Fig. 2.61

   Determine (i) Characteristic equation (ii) order (iii) type (iv) poles and zeros in s-plane.

3. A second order system is given by

$$y(t) = 2\frac{d^2 z(t)}{dt} + 10\frac{dz(t)}{dt} + 5\,z(t)$$

   Obtain transfer function, characteristic equation, pole and zero behavior of the system.

4. Obtain the transfer function of the system shown in Fig. 2.62.

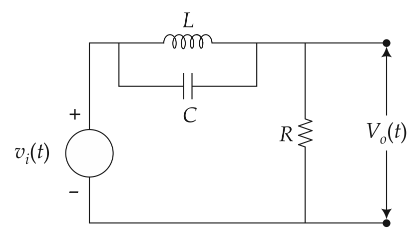


Fig. 2.62

5. Derive the transfer function of the electric circuit shown in Fig. 2.63.

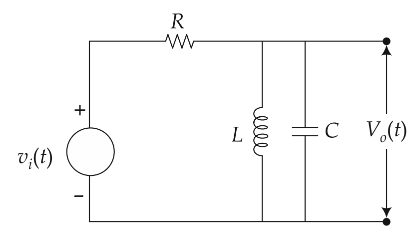


Fig. 2.63

6. Obtain the transfer function of the system shown in Fig. 2.64.

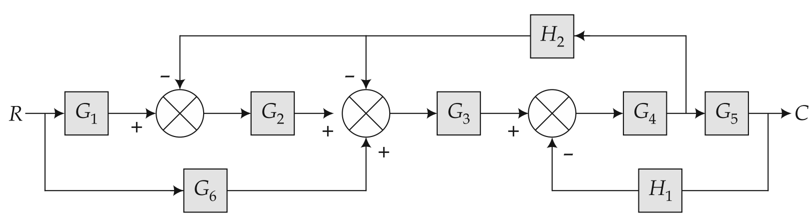


Fig. 2.64

7. Obtain the transfer function of the system shown in Fig. 2.65 by block diagram reduction method and by signal flow graph method.

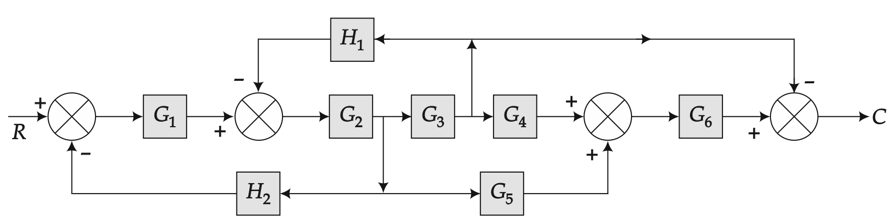
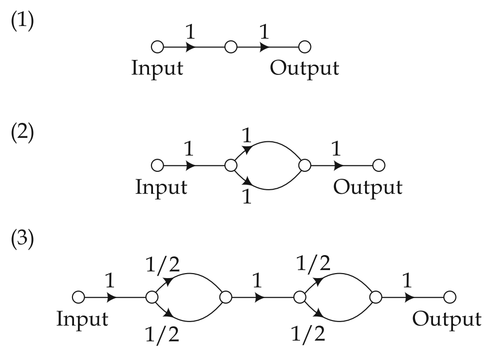


Fig. 2.65

Objective Type Questions

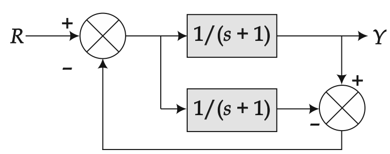
1. Consider the following signal flow graphs. The value of gain is 2 for



(a) 2 and 3 (b) 1
(c) 3 (d) 2

2. Which of the following is open loop system?
   (a) The respiratory system of a man
   (b) A system for controlling the movement of the slide of a copying milling machine.
   (c) Traffic light control
   (d) A thermostatic control

3. The transfer function Y/R of the system shown is



(a) 0 (b) $1/(s + 1)$
(c) $2/(s + 1)$ (d) $2/(s + 3)$

4. The block diagram shown in Fig. 1 is equivalent to

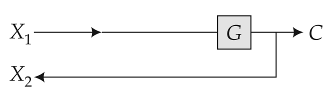
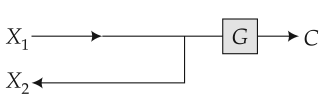
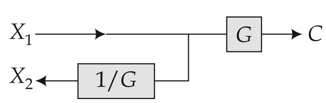


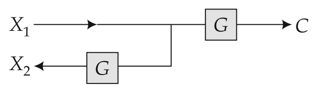
Fig. 1



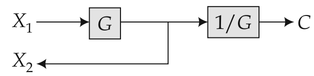
(a)



(b)



(c)



(d)

5. The transfer function of the system is $\dfrac{2s^2+6s+5}{(s+1)^2\,(s+2)}$. The characteristic equation is
   (a) $2s^2 + 6s + 5 = 0$
   (b) $(s + 1)^2\,(s + 2) = 0$
   (c) $2s^2 + 6s + 5 + (s + 1)^2\,(s + 2) = 0$
   (d) $2s^2 + 6s + 5 - (s + 1)^2\,(s + 2) = 0$
6. Transfer function of the integrator shown in Fig. 2 is

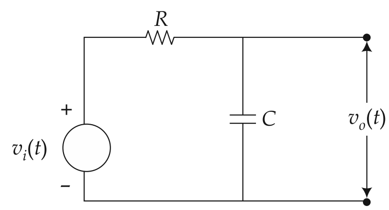
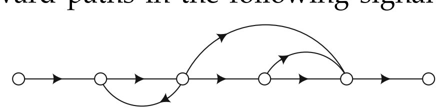


Fig. 2

   (a) $\dfrac{RCs}{RCs+1}$ (b) $\dfrac{1}{RCs+1}$
   (c) $\dfrac{1}{RC+s}$ (d) $\dfrac{RC}{1+s}$
7. Type-2 system has
   (a) two poles at origin (b) one pole at origin
   (c) three poles at origin (d) two zeros at origin
8. Roots of characteristic equation is
   (a) zeros of the system (b) order of the system
   (c) poles of the system (d) type of the system
9. Which of the following transfer functions indicate type-0 system?
   (a) $\dfrac{(s+3)\,(s+7)}{(s+6)\,(s+8)}$ (b) $\dfrac{(s+3)\,(s+7)}{s\,(s+6)\,(s+8)}$
   (c) $\dfrac{(s+3)\,(s+7)}{s^2\,(s+6)\,(s+8)}$ (d) $\dfrac{(s+3)\,(s-7)}{s^3\,(s+6)\,(s+8)}$
10. The number of forward paths in the following signal flow graph are



d =
 1 17 104 268 240
Transfer function:
 10 s^2 + 30 s + 20
----
s^4 + 17 s^3 + 104 s^2 + 268 s + 240

P4. Plot pole-zero plot of the system having transfer function $T = \dfrac{s^3 + 2s^2 + s + 3}{s^4 + 5s^3 + 3s^2 + 4s + 1}$

Program:

```
n=[1 2 1 3];      % Numerator of transfer function
d=[1 5 3 4 1];    % Denominator of transfer function
T=tf(n, d)        % Transfer function
pzmap(T)          % Plot of pole-zero map
```

Execution:

Transfer function:
 s^3 + 2 s^2 + s + 3
----
s^4 + 5 s^3 + 3 s^2 + 4 s + 1

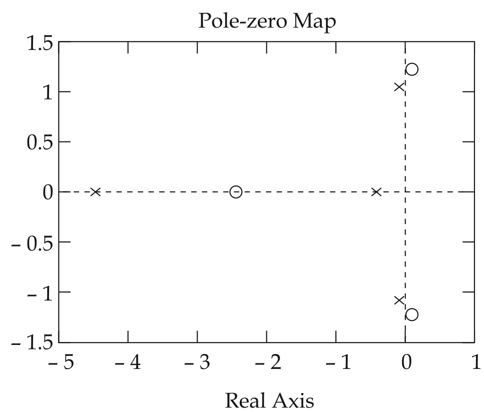


Fig. F.1

P5. Find the overall transfer function for the system shown in Fig. F.2.

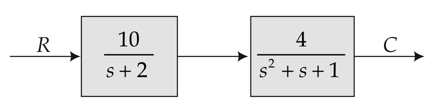


Fig. F.2

Execution:

Transfer function:
```
 2 s^2 + 44 s + 42
---------------------
s^3 + 3 s^2 + 3 s + 1
```

P7. Find the overall transfer function for the system shown in Fig. F.4.

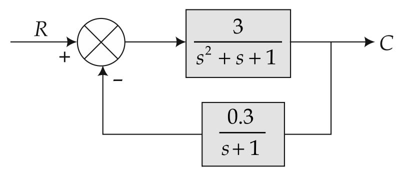


Fig. F.4

Program:

```
% To find G
n1=[3];
d1=[1 1 2];
G=tf(n1,d1);
% To find H
n2=[0.3];
d2=[1 1];
H=tf(n2,d2);
% To find overall transfer function
T=feedback(G,H)
```

Execution:

Transfer function:
```
 3 s + 3
-----------------------
s^3 + 2 s^2 + 3 s + 2.9
```

P8. Find the overall transfer function for the system shown in Fig. F.5.

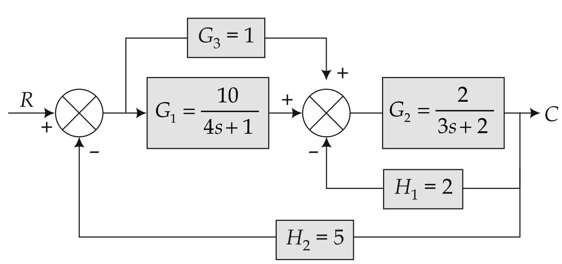


Fig. F.5