CHAPTER 2

Vibration Control

2.1 TECHNIQUES USED FOR VIBRATION CONTROL

Usually, vibrations are undesirable and need to be reduced.

Following techniques can be used for the control of vibrations.

- Reduction of excitation force
- Avoiding resonance
  (Change of mass and stiffness) to avoid coincidence of natural frequencies and excitation frequencies
- Use of absorbers
- Isolation

For wide frequency range excitations:

- Damping: viscous, coulomb, hysteresis, viscoelastic polymeric damping.
- Active control

Techniques as above are given on the following pages (Figs. 2.1-2.18).

These range from improved support, rotor balancing use of vibration absorbers, damping treatments of various types on varied applications.

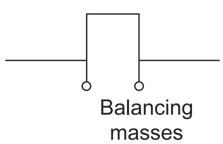


Fig. 2.1 Engine crankshaft balancing

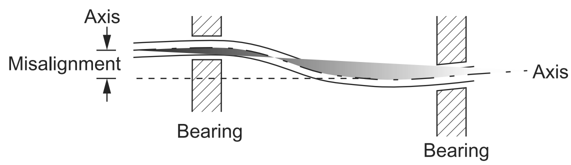


Fig. 2.2 For shaft misalignment

Use of vibration absorber is shown in Figs. 2.3 and 2.4.

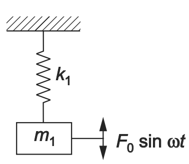


Fig. 2.3 Main system

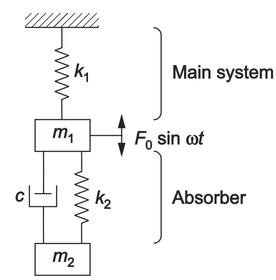


Fig. 2.4 Vibration control by absorber

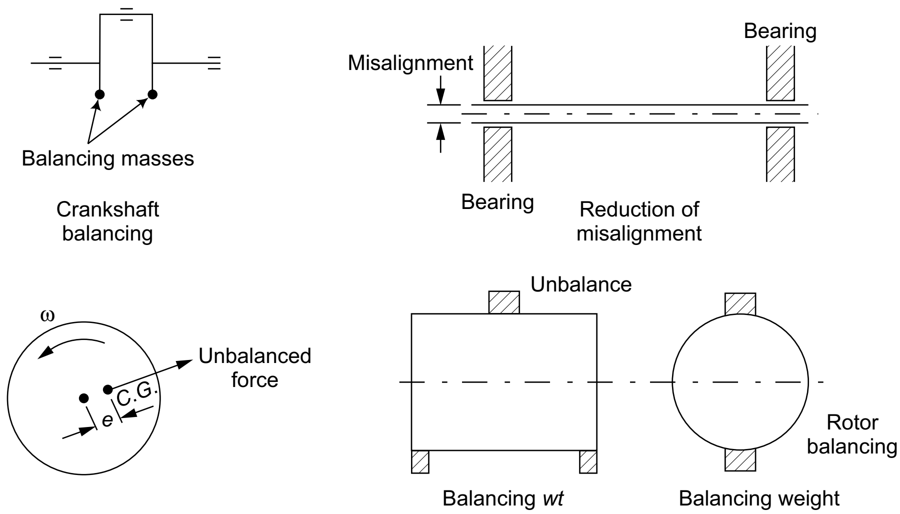


Fig. 2.7 Rotor balancing

The balancing weights are shown in above figures.

VIBRATION DAMPING

"Q" Factors

Q=Ampl./static defl.

Welded Steel Structure.............100 to 300.
Jointed Structure.......................25 to 60.
MN-CU Alloy..............................20 to 40.
Viscoelastic Material..................1 to 10.

Fig. 2.8 "*Q*" factor for various materials (measure of damping)

2.4 DAMPING

Damping is a mechanism by which energy is dissipated from a vibrating system. This is useful for limiting resonant amplitudes of vibrations and may be of several types, i.e., coulomb, viscous, hysteresis, viscoelastic, etc. A coulomb (Fig. 2.13) type Lanchester damper is used for damping torsional vibrations in engines. Viscous type dampers are used in automobiles for improving riding comfort. (Figs. 2.11 and 2.12)

Figure 2.12 and 2.13 show a viscous type torsional damper used in engines and Fig. 2.14 shows a stockbridge damper used to control flow induced vibrations in overhead transmission lines where damping is due to coulomb friction between the stranded steel cables. In some of the copper-manganese alloys. Gears with high damping materials are less prone to vibrations. Some of the plastic materials which are viscoelastic in behaviour, have high damping which may be suitably incorporated to dissipate energy in vibratory systems.

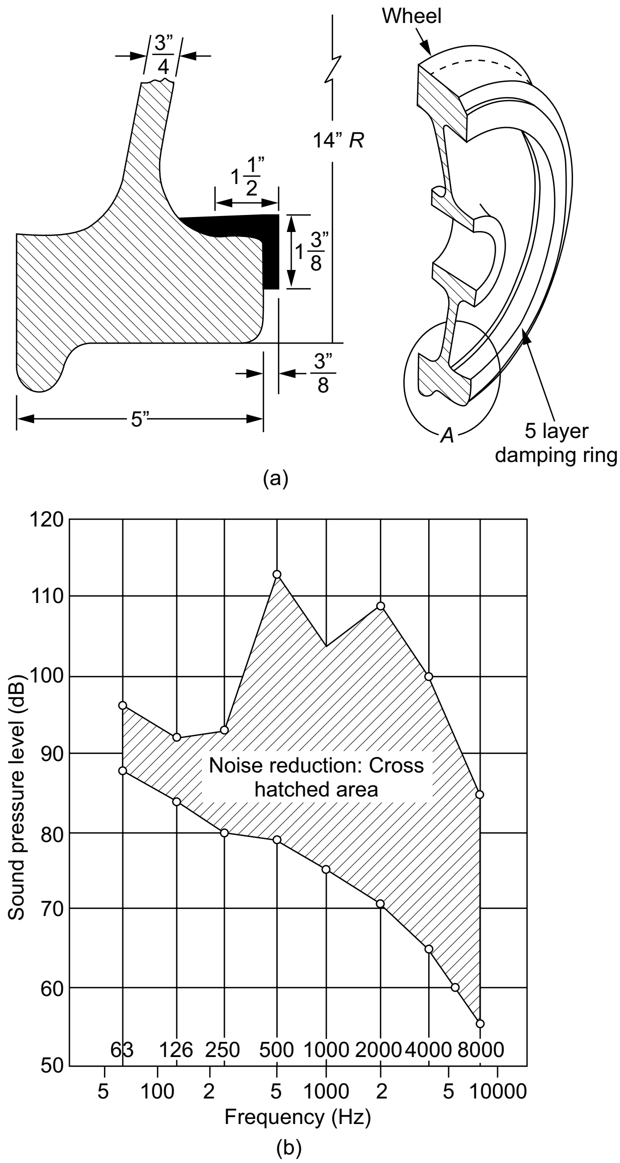
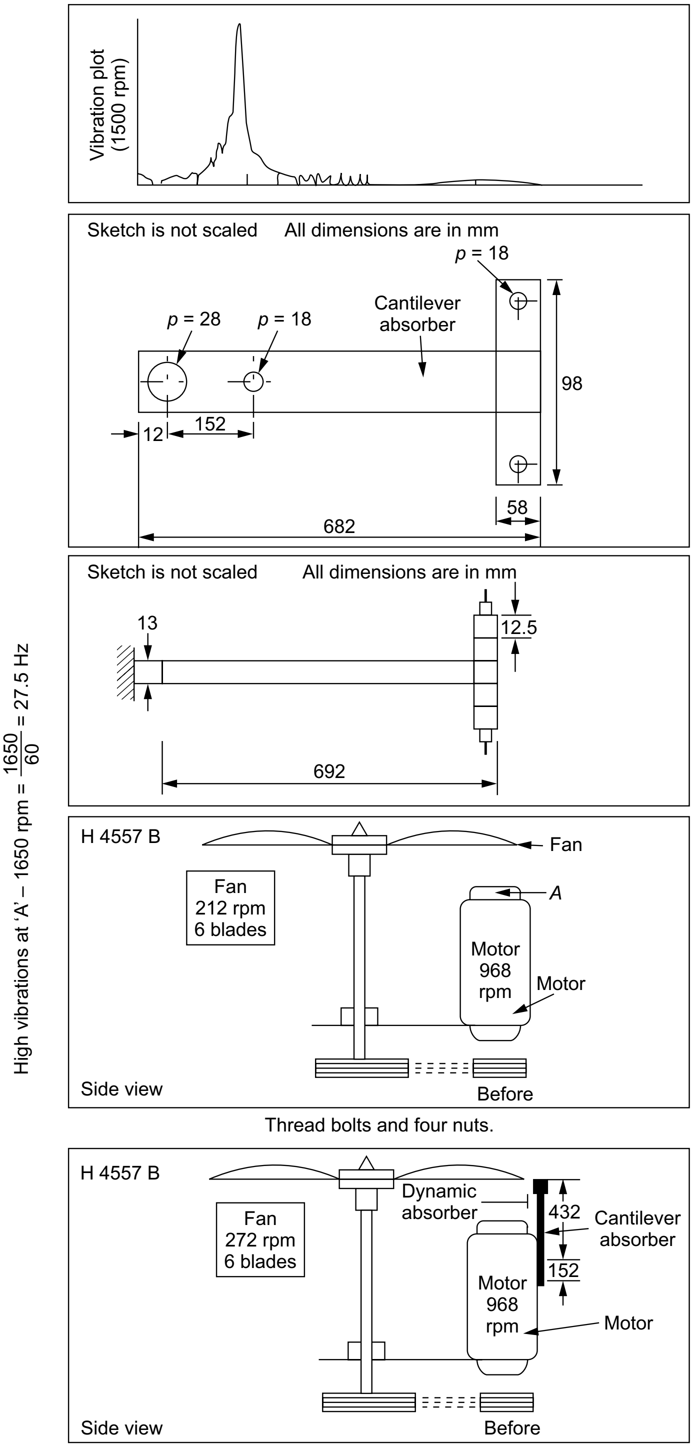


Fig. 2.9 Damping treatment applied to railway wheel (Unconstrained treatment for reduction of noise + vibration)

Use of damping material helps in reducing vibration and noise of a railway wheel as shown in Fig. 2.9.



$$\text{Excitation } 'F' = \frac{272 \times 6}{60} = 27.2 \text{ Hz}$$

Fig. 2.10 Control of vibration of a fan by absorber control treatments

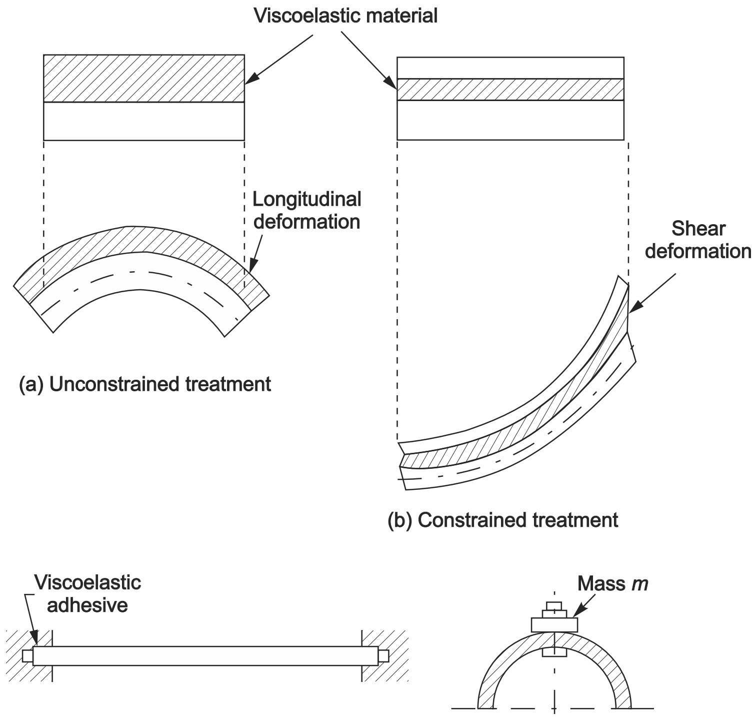


Fig. 2.15 Viscoelastic damping treatments

Some other applications are given in Figs. 2.16 to 2.18.

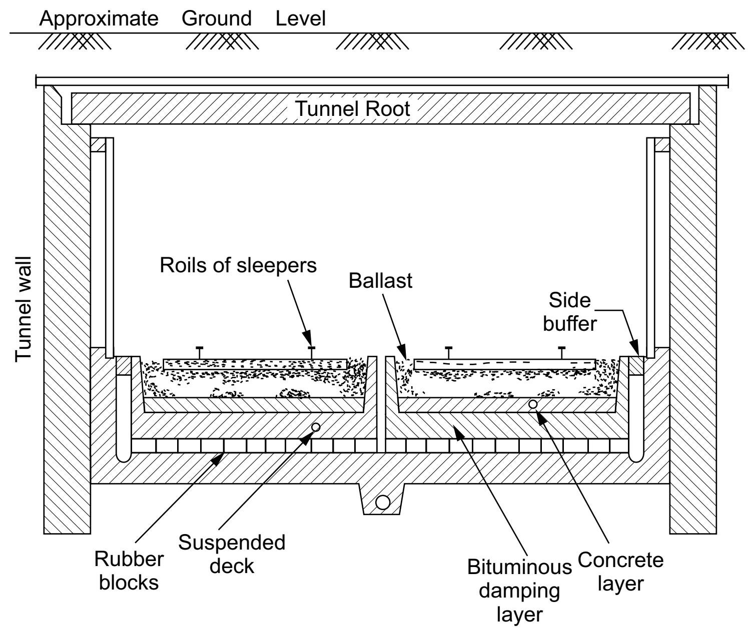


Fig. 2.16 Railway vibration control on suspended deck

P.H. Alloway and P. Grootenhuis – Unusual techniques to control of vibrations, Proc. International Congress on Acoustics, LIEGE Sept. 1965

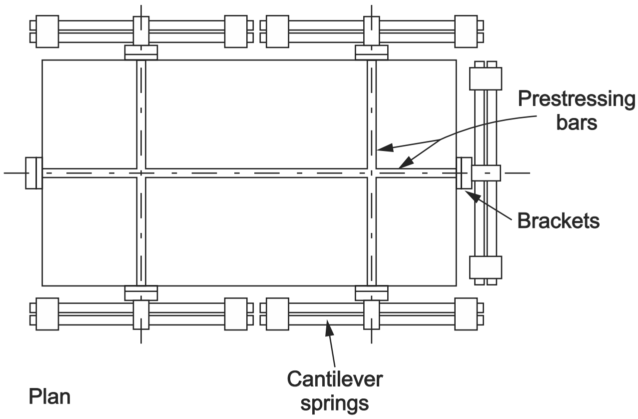


Fig. 2.17 Compressor foundation: Use of vibration absorbers (585 rpm, 7 ton unbalanced force-2 reciprocating compressors) structural resonances in the building). Vibrations reduced considerably

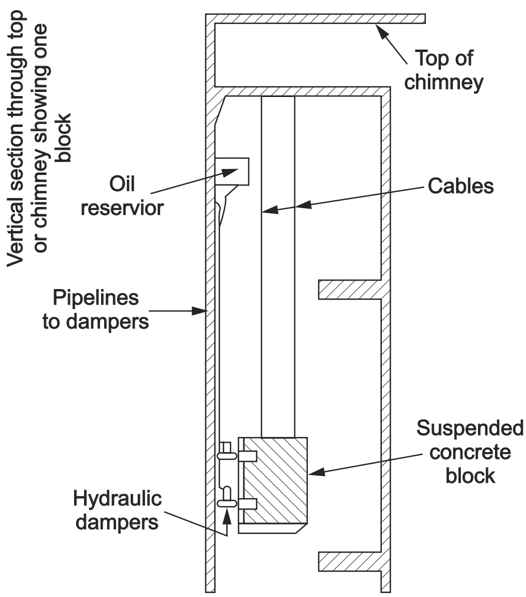


Fig. 2.18 Chimney vibration: Wind excited, 200 metres high 18 metres square chimney ω_n = 0.455 CPS. Damping introduces – 6 hydraulic damper between each concrete block + chimney, each block 25 ton on 40 ft. long cables. Natural frequency (horizontal) of mass-cable = 14 CPS.

2.5 VIBRATION ABSORBERS

A single degree of freedom with harmonic excitation has large vibration amplitude when excitation frequency equals the natural frequency of a system or when $\omega = \omega_n$, where ω is the excitation frequency and ω_n is the natural frequency of the system.

To control the vibration of the system, an auxiliary or absorber system, another mass and stiffness are added to the main system as shown in Fig. 2.19.

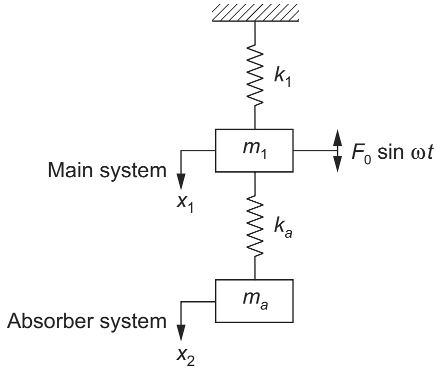


Fig. 2.19

The equations of motion of the system are:

$$m_1\ddot{x}_1 + (k_1 + k_a)\,x_1 = F_o \sin \omega t$$
$$m_a\ddot{x}_2 - k_a x_1 + k_2 x_a = 0 \qquad \ldots(2.1)$$

From the above equations, we can get the amplitudes of vibrations of m_1 and m_a.

Taking the harmonic solution, $X_1 = x_1 \sin(\omega t + \phi)$

$$X_a = x_a \sin(\omega t + \phi) \qquad \ldots(2.2)$$

(ϕ is phase difference depending on initial conditions

$$(-m_1\omega^2 + k_1 + k_a)\,(-m_a\omega^2 + k_a) - k_a^2) \quad \begin{matrix} x_1 \\ x_2 \end{matrix} \quad = \quad \begin{matrix} F_o \\ o \end{matrix} \qquad \ldots(2.3)$$

Solving,

$$U_1 = \frac{(-\omega^2 m_a + k_a)\,F_o}{\Delta}$$

$$U_2 = \frac{k_a\,F_o}{\Delta}$$

where $\Delta = m_1 m_a\,\omega^2 - (k_a m_1 + k_1 m_a + k_a m_a)\,\omega^2 + k_1 k_a \qquad \ldots(2.4)$

The amplitude of the main mass is zero,
when

$$\omega = \sqrt{k_a / m_a} \qquad \text{...(2.12)}$$

The amplitude of the absorber mass is

$$X_2 = F_o / k_2 \qquad \text{...(2.13)}$$

The vibration absorber thus eliminates the vibrations of the main mass. The absorber mass keeps on vibrating and it has to be designed to avoid fatigue failure.

As seen, tuning is done with excitation frequency. Figure 2.20 shows that one frequency is below the absorber natural frequency another one is above the absorber natural frequency.

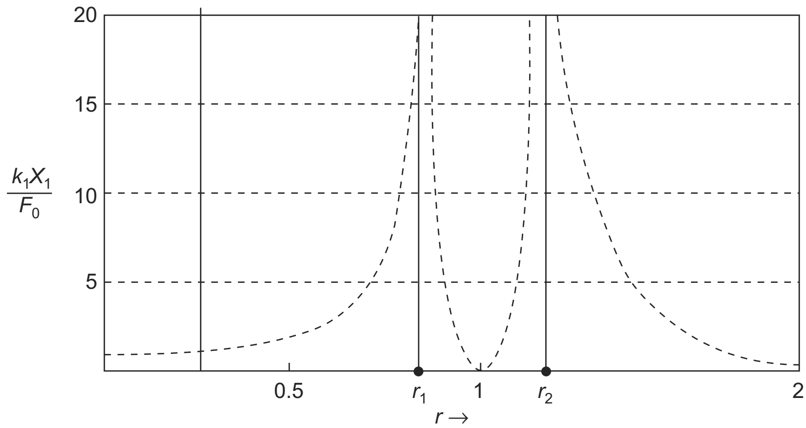


Fig. 2.20 Frequency response of the main mass by adding a vibration absorber (μ = 0.15, q = 1.0)

In Fig. 2.20 it may been that r_1 = 0.15 and r_2 = 1.25.

The amplitudes X_1 of the main mass are very high at $r = r_1$ and r_2.

If the absorber is tuned to excitation frequency, the maximum value of $k_2 = \mu m_1 \omega$

Minimum steady state amplitude of the absorber mass is

$$X_2(\text{min}) = \frac{F_o}{\mu m_1 \omega}. \qquad \text{...(2.14)}$$

The amplitude of the main mass can be reduced as seen from Fig. 2.20.

Problem 2.1 A machine of mass m_1 of 160 kg has stiffness $R_1 = 3 \times 10^6$ N/mm. If the machine speed is 1200 rpm, find the natural frequency and excitation frequency of the system. Force F_o on the main m_1 = 10,000 N. Find parameters of a vibration absorber to be used so that the vibrations of the main systems are minimized and the amplitude of the absorber will be less than 25 mm. What are the system natural frequencies with the added absorber; what is operating range so that the main system amplitude does not exceed 5 mm with the added absorber?

The equations of motion may be written as:

$$\begin{bmatrix} m_1 & o \\ o & m_a \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_a \end{bmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\begin{bmatrix} \dot{x}_1 \\ \dot{x}_a \end{bmatrix} + \begin{bmatrix} k_1 + k_a & -k_a \\ -k_a & k_a \end{bmatrix}\begin{bmatrix} x_1 \\ x_a \end{bmatrix} = \begin{bmatrix} F_o \sin \omega t \\ 0 \end{bmatrix} \quad \text{...(2.15)}$$

Taking
$$x_1 = x_1 \sin(\omega t + \phi_1)$$
$$x_a = x_a \sin(\omega t + \phi_2)$$

and substituting, we get

$$x_1 = \sqrt{\frac{(cw)^2 + (k_a - m_a\omega^2)}{A^2 + B^2}} \quad \text{...(2.16)}$$

$$x_1(t) = x_1 \sin(\omega t - \phi_1) \quad \text{...(2.17)}$$

$$x_1 = \sqrt{U_1^2 + \left(\frac{V_1}{\omega}\right)^2} \quad \text{...(2.18)}$$

$$x_1 = \sqrt{U_1 + \left(\frac{V_1}{\omega}\right)^2} \quad \text{...(2.19)}$$

$$= \sqrt{\frac{(c\omega)^2 + (k_2 - m_2\omega^2)}{U_1^2 + V_1^2}}$$

$$\phi_1 = \frac{\omega U_1}{V_1}$$

$$U_1 = \frac{F_o[c\omega U_1(\omega) - (k_2 - m_2\omega^2)^2\, U(\omega)]}{U_1^2(\omega) + V_1^2(\omega)}$$

$$V_1 = \frac{F_o(\omega)\,[(k_2 - m_2\omega^2)\,\omega_1(\omega) + c\omega V_1(\omega)]}{U_1^2(\omega) + V_1^2(\omega)} \quad \text{...(2.20)}$$

where
$$U_1(\omega) = m_1 m_2 \omega^2 - (m_1 k_2 + k_1 m_2 + m_2 k_2)\, \text{w}^2 + k_1 k_2$$
$$V_1(\omega) = -(m + m_2)\, c\omega^3 + k_1 c\omega$$

similarly for
$$x_2 = U_2 \cos \omega t + \frac{V_2}{\omega} \sin \omega t \quad \text{...(2.21)}$$

or
$$x_2(t) = x_2 \sin(\omega t - \phi_1) \quad \text{...(2.22)}$$

$$U_2 = \frac{F_o[k_2 M(\omega) - c\omega N(\omega)]}{M^2(\omega) + N^2(w)} \quad \text{...(2.23)}$$

$$V_2 = \frac{F_o \omega\,[c\omega M(\omega) + k_2 N(\omega)]}{M^2(\omega) + N^2(w)} \quad \text{...(2.24)}$$

In Fig. 2.23, the addition of absorber mass (damped) to main mass has similar reduction with different parameters μ + q.

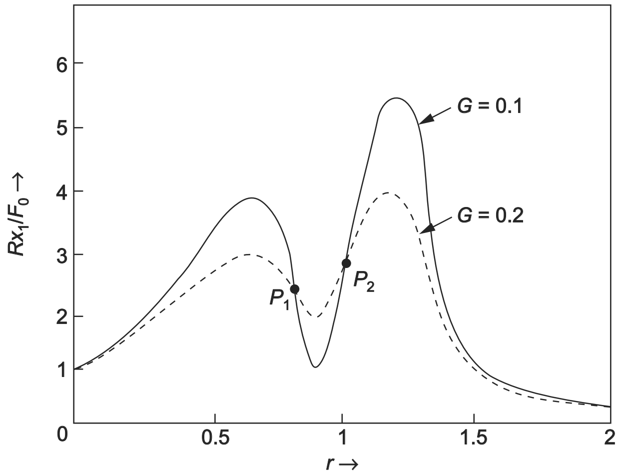


Fig. 2.23

Frequency response of the main mass with addition of damped vibration absorber (μ = 0.25, q = 0.8)

A damped absorber should be designed so that its performance is better than that over the entire operating range.

An optimum absorber may preferably have the same magnitude, over the range it is tried to tune the absorber so that the points P_1 and P_2 of Fig. 2.24, are at the same height. The location depends on q. See curves for various values of ς pass through the fixed points. It is possible to find a value of ς so that the fixed points are near peaks.

Addition of a suitable damped absorber reduces the steady state response over a wide frequency range.

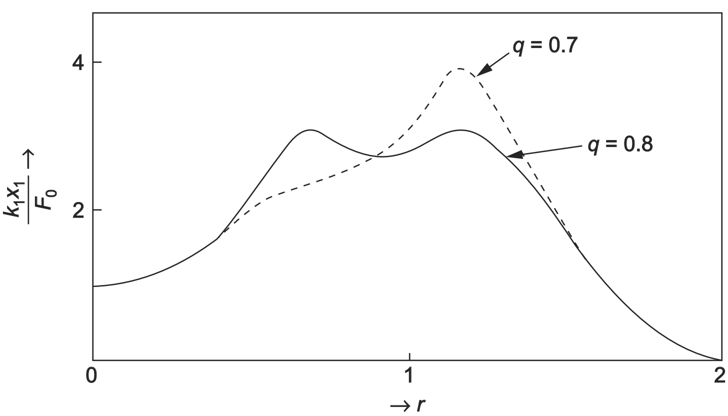


Fig. 2.24

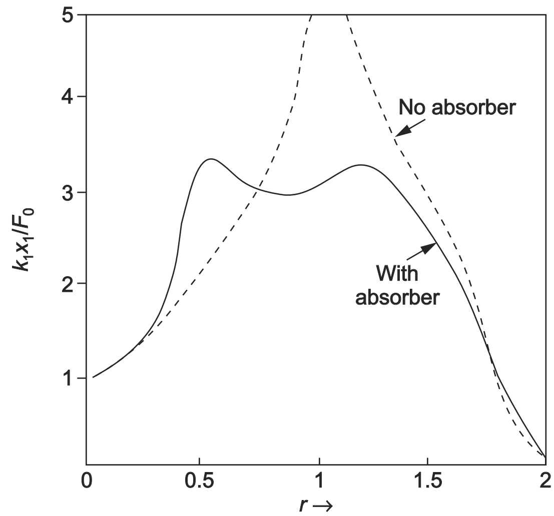


Fig. 2.25 Optimum damped absorber

Though the peaks in Fig. 2.24 are lower than those in Fig. 2.23, the response is independent of ς. The value of a function say

$$Y = \sqrt{\frac{A+B}{C+D}} \text{ can be same} \qquad \text{...(2.27)}$$

A, *B*, *C* and *D* are functions of *m* and *q*

Since Eqn. (2.27) holds for all values of ς and powers of ς are linearly independent

$$\frac{A}{B} = \frac{B}{D} \qquad \text{...(2.28)}$$

Using Eq. (2.27) to get the forms of *A*, *B*, *C* and *D* and substituting in Eqn. (2.23), after rearranging gives $r_1^4\left(1+\frac{\mu}{2}\right) - [1 + q^2(1 + \mu)]\, r_1^2 + q^2 = 0$ ...(2.29)

Solution of Eqn. (2.24) places the fixed points at

$$r_1 = \sqrt{\frac{1+(1+\mu)\,q^2 I\sqrt{1-2q^2+(1+\mu)^2\,q^4}}{2+\mu}} \qquad \text{...(2.30)}$$

Since Eqn. (2.27) yields the same value of *Y*, independent of ς for r_1 given by Eqn. (2.30), taking ς → ∞ gives:

$$Y = \sqrt{\frac{1}{1-r_1^2(1+\mu)^2}} \qquad \text{...(2.31)}$$

$$m_{n1} = \sqrt{k_1/m_1} = 150.7 \text{ rad/sec}$$

An excitation frequency should equal the absorber natural frequency for tuning. Substituting the data in the Eqn. above we get

$$X_a = 0.57 \text{ mm}$$

F_o is given as 20,000 N

$$\varsigma_{\text{opt}} = 0.06$$

2.7 VIBRATION ISOLATION

Vibratory forces generated by machines are unavoidable. If a machine is rigidly mounted to its foundation, the foundation experiences the full effect of the excitation. However, if an elastic element is placed between the machine and the foundation (called vibration isolator), the magnitude of the transmitted force can be reduced. The elastic foundation is modeled as springs in parallel with a viscous damper. Theoretical details of vibration transmissibility ratio are given in Chapter 1. If the transmissibility ratio is less than one, the magnitude of the force transmitted is less than the magnitude of the excitation force and the vibrations are isolated. Proper design of vibration isolators used in machines such as pumps, compressors, turbines, engines, etc., reduces structural vibrations and protects the structures from damage.

Exercise

1. A rotating machine has a mass of 1000 kg. It is mounted on springs and a damper, having total stiffness and damping ratio of 5×10^6 N/m and 0.2 respectively. The machine rotates at 3000 rpm and creates a force of 2500 N, due to unbalance.

   Find (i) amplitude of vibration of machine, due to unbalance force

   (ii) the phase lag

   (iii) the transmitted force to the machine foundation

   [Ans. 2.5×10^{-2} mm, 50.6°, 507.5 N]

2. A machine of mass 20 kg is mounted on a spring with stiffness $k_1 = 2.5 \times 10^6$ N/m. If the speed of rotation of the machine is 3000 rpm, find the parameters of an absorber with spring of stiffness k_a and mass M_a, which will suitably control the vibrations of the machine. Mass M_a may be suitably selected.

   [Ans. M_a = 20 kg (selected), K_a = 2.5 × 106 N/m]

3. A machine has mass of 3000 kg and is subjected to a harmonic force of 400 sin 314.6 t newton.

   To reduce the vibrations, damped absorber is used. Its mass is 300 kg. Find optimum values of absorber stiffness and damping ratio.

   [Ans. $k_a = 29.692 \times 10^6$ N/m, $\varsigma = 0.028$]