CHAPTER
Shear Force and Bending Moment in Beams and Frames

2.0 INTRODUCTION

To bridge a gap in land on earth is most pressing problem for structural engineers. Slabs, beams or any combination of these are extensively used in bridging gaps. Construction of bridges, covering of door and window openings and covering of roof of enclosures are examples where beams and slabs are used. The space below a beam is available for other use. Structural actions of beams and slabs are slightly different. A beam collects floor load and transfers it longitudinally to the supports, which are usually provided either at both ends (beam action) or at one end only (cantilever action). Cantilevers are used in construction of balconies in homes and footpaths in bridges. A slab transfers floor load in both directions whereas a beam transfers floor load in only longitudinal direction. Therefore, a beam is a one-dimensional structural element and it can be represented by a line as shown in Fig. 2.1(b) in which arrow shows the direction of load transfer. Mechanical engineers extensively use cantilevers. The boom of a crane, the cutting blade of a bulldozer and wings of a fan are few examples of cantilevers. The span of beam or cantilever, which is defined as distance between adjacent supports, governs the complexity of its design. Shear force and bending moment diagrams quantify structural action of beams and cantilevers and are required for their rational design. Beams and cantilevers shown in Fig. 2.1 are known as flexural elements.

Fig. 2.1  Conventional simple beam supported at both ends. The beam supported at one end only is known as cantilever. Arrow shows the direction of load transfer.
Load transfer in a frame involves axial action as well as flexural action. The load from beams goes to columns and then to the foundations. In general, columns are subjected to axial force as well as bending moment. A frame member is a combination of columns and beams and therefore, is known as beam-column. Several single-storey and multi-bay, and multi-storey and multi-bay frames are shown in Chapter 1 of this book.

2.1 DEFLECTED SHAPES AND POINT OF COUNTER FLEXURE

Figure 2.2 shows several commonly encountered beams. Dotted lines show its expected (exaggerated) deflected shape under a uniform downward loading applied on the top surface of the beam. The deflected shape of a propped cantilever, built-in beam and continuous beams requires further explanation. This is done in Fig. 2.3. The deflected shape of a beam must satisfy certain conditions, for example, it must remain attached to the supports and it must form a continuous curve without any discontinuity. When deflected shapes contain sagging as well as hogging type deflections, the point which separates these different types of deflections is known as point of counter flexure or inflexion point. The bending moment changes sign at the location of inflexion point. There is hogging type of bending at one side of inflexion point and sagging type of bending on its other side. Zero must occur in crossing from positive to negative. Hence, value of bending moment is nil at the location of inflexion point.

![Deflected shapes of beams](image)

**Fig. 2.2** Deflected shapes of beams when subjected to uniform gravity load. The deflected shape must remain attached to the supports when supports do not yield. The deflected shapes in the middle of span and in the region of fixed support/internal support are of opposite type.

In the case of a single span simply supported beam, the inflexion point coincides with the supports so that bending moment is nil at supports. Knowledge of the expected
deflected shape of a beam under the action of external loads is helpful in visualization of bending action.

![Diagram showing types of bending: Sagging, Hogging, Point of counter flexure, No moment](image)

**Fig. 2.3** The type of bending changes at an inflexion point and its consequence is that there cannot be any moment at the inflexion point.

### 2.2 SIGN CONVENTION

The results of flexural analyses, i.e., shear force and bending moment, are presented graphically for effective visualization of response of beam to external loading. Bending can be of sagging and hogging types and shearing action can be in upward or downward directions. It is necessary to establish a sign convention so that results of analysis can be correctly presented and correctly interpreted. Figure 2.4 shows the sign convention for shear force and bending moment. Left up and right down defines the positive shear force. Bending moment which causes sagging type of bending is considered positive. Different authors choose different sign conventions. A reader must be careful while consulting different textbooks.

![Sign conventions for shear force and bending moment](image)

**Fig. 2.4** Sign conventions for shear force (left up and right down is positive). The bending moment which causes sagging is positive. Hogging bending moment is negative.

### 2.3 ASSUMPTIONS

Following assumptions are made in the subsequent analysis.

- A beam shall be considered mass less in the subsequent analysis. The weight of the beam may be clubbed with the magnitude of external actions if it is necessary to take beam weight into account.

- The beam cross-section is uniform so that it is drawn as a line, which coincides with the axis of the actual beam. The size of beam can be taken into account by assigning finite moment of inertia to the line. Value of moment of inertia is not needed in the calculation of shear force and bending moment in simple beams and cantilevers. But it is needed in calculation of deflection.

- Deflections are assumed to be smaller as compared with the dimensions of structure.
2.4 DIFFERENTIAL EQUATION OF A BEAM SEGMENT

2.4.1 Analytical Derivation

Figure 2.5 shows a beam element of incremental length \( dx \). All external and internal actions are shown in their positive sense. The external load acting towards the beam is positive. The beam from which this element is derived is in equilibrium under the external load and support conditions so its element must also be in equilibrium. The three equations of equilibrium are applied to this beam element.

\[
\Sigma F_X = 0 + \rightarrow \text{Horizontal equilibrium is automatically satisfied because all the forces are vertical.}
\]

\[
\Sigma F_Y = 0 + \uparrow \text{Vertical equilibrium gives } V - (V + dV) - w \, dx = 0 \tag{2.1}
\]

Equation 2.1 on simplification gives,

\[
\frac{dV}{dx} = -w \tag{2.2}
\]

Take moment of all forces acting on the beam element (Fig. 2.5) about right hand bottom edge (point \( O \)) with clockwise moments as positive. The distributed load on beam is replaced by its statically equivalent load of magnitude \((w \cdot dx)\) which acts at the centroid of the load diagram.

\[
M - (M + dM) + V \cdot dx - w \cdot dx \cdot dx / 2 = 0 \tag{2.3}
\]

Simplification of Equation 2.3 gives,

\[
-dM + V \cdot dx - w \cdot dx^2 / 2 = 0 \tag{2.4}
\]

Neglect high order terms such as the one containing product of two small quantities \((dx^2)\). Equation 2.5 is an intimate relationship between bending and shearing actions.

\[
\frac{dM}{dx} = V \tag{2.5}
\]

2.4.2 Differential and Integral Forms of Differential Equation

Equations 2.2 and 2.5 are known as differential equations of equilibrium of beam element. These have integral form also which are as follows.
Differential form
\[
\frac{dV}{dx} = -w \quad \frac{dM}{dx} = V
\] (2.6)

Integral form
\[
V = \int -w \, dx + C \quad M = \int V \, dx + C
\] (2.7)

Alternate integral form
\[
V_2 - V_1 = \int -w \, dx \quad M_2 - M_1 = \int V \, dx
\] (2.8)

The first differential equation of Equation 2.6 relates to the slope of shear force diagram with the intensity and orientation of external loading. The second differential equation of Equation 2.6 relates to the slope of bending moment diagram with the magnitude and sense of shear force. The usual problem of structural engineering is to derive shear force and bending moment diagrams for a given flexural member and given loading. This can be achieved by using Equations 2.7 and 2.8. However, loading on the beam for a given bending moment diagram can also be derived by using Equations 2.6. Second expression in Equation 2.6 converts the given bending moment diagram into the corresponding shear force diagram. Now first expression in Equation 2.6 converts shear force into external loading. This procedure shall be illustrated in the following section.

Given the loading on a beam, first expression in Equation 2.8 is used to construct shear force diagram. Then second expression in Equation 2.8 gives bending moment. The first integral expression Equation 2.8 says that the change in shear from station \(x = x_1\) to station \(x = x_2\) is equal to the area under the load diagram between these limits. Similarly, the second expression of Equation 2.8 says that change in bending moment between station \(x = x_1\) to station \(x = x_2\) is equal to the area under the shear force diagram between these limits. This procedure can be successively applied to construct shear force and bending moment diagrams from given loading. The second expression of Equation 2.6 shows that maximum moment occurs at the location where shear force is zero (i.e., \(dM/dX = 0\)). This statement is of great practical significance.

2.4.3 Study of Differential Equations in Differential Form

Equation 2.6 shows that magnitude of external load is equal to slope of line denoting shear force and magnitude of shear is slope of line denoting bending moment. Loading is zero between point loads acting on a beam. When \(w = 0\), then slope of line denoting shear force is zero (\(V = \text{constant}\)), i.e., it is parallel to the x-axis. The corresponding line denoting bending moment is inclined at a uniform slope (\(M = Vx + \text{constant}\)) the slope of which is derived from the sign of shear force.

When \(w = \text{constant}\) (uniformly distributed load), the slope of shear force line is constant and linear (\(V = -wx + \text{constant} C_1\)). The slope of line denoting bending moment varies linearly and so it is non-linear (parabolic: \(M = -wx^2/2 + C_1x + C_2\)). Similar conclusions can be drawn for other variety of loading such as a triangular load in which the load is a linear function of distance (= \(-wx + C_1\)). Now, shear force variation is parabolic (= \(-wx^2/2 + C_1x + C_2\)). Variation of bending moment is cubic (= \(-wx^3/6 + C_1x^2/2 + C_2x + C_3\)).
2.4.4 Study of Differential Equations in Integral Form

Consider that \( w = 0 \), i.e., the beam segment has no loading. It would happen in the region between point loads. Equation 2.7 predicts that the shear force is constant whereas bending moment varies linearly. First expression in Equation 2.8 predicts that the value of shear force from station 1 to 2 shall not change. Second expression in Equation 2.8 predicts that change in bending moment from location 1 to location 2, however, is equal to the area under the shear force diagram between locations 1 and 2.

Consider that \( w = \) constant, i.e., the beam segment has uniform loading. It would happen in a beam subjected to self weight. Equation 2.7 predicts that the shear force shall vary linearly whereas bending moment will have parabolic variation. First expression in Equation 2.8 predicts that the change in shear force between stations 1 and 2 is equal to the area under the load diagram between locations 1 and 2. Similarly, second expression in Equation 2.8 predicts that change in bending moment between locations 1 and 2 is equal to the area under the shear force diagram between locations 1 and 2.

Similar conclusions can be drawn if the applied external loading has any other variation. These observations can be used to construct shear force and bending moment diagrams for given loads. Also, if shear force and bending moment diagrams are available, the above observations provide an efficient and quick means to check their accuracy. Following checks can be applied.

1. Change in shear force between locations 1 and 2 is equal to the area of loading diagram between these locations.
2. Change in bending moment between locations 1 and 2 is equal to the area under the shear force diagram between these locations.
3. Maximum bending moment occurs at the location where shear force is zero.

2.5 TIPS ON SHEAR FORCE

The findings of previous sections on shear force diagrams offer the following tips. These can be used in the solution of a general problem containing a combination of several load types. Differential equations of equilibrium can be used to justify several of the following tips. The positive shear is defined in Fig. 2.4.

- The shear force at the end supports of a simply supported beam is equal to the value of reaction forces at the respective locations if no external force acts at the end supports.
- The value of any externally applied load at the support must be added to the reaction force due to other loads at that location with proper sign. Support load adds to column load and causes no shear in beam.
- Point load at the support does not contribute to span shear.
- When a portion of simply supported beam or cantilever is not loaded, shear force is constant in this range.
• When a portion of simply supported beam or cantilever has uniform load, the shear force varies linearly in this range.
• Shear force diagram contains a jump/drop at the location of point load and this jump/drop is equal to the value of point load.
• The reaction system corresponding to a moment load must form a couple of equal magnitude but opposite in direction. This reaction system causes uniform shear force in simply supported beam.
• The end portion of a cantilever without any load carries no shear force.

2.6 TIPS ON BENDING MOMENT (BM)

The findings of previous sections on bending moment diagrams can be used in the solution of a general problem containing a combination of several load types. Differential equations of equilibrium can be used to justify several of the following tips. The positive bending is defined in Fig. 2.4.

• The bending moments at the hinge and roller supports at the ends must be zero if no moment is externally applied.
• The net bending moment at the end support must be equal to the value of externally applied moment at that location with proper sign.
• When a portion of simply supported beam or cantilever is not loaded, bending moment varies linearly in this range.
• When a portion of simply supported beam or cantilever has uniform load, the bending moment has quadratic variation in this range.
• The bending moment diagram contains a change of slope (discontinuity or kink) at the location of point load.
• Maximum bending moment occurs where shear force is zero or shear force line crosses the zero line.
• The location of moment load on a simply supported beam is an inflexion point. The bending moment diagram crosses zero axis at this location. Total jump in bending moment diagram at the location of moment load is equal to the value of moment load.
• The end portion of a cantilever without any load carries no bending moment.
• If a beam is supporting several point loads, maximum BM occurs under one of the point load.

2.7 USE OF DIFFERENTIAL EQUATIONS IN DRAWING OF SFD AND BMD

Equations 2.7 and 2.8 presented in Section 2.4 of this chapter give an alternate tool to construct shear force (SFD) and bending moment diagrams (BMD). It is possible to use Equation 2.2 to draw shear force diagram from the load diagram. Similarly, Equation 2.8 can be used to draw bending moment diagram entirely from the shear force diagram.
The procedure to be developed in this section is based on the following facts. 

(a) Equation 2.8 shows that change of ordinate is equal to the area under the diagram. Successive application of first expression of Equation 2.8 converts load diagram into shear force diagram. Then second expression of Equation 2.8 converts shear force diagram into bending moment diagram.

(b) The order of slope of various curves increases by one from load to shear force to bending moment. A uniform load is a curve of zero slope. It leads to linear variation in shear force and a quadratic variation in bending moment.

(c) An unloaded span may be given slope of –1 so it gives uniform shear and linear variation in bending moment.

(d) A point load may be considered a uniform load which is distributed over a very small area. Therefore, area under a point load may be taken as the value of point load itself.

(e) A triangular load varies linearly. It gives quadratic variation in shear force and bending moment has cubic variation.

Consider the beam shown in Fig. 2.6. Its shear force and bending moment diagrams are to be drawn. Standard procedure is used to calculate reactions of this beam. The distributed load is replaced by its statically equivalent load. This is shown by a dotted arrow in Fig. 2.6(b). The final result of calculation is \( H = 0, R_A = 53.1 \text{ kN} \) and \( R_B = 68.6 \text{ kN} \).

**Observations:** Following qualitative and quantitative observations of the load diagram shall be helpful in drawing of shear force and bending moment diagrams.

**Construction of shear force diagram:**
- Negative shear force of 40 kN at end C of beam shall occur.
- The shear force in span CA is constant.
There is a jump in shear at $A$. The jump is equal to magnitude of reaction $R_A = 53.1$ kN. This jump is divided into 40 kN below the axis and remaining above the axis.

Shear force in range $AE$ varies linearly. Difference in ordinates between location $A$ and $E$ is $0.4 \times 0.75 = 0.3$ kN. This is equal to the area under the load diagram between location $A$ and $E$. The shear force at $E$ is then $13.1 - 0.3 = 12.8$ kN.

There is a drop in shear at $E$ of magnitude 20 kN so net shear force at $E$ is $-7.2$ kN.

The shear force varies linearly between $E$ and $B$. The difference in ordinates between $E$ and $B$ is equal to the area of distributed load between $E$ and $B$. This is equal to 0.8 kN. Therefore, shear at $B$ is $-8.0$ kN.

There is a jump at location $B$ of magnitude 68.6 kN. Therefore, shear at support $B$ jumps from $-8.0$ kN to $+60.6$ kN.

Shear from location $B$ to $F$ decreases because of UDL. The magnitude of decrease is equal to the area under the load diagram which is $0.4 \times 0.75 = 0.3$ kN. So shear at $F$ is $60.3$ kN.

There must be a drop at $F$ corresponding to the point load of magnitude 60 kN. Shear of magnitude $0.3$ kN remains at $F$.

The shear must decrease from location $F$ to $D$ by a magnitude $0.4 \times 0.75 = 0.3$ kN. Thus, shear force at location $D$ becomes identically zero.

**Construction of bending moment diagram**

- The overhangs carry heavy loads as compared with the span load. So the beam is likely to hog and develop negative moment throughout the entire span.
- The ends of beam do not carry any external bending moment so moment at $C$ and $D$ must be zero.
- Beam does not have any concentrated moment at any location.
- The difference in ordinates in bending moment diagram between $C$ and $A$ is equal to the area under shear force diagram between these locations. This area is 40 kN-m. The variation is linear because shear force is constant in this range.
- The difference in ordinates in bending moment between $A$ and $E$ is equal to the area under the shear force diagram between these locations. This area is $12.8 \times 0.75 + 0.3 \times 0.75/2 = 9.7125$ kN-m. The ordinate at location $E$ is $40 - 9.7125 = 30.2875$ kN-m. The variation is quadratic because shear force varies linearly in this range.
- Starting from right end $D$ the difference in ordinate in bending moment between $F$ and $D$ is equal to the area under shear force diagram between these locations. This is $0.7580.3/2 = 0.1125$ kN-m. The variation is quadratic.
- The change in ordinates in bending moment between $B$ and $F$ is equal to the area under the shear force diagram between these locations. This is $60.3 \times 0.75 + 0.3 \times 0.75/2 = 45.3375$ kN-m.
Bending moment in the range $AD$ had parabolic variation.

Bending moment is either minimum or maximum when shear force is zero.

### 2.8 LOAD TRANSFER AND BENDING MOMENT IN COMPLEX BEAMS

Free body diagrams are essential to understand load transfer in structures. Consider the following statically determinate structures. Figure 2.4 describes sign convention for SF and BM. **Bending moment is drawn on the compression side of member.** Study Fig. 2.8.

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**Contd....**
Fig. 2.8 Several statically determinate complex beams. The reactions are calculated and then free body diagram is drawn by dissecting the structure. Consider equilibrium of each component. The bending moment diagram is then drawn. The tips given in Sections 2.5 and 2.6 must be remembered.
2.9 LOAD TRANSFER AND BENDING MOMENT IN SIMPLE FRAMES

Free body diagrams are essential to understand load transfer in structures. Consider the following statically determinate structures. Figure 2.4 described sign convention for SF and BM. **BENDING MOMENT IS DRAWN ON THE COMPRESSION SIDE OF MEMBER.** Study Fig. 2.9.

(a) Structure, free body diagram and bending moment diagram of a simple frame

(b) Structure, free body diagram and bending moment diagram of a simple frame

(c) Structure, free body diagram and bending moment diagram of a simple frame

(d) Structure, free body diagram and bending moment diagram of a simple frame

Contd....
Several statically determinate complex frames. The reactions are calculated and then free body diagram is drawn by dissecting the structure. The bending moment diagram is then drawn.

**2.10 LOAD TRANSFER AND BENDING MOMENT IN COMPLEX FRAMES**

Study Fig. 2.10

(a) Structure, free body diagram and bending moment diagram of a simple frame

(b) Structure, free body diagram and bending moment diagram of a simple frame. This frame is statically indeterminate. The ends of beam in this frame are partially fixed. Analysis is fixed ended beams is given in the next chapter. Drawing of shape of bending moment diagram is still possible.

**Fig. 2.10** Several complex frames. The reactions are calculated and then free body diagram is drawn by dissecting the structure. The bending moment diagram is then drawn.
A typical structure may be subjected to a variety of load types. Important load varieties are as follows:

**Dead Load:** This load type is usually proportional to the shape of structural element and its magnitude is derived by multiplication with the unit weight of material. In addition, this load type always has fixed distribution as long as the shape of structural element remains fixed. This load always acts in the direction of gravity.

**Live Load:** This load has variable distribution in space and time. The live load in a classroom building varies from day to day and from time to time because students come and go depending upon time table of classroom teaching. Most bridge structures are subjected to variable loads. A vehicle moves across the road bridge and a train passes over a railway bridge. This loading occurs when a train comes, moves across the span and bridge is unloaded thereafter. This load type is usually vertical.

**Wind Load:** This load is horizontal and varies in space and time. But this load is not a rolling load. This load is associated with cyclones and storms. The duration and magnitude of this loading is usually predictable with sufficient accuracy. Also, storms provide sufficient warning before they strike.

**Earthquake load:** This load is most dangerous. It occurs once in a while but can cause severe damage when it occurs because severity of this loading as well as its timing cannot be predicted. The damage is usually determined by the state of structure at the time of loading. This also is not a rolling load.

**Traffic Load:** Most railway and road bridges are subjected to moving or rolling loads. This variety of loading is different from the above loads.

A designer of a structure is always interested in maximum values of internal actions such as shear and bending moment. When loading is fixed, shear force diagram and bending moment diagram are drawn to answer these questions. Rolling loads such as live and traffic loads present additional difficulties. (1) What is the distribution of rolling loads? (2) What is the position of load which would cause maximum shear and bending in the structure? (3) What is the magnitude of maximum shear and bending? The maximum shear and bending moment at a given section and the absolute maximum values are also different. In the case of absolute maximum, both the location and magnitude are unknown.

**Distribution of Rolling Loads** is readily available in the publications of Indian Roads Congress. The loading under a wheel is known as axel load. The magnitude of axel load and spacing between loads is specified for a variety of vehicle types. A typical distribution of rolling loads consists of number of loads, their magnitude and spacing between them. It appears as shown in Fig. 2.11.

The shear force and bending moment diagram are drawn for a **FIXED** distribution of load. The ordinate of these diagrams gives magnitude of action at that location. Therefore, visual inspection is sufficient to locate magnitude of maximum action. Visual inspection does not work when position of loading is variable. **Influence line**
diagrams are drawn to answer the questions regarding POSITION of load which causes maximum actions and MAGNITUDE of maximum actions under rolling loads.

2.11.1 Mechanics of Influence Lines

Influence line diagram is drawn for a particular function which could be any of the following.

- **Reaction component** at a support, for example, vertical reaction at a support in a beam or horizontal thrust in an arch.
- **Shear force** at a particular location.
- **Bending moment** at a particular location.
- **Deflection** at a particular location.
- **Force** in a particular member of a truss.

Influence line diagram is drawn for a unit load which is moved across the span of the structure. It can take any position in a beam but it has to move from joint to joint in a truss. The value of the above particular function is plotted under the load position which causes this value. Thus, an ordinate of an influence line diagram gives value of function for a unit load acting at that location. It is conventional to draw influence line diagrams for a unit load. The advantage is that the ordinate of influence line diagram is multiplied by the value of load to obtain value of function for that value of load.

**Definition:** An influence line diagram is a curve the ordinate of which at any location equals the value of some particular function due to a unit load acting at that location.

An influence line diagram of a function provides answers to the following questions.

1. What is the position of the given rolling load which causes maximum value of function?
2. What is the maximum value of the function?
3. What is value and position of load for absolute maximum value of function?

Influence line diagrams can be drawn for a beam, a truss and an arch. The discussion in this section is restricted to statically indeterminate structures only. Influence lines for determinate structures are covered in a companion book on determinate structures. Influence line diagrams can be drawn in two varieties (1) quantitative and (2) qualitative. Quantitative method for determinate structures has two varieties (1) tabulation and (2) analytical equation. Qualitative method is used for drawing influence line diagrams for indeterminate beams.

2.11.2 Properties of Influence Lines

- Influence line is drawn for a particular function or action such as shear force at a location, bending moment at a location, reaction force at a support, etc.
Strength and Deformation of Statically Indeterminate Structures

- Influence line is constructed by moving a unit load across the span of the structure. The value of action at the specified location is computed for each placement of unit load.
- Ordinate of influence line at a location gives value of function or action if a unit load was placed at that location.
- Live loads are usually movable. Live loads can be in the form of a concentrated load, a pair of concentrated loads, a train of concentrated loads or a distributed load on a specified length.
- Value of loads and their spacing for use in design of real structures are specified by Indian Road Congress specifications. Values of live loads for buildings are also given by Bureau of Indian Standards.

2.11.3 Construction of Influence Lines

Analytical expression for the required function is derived. Its plot then gives the influence line. Quantity \( z \) is variable in the following derivations. Some simple solutions are subsequently derived. These shall then be derived by a graphical method to illustrate and establish this method.

**Solution 1:** Draw influence line for reaction at supports for a simply supported girder of span \( L \). For any placement of a unit load at a distance \( z \) from the left end, a simple analysis shows that \( R_A = (L - z)/L \) and \( R_B = z/L \). A plot of these functions for various values of \( z (0 < z < L) \) gives the influence line for \( R_A \) and \( R_B \), respectively. The unit of reaction shall be force per unit force.

![Influence line diagram for support reaction](image)

**Fig. 2.12** Plots influence line diagram for support reaction.
Solution 2: Draw influence line for shear at section C in a simply supported girder of span L. The free body diagrams must be drawn to determine shear at section C for placement of a unit load at distance z from the left end. The unit load can be placed either to the left of section C or to its right-hand side. The shear force in both the cases is determined from the following free body diagrams. The sign of shear force is determined from the sign convention that “left up and right down” is positive. These equations are now plotted with z as variable to obtain influence line for shear force at section C.

Fig. 2.13 Influence line diagram for shear at section C.

Shear force at section C = -z/L for (0 < z < x) and (L - z) for (x < z < L).

Solution 3: Draw influence line for bending moment at section C in a simply supported girder of span L. The procedure is again same as has already been described. Fig. 2.14 has the required details.

Bending moment at section C = z (L - x)/L for (0 < z < x) and = x (L - z)/L for (x < z < L)

Bending moment at section C when (z = x) = x (L - x)/L
The above expressions are plotted to obtain influence line of bending moment at section C. Notice that z is the variable. Figure 2.14 gives the plot of influence line.

**Solution 4:** Draw influence lines for cantilevers. These are shown in Fig. 2.15.

**Solution 5:** Draw influence line for vertical deflection at section C in a simply supported beam.

Let $\delta$ be the deflection. The following terminology is introduced. The deflection at a point shall contain two subscripts such as $\delta_{mn}$. The first subscript denotes the point where the deflection is measured. The second subscript identifies the point where the
load causing deflection is applied. Thus, deflection at point \( m \) due to a load action at point \( n \) is denoted as \( \delta_{nm} \).

![Diagram of influence line for deflection at section C.]

Consider the simply supported beam. The influence line for deflection at section C is to be drawn. As described in Figs. 2.16(a) and 2.16(b), the procedure requires a unit load to be placed at various points along the beam and to measure deflection at section (C) for each case. The unit load placed at points \( m \) and \( n \) produces influence coefficients \( \delta_{cm} \) and \( \delta_{cn} \).

Now apply the Maxwell reciprocal theorem of deflections. In any structure, the material of which is elastic and follows Hooke’s law, and in which the supports do not yield and temperature is constant, deflection of point 1 in the direction \( ab \) due to a load at point 2 in a direction \( cd \) is numerically equal to the deflection of point 2 in a direction \( cd \) due to a load \( P \) at point 1 in a direction \( ab \).

Simply stated, \( \delta_{mn} = \delta_{nm} \)

Refer to Fig. 2.16, \( \delta_{nc} = \delta_{cn} \) and \( \delta_{nc} = \delta_{cn} \)

\( \delta_{nc} \) and \( \delta_{nc} \) are deflections produced at points \( n \) and \( m \), respectively by a unit load placed at section C.

In other words, the elastic curve of the beam when a unit load placed at section C is the influence line for vertical deflection at section C.

### 2.11.4 Muller–Breslau Principle

This principle gives qualitative shape of influence line diagrams for determinate and indeterminate beams. This is presented in Chapter 8 of this book.

### 2.12 HIGH ALERT

- Flexural elements consist of beams, cantilevers and their combination.
- Simple beam contains one hinge and one roller support.
- A continuous beam contains several supports and several spans.
Strength and Deformation of Statically Indeterminate Structures

• A simple cantilever contains one fixed support.
• External load on a flexural element travels longitudinally. It travels in both directions in a beam whereas it travels towards the supported end in a cantilever.
• A cantilever attachment with end span of a beam is called overhang.
• Plane of bending divides a flexural element in two longitudinal halves, and deflections and structural actions occur within this plane.
• Flexural action causes development of bending moment and shear force.
• Left up and right down shear force is positive.
• Bending moment which causes sagging is positive whereas bending moment which causes hogging is negative.
• Shear action always occurs in pair and tends to cause splitting.
• Bending moment causes rotation of imaginary vertical lines in the flexural element.
• The deflected shape of a flexural element must be continuous without a discontinuity and must remain attached to the supports.
• The deflected shape of a beam may consist of hogging and sagging portions. Locations where hogging and sagging portions join are called inflexion point or point of counter flexure.
• Bending moment is zero at inflexion point because zero must occur while crossing over from positive to negative.
• A diagram which shows variation of shear force along the flexural element on account of external load is called shear force diagram.
• A diagram which shows variation of bending moment along the flexural element on account of external load is called bending moment diagram.
• The differential equations of bending relate magnitude of external load with the slope of shear force diagram and magnitude of shear with the slope of bending moment diagram.
• The change in shear force between two locations on a flexural element is equal to the area under the load diagram between these locations.
• The change in bending moment between two locations on a flexural element is equal to the area under the shear force diagram between these locations.
• If loading on some segment of a flexural element is zero, then shear force on this segment is constant and bending moment on this segment varies linearly.
• If loading on some segment of a flexural element is uniform, then shear force on this segment has linear variation and bending moment on this segment has parabolic variation.
• If bending moment on a segment of a flexural element varies linearly, the shear force on this segment must be constant and external loading on this segment must be zero.
• If a flexural element is in equilibrium under the action of external loads, any of its part must also be in equilibrium. This fact is used in calculation of internal actions in the flexural element.

• A point load may be considered a uniform load which is distributed over a very small area.

• Shear force and bending moment diagrams are drawn for fixed loading arrangements. These diagrams provide design information to structural engineers for design of structures.

• Ordinates of shear force and bending moment diagrams provide value of action at a location for the fixed load arrangement.

• Bridge structures are subjected to rolling loads. Indian Road Congress and Bureau of Indian Standards provide complete information on distribution and magnitude of rolling loads.

• The design variables like shear force and bending moment now become a factor of position of load. A large number of shear force and bending moment diagrams will be required if conventional design approach is used.

• Influence line diagrams are drawn to overcome this difficulty and are exclusive for rolling loads.

• Design of a structure requires maximum value of shear force and bending moment at a location as the load rolls on the structure.

• Absolute maximum and minimum values of these actions and their location are also needed.

• Influence lines are drawn for particular function. Such functions are support reaction, shear force at a location, bending moment at a location and deflection at a location.

• Value of the above-mentioned functions at a location is calculated as a unit load rolls on the structure. Graphical presentation of this information is known as influence line diagram.

• The ordinate of influence line diagram at a location gives value of the function of influence line if a unit load is applied at that location. This value is known as influence coefficient.

• Quantitative and qualitative methods are available to draw influence lines.

• Quantitative methods are easy to apply to determinate structures but become cumbersome for indeterminate structures.

• Qualitative method such as Muller-Breslau principle applies to determinate and indeterminate structures.
2.1 Use principle of superposition to determine support reactions in the following beams. Apply equilibrium analysis to cross-check your results of computations.

Fig. P.2.1

2.2 Study the following loaded beams. Verify by calculations that shear force and bending moment diagrams as shown here, are correct. List the necessary corrections if any.

Fig. P.2.2 (Contd.)
2.3 Study the beams and loading in Fig. P.2.2 and list features which are expected in corresponding shear force and bending moment diagrams.

2.4 Study bending moment diagrams in Fig. P.2.2 and list the features which are expected in the corresponding shear force and load diagrams.

2.5 Use differential equations of equilibrium (no calculations) to construct shear force and bending moment diagrams of the following beams.
2.6 Use the differential equations of equilibrium to construct shear force and load diagram of the beams for which the bending moment diagrams are as follows.

![Fig. P.2.6](image)

2.7 Draw shear force and bending moment diagrams of the following beams and cantilevers by any method.

![Fig. P.2.7](image)
2.8 Draw free body and BM diagrams in following frames.

![Fig. P.2.8](image-url)