CHAPTER 2

Basic Principles of Hydraulic Flow and Jet Theory

2.1 INTRODUCTION

Hydraulic machines handle water. Machines changing fluid energy into mechanical energy are called hydraulic turbines or hydraulic motors. Machines designed to more liquids and add energy to them are called pumps. There are also special systems dealing with the transmission of hydraulic power. The hydraulic machines are classified in Fig. 2.1.

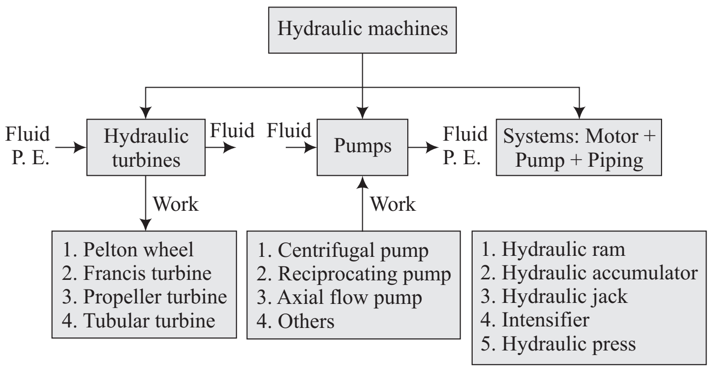


Fig. 2.1 Classification of hydraulic machines

The hydraulic mechanics are designed on the principles of fluid machanics, hydraulic flow and water jet forces.

2.2 PRINCIPLES OF FLUID MECHANICS

Important principles of fluid mechanics required for the design of fluid (hydraulic) machines are summarized below.

2.2.2 Kinematics of Fluid Flow

For a steady flow, the fluid properties like pressure, density, velocity, etc., do not change at a point with respect to time

$$\frac{dV}{dt} = 0, \quad \text{steady flow}$$

For uniform flow of fluid, the velocity does not change with respect to space (length of direction of flow)

$$\frac{dV}{ds} = 0, \quad \text{uniform flow}$$

The density of fluid remains constant for incompressible flow.

$$\rho = \text{constant: Incompressible flow.}$$

For laminar flow in a pipe, Reynolds number is less than 2000. For turbulent flow in a pipe, Reynolds number is more than 4000, Volumetric flow rate,

$$\dot{Q} = A_1V_1 = A_2V_2 = A_3V_3 \quad [\text{m}^3/\text{s}]$$

This is called continuity equation.

2.2.3 Dynamics of Fluid Flow

1. Equation of motion (Newton's second law of motion)

$$F_x = m\,a_x \quad [\text{N}]$$

2. Euler's equation of motion along a stream line,

$$\frac{\partial p}{\rho} + g\partial z + V\partial V = 0$$

3. Bernaulli's equation (integration of Euler's equation of motion for a steady, ideal flow of an incompressible fluid), states that total energy consisting of pressure energy, kinetic energy and potential energy at any point of fluid is constant.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_L\,[m]$$

where $\frac{p_1}{\rho g}$ = pressure head = pressure energy per unit weight,

$\frac{V_1^2}{2g}$ = kinetic head = kinetic energy per unit weight

z_1 = datum head = datum energy per unit weight

h_L = loss of energy head between sections 1 and 2

4. Momentum equation states that net force acting on a fluid mass is equal to change in momentum per second in that direction

$$F = \frac{d}{dt}(\text{mV}) \quad [\text{N}]$$

5. The impulse momentum equation is given as:

$$F\,dt = d\,(\text{mV}) \quad [\text{N-s}]$$

6. The force exerted by the nozzle on water

$$F_x = \rho Q(V_{2x} - \text{V}_{1x}) \quad [\text{N}]$$

7. Moment of momentum equation states that the resultant torque on a rotating fluid is equal to the rate of change of moment of momentum

$$T = \rho Q(V_2 r_2 - V_1 r_1) \quad [\text{N-m}]$$

8. Loss of pressure head for viscous flow through circular pipe

$$h_f = \frac{32\mu \bar{u} L}{\rho g D^2} \quad [\text{m}]$$

where μ = coefficient of viscosity

$\bar{u}$ = average fluid velocity

$$= \frac{Q}{\pi R^2} \text{ (m/s)}$$

D = diameter of pipe [m].

9. *Darcy formula*. Energy loss due to friction,

$$h_f = \frac{4 fLV^2}{2gD} \quad [\text{m}]$$

10. *Darcy Weisbach Equation.* Head loss due to friction in pipes,

$$h_f = \frac{4 fLV^2}{2gD} \quad [\text{m}]$$

where f = Coefficient of friction

$$= \frac{16}{\text{Re}} \text{ for laminar flow}$$

$$= \frac{0.0791}{(\text{Re})^{1/4}} \text{ for turbulent flow.}$$

11. The velocity of water at the outlet of the nozzle,

$$V = \sqrt{\frac{2gH}{1 + \frac{4fL}{D} \cdot \frac{a^2}{A^2}}} \quad \text{[m/s]}$$

where H = head at inlet of pipe [m]

L = length of pipe [m]

D = diameter of pipe [m]

a = area of nozzle outlet [m^2]

A = area of pipe [m^2]

12. The power transmitted through nozzle,

$$P = \frac{\rho g Q}{1000}\left[H - \frac{4fLV^2}{2gN}\right] \quad \text{[kW]}$$

2.3 BASIC CONCEPTS OF HYDRAULIC FLOW

The hydraulic turbines utilize the potential energy of water to produce mechanical work. The pumps are required to transfer water and other liquids. The fluid couplings, torque converters and fluid system work with special oils.

2.3.1 Fluid Characteristics

The following fluid characteristics are assumed for the working substance used in hydraulic machines and systems.

1. Ideal fluid

An ideal fluid is an imaginary fluid which is both incompressible and non-viscous. Such liquids do not exist in nature. However, water is assumed to be incompressible and has very low value of viscosity. Therefore, it is nearly an ideal fluid. For incompressible fluid, the water density is constant.

$$\rho = \text{const.}$$

2. Newtonian fluid

The oils used in power transmission machines and fluid systems are also incompressible. These oils are assumed to be Newtonian fluids whose viscosity is independent of velocity gradient,

$$\tau = \mu \frac{du}{dy}$$

2.3.2 Closed and Open Systems

A system is a quantity of matter in space upon which attention is made in the study of changes of properties and analysis of a problem. Everything external to the system is called the surrounding. The boundary separating the system from the surrounding may be real (solid) or imaginary.

There may be energy transfer into or out of the system.

1. *Closed system.* A closed system has fixed identity with fixed mass. There is no mass (fluid) transfer across the system boundary.

   All positive displacement machines such as reciprocating pumps, gear pumps, etc. torque converters, fluid couplings, various fluid power systems and fluid control systems will be analyzed as constant mass closed systems.
2. *Open system.* In an open system, the fluid crosses the boundary of the system in addition to interaction of energy between the system and the surrounding. The mass of an open system may or may not change. The identity of the fluid changes continuously. The boundary of the open system is kept fixed without any change in its volume. An open system is also referred to as *control volume system*. The closed boundary of a control volume is called the *control surface*. There is a transfer of both mass and energy across the control surface. All types of hydraulic turbines, centrifugal pumps, axial flow pumps, slurry pumps, jet pumps, pneumatic lift pumps, various supply and disposal fluid systems will be analyzed and studied as control volume systems.

2.3.3 Macroscopic Properties

Matter is composed of several molecules and description of the motion of a fluid will consider the behaviour of discrete molecules which constitute the fluid. The microscopic properties can be found out by statistical summation of properties of individual molecules.

Liquids have extra-strong intermolecular attractive forces. Therefore, molecular description is not required. The entire liquid mass behaves as a continuous mass. The time-averaged values of properties are sufficiently accurate and valid. The matter of the system is assumed as a continuous distribution of mass with no empty space and no conglomeration of separate molecules. The fluid machines and system will be designed and analyzed on the basis of time-averaged macroscopic properties measured with the help of instruments. This is called the concept of *continuum* model.

2.3.4 Conservation of Mass

In non-nuclear processes, the matter can neither be created nor destroyed. The conservation of mass is inherent to the concept of a closed system. For a control volume, the rate of mass entering the system must be equal to the rate of mass leaving the system plus the rate of storage of mass in the system. If the fluid flow is steady, the rate of liquid stored is zero.

2.3.6 Governing Equation

The flow of liquid through hydraulic machines is divided in imaginary stream tubes and the behaviour of the hydraulic flow is studied with the help of the following governing equations.

1. *Continuity Equation*

   This is based on the principle of conservation of mass flow ($\dot{m}$).
2. *Energy Equation*

   This is derived from the principle of conservation of energy ($\dot{m}$ V^2)
3. *Equation of Motion*

   This is based on the principle of conservation of momentum ($\dot{m}$ V).

2.4 CONTINUITY EQUATION

The continuity equation of flow is derived on the principle of conservation of mass. For an incompressible, uniform and steady state flow, the quantity of fluid entering at one end of stream tube and leaving at the other must be same provided there is no sink or source in the tube.

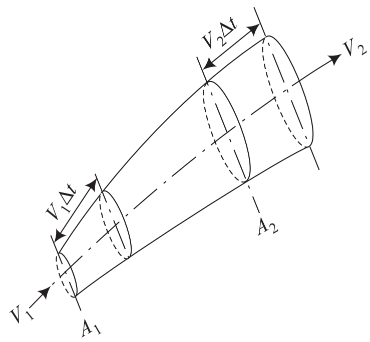


Fig. 2.2 Flow through a stream tube

Consider a stream tube with two arbitrary cross-sections A_1 and A_2, the mass of fluid enters the tube through A_1 with a velocity V_1 and leaves through A_2 with velocity V_2. In an increment of time Δt, the fluid particles entering A_1 have moved an infinitesimal distance $V_1\Delta t$. The particle at A_2 have moved a distance $V_2\Delta t$. By the definition of a stream tube no fluid can cross the stream tube. The mass of fluid entering the stream tube at A_1 in time Δt must be exactly equal to the mass of the fluid leaving the stream tube at A_2 during the same interval of time. For a steady flow, the rate of fluid stored is zero. Mathematically, the statement can be expressed as follows:

$$\dot{m}_1 = \dot{m}_2$$

$$\rho\dot{Q}_1 = \rho\dot{Q}_2$$

$$\rho A_1 V_1 \Delta t = \rho A_2 V_2 \Delta t$$

For incompressible flow, density ρ is constant.

$$\therefore \quad V_1 A_1 = V_2 A_2 = VA = \text{Constant} = \dot{Q} \quad [\text{m}^3/\text{s}]$$

This is called equation of continuity and the constant $\dot{Q}$ represents the volumetric flow rate. Therefore, the volume of fluid which passes through each cross-section of the stream tube per unit time remains constant for a steady flow and incompressible fluid.

2.5 THREE-DIMENSIONAL FLOW

In a three-dimensional fluid flow, the fluid properties (velocity, pressure, density, viscosity) may vary in all directions.

Examples are: flow in a river, flow within a fluid machine. However, for simplicity, the fluid machines are analyzed as one-dimensional fluid flow.

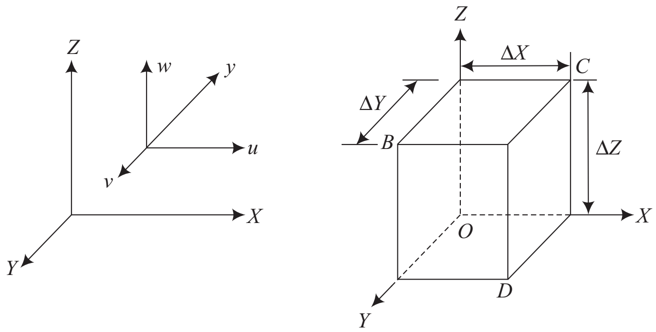


Fig. 2.3 Three-dimensional flow

Consider an elementary cube Δx, Δy, Δz in the fluid body. The difference between the amounts of fluid which flows into and out of its faces during time Δt, must be equal to the increase in the mass which the edges enclosed.

The mass of fluid of density ρ entering the cross-section across the face *OB* in time $\Delta t = \rho u(\Delta y.\Delta z)\Delta t$.

The mass of fluid leaving across the face *CD* in time $\Delta t = \rho u(\Delta y.\Delta z)\Delta t + \frac{\partial}{\partial x}(\rho u \Delta y \Delta z \Delta t)\Delta x$

$\therefore$ Gain of mass across the above faces $= -\frac{\partial}{\partial x}\rho u(\Delta x.\Delta y.\Delta z)\Delta t$

Similarly, gain of mass across the faces *BD* and *OC* $= -\frac{\partial}{\partial y}\rho v(\Delta x.\Delta y.\Delta z)\Delta t$

And, gain of mass across the faces *BC* and *OD* $= -\frac{\partial}{\partial z}\rho w(\Delta x.\Delta y.\Delta z)\Delta t$

$$\text{Total gain} = -\left(\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w\right)(\Delta x.\Delta \text{y}.\Delta z)\Delta t. \qquad \text{...(1)}$$

Mass of fluid contained at time $t = \rho(\Delta x.\Delta y.\Delta z)$

Mass of fluid contained at time $t + \Delta t = \rho(\Delta x.\Delta y.\Delta z) + \frac{\partial}{\partial t}(\rho.\Delta x.\Delta y.\Delta z)\Delta t$

2.6 MOMENTUM EQUATION

The momentum equation is based on Newton's second law of motion which states that the time rate of change of momentum is proportional to the applied force and takes place in the direction of force.

Assume a resultant force F_x acts on a mass m along x-axis. The change in velocity of mass m in time dt be dV.

$$\therefore \qquad F_X = m.\frac{du}{dt} \qquad ...(3)$$

or

$$F_X \, dt = m \, du \qquad ...(4)$$

Equation (3) is called linear momentum equation. Equation (4) is called impulse-momentum equation which shows that impulse of applied force ($F_X.dt$) is equal to change of momentum ($m.du$).

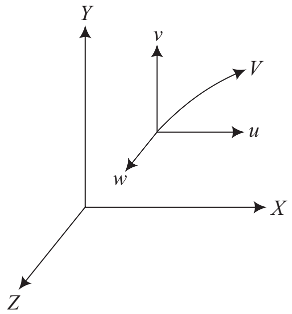


Fig. 2.6(a) Velocity components

Consider a stream tube. For steady state flow, mass entering the tube is equal to mass leaving the tube.

$$\rho_1 A_1 V_1 \Delta t = \rho_2 A_2 V_2 \Delta t$$

Momentum of fluid entering

$$= (\rho_1 A_1 V_1 \Delta t) V_1$$

Momentum of fluid leaving

$$= (\rho_2 A_2 V_2 \Delta t) V_2$$

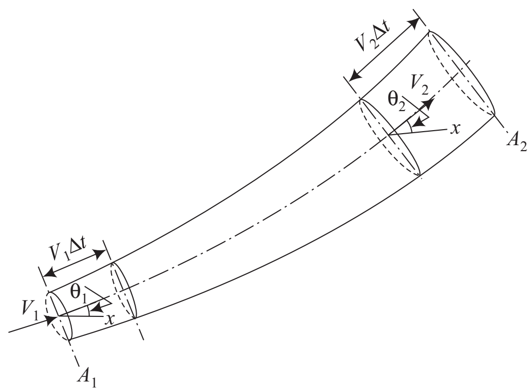


Fig. 2.6(b) Momentum equation

$\therefore$ Change of momentum in x-direction,

$$= [(\rho_2 A_2 V_2 \Delta t) V_2 \cos \theta_2] - [(\rho_1 A_1 V_1 \Delta t) V_1 \cos \theta_1]$$

But for a steady state, equation of continuity:

$$(\rho_1 A_1 V_1 \Delta t) = (\rho_2 A_2 V_2 \Delta t) = \rho Q \Delta t$$

∴ Change of momentum in x-direction,

$$= \rho Q \Delta t\,(V_2 \cos \theta_2 - V_1 \cos \theta_1)$$

From impulse-momentum equation,

$$\sum F_x \,\Delta t = \sum \rho Q\,\Delta t (V_2 \cos \theta_2 - V_1 \cos \theta_1)$$

$$F_x = \rho Q (V_2 \cos \theta_2 - V_1 \cos \theta_1) = \rho Q (v_{x_2} - v_{x_1}]$$

Similarly, $$F_y = \rho Q (V_2 \sin \theta_2 - V_1 \sin \theta_1) = \rho Q (v_{y_2} - v_{y_1})$$

Resultant force acting on the flowing fluid,

$$F = \sqrt{F_x^2 + F_y^2}$$

The angle between F and F_x can be found out as:

$$\tan \theta = \frac{F_y}{F_x}$$

The flowing fluid will also exert an equal and opposite force on the boundary of the stream tube. The forces may include dynamic force due to change of momentum, static pressure, weight of fluid, drag force due to friction, gravity and inertia forces due to centrifugal force.

2.7 APPLICATIONS OF MOMENTUM EQUATION

There are two types of applications of Impulse-Momentum equation.

1. Determination of forces exerted by the flowing fluid on the boundaries of flow passage of hydraulic machines due to change of momentum. The examples are:
   (i) Forces caused by a fluid jet striking a surface, i.e., fixed and moving blades of hydraulic machines.
   (ii) Jet propulsion.
   Propulsion of ships, boats, rockets, turbojet, ramjet, etc.
   (iii) Propellers.
   Marine propellers, helicopters.
   (iv) Pipe bends and reducers
2. Determination of flow characteristics due to energy loss in the flow systems. The examples are:
   (i) Sudden enlargement or constriction in a pipe such as orifices and mouth pieces.
   (ii) Hydraulic jump in open channel.

But rate of change of momentum is equal to torque.

$$\therefore \qquad T = \frac{m}{t}(V_2 \cos \alpha_2 r_2 - V_1 \cos \alpha_1 r_1)$$

$$= \rho Q(V_2 \cos \alpha_2 r_2 - V_1 \cos \alpha_1 r_1)$$

For a circular path, $r_1 = r_2 = r_3$ and $\alpha_1 = \alpha_2 = \alpha_3 = 0$

$$\therefore \qquad T = \rho Qr(V_2 - V_1)$$

Power $\qquad P = Tw = \rho Qr(V_2 - V_1)w$

where w = angular velocity.

Blade velocity, $\qquad V_b = wr$

$$\therefore \qquad P = \rho Q(V_{w_2} V_{b_2} - V_{w_1} + V_{b_1})$$

The moment of momentum equation is applied for:

1. Analysis of flow problems in turbines and centrifugal pumps.
2. Finding torque exerted by water on sprinkler.

2.9 EULER'S FUNDAMENTAL EQUATION

Euler's fundamental equation is the equation of motion for a fluid with the following assumptions.

Assumptions

1. Fluid is non-viscous and frictional losses are zero.
2. Fluid is homogeneous and incompressible. ρ = constant
3. Flow is steady.
4. Flow is one-dimensional along the streamline.
5. Velocity is uniform over the section
6. Flow is continuous
7. Except gravity and pressure forces, no other forces are involved. Other forces like forces due to viscosity, turbulence and compressibility are neglected.

Energy is defined as ability to do work. It manifests in various forms and can change from one form to another.

1. Gravitational potential energy or elevation energy Z in metres of liquid column.
2. Kinetic energy due to mass and velocity of fluid $\frac{V^2}{2}$ in N-m.
3. Pressure energy required to move the fluid against its pressure.

   Pressure energy = p $\quad$ [N/m^2].

From equation (5),

$$-\frac{\partial p}{\partial s} dA\, dS - \rho g\, dA\, dS \cos\theta = \rho dA\, dS \frac{V \partial V}{\partial S}.$$

Dividing throughout by $\rho dS\, dA$

$$\therefore \qquad -\frac{\partial p}{\rho \partial S} - g\cos\theta + V\frac{\partial V}{\partial S} = 0$$

But $\qquad \cos\theta = \dfrac{dZ}{dS}$

$$\therefore \qquad \frac{1}{\rho}\frac{\partial p}{\partial S} + g\frac{dZ}{dS} + V\frac{dV}{dS} = 0$$

or $$\frac{\partial p}{\rho} + g\,dZ + V\,dV = 0$$

or $$\frac{\partial p}{\rho} + g\,dZ + V\,dV = 0 \qquad \text{...(6)}$$

Equation (6) is called Euler's equation of motion.

2.9.1 Bernoulli's Equation

Bernoulli's equation is also called energy equation and is obtained by integration of Euler's equation of motion

$$\int \frac{dp}{\rho} + \int g\,dZ + \int V\,dV = \text{constant}$$

For incompressible flow, ρ = constant

$$\therefore \qquad \frac{p}{\rho} + gZ + \frac{V^2}{2} = \text{constant}$$

or $$\frac{p}{\rho g} + Z + \frac{V^2}{2g} = \text{constant} \qquad \text{...(7)}$$

Equation (7) is called Bernoulli's equation.

$\dfrac{p}{\rho g}$ = Pressure head = Pressure energy per unit weight of fluid.

$\dfrac{V^2}{2g}$ = Kinetic head = Kinetic energy per unit weight

Z = Potential head = Potential energy per unit weight.

The Bernoulli's equation can also be written as

$$\frac{p_1}{\rho g} + z_1 + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g}.$$

2.10 JET THEORY

When a nozzle is fitted at the end of a pipe, a jet of liquid comes out with high velocity utilizing the pressure of liquid in the pipe. Applying continuity equation to the sections 1 and 2 of the nozzle,

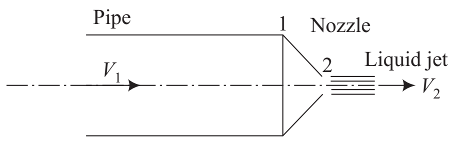


Fig. 2.9 Liquid jet

$$A_1 V_1 = A_2 V_2$$

The velocity of liquid jet,

$$V_2 = \frac{A_1}{A_2} V_1$$

The kinetic energy of the jet,

$$KE_j = \frac{1}{2} \dot{m} V_2^2$$

Let jet velocity = $V_j = V_2$

$$\therefore \quad KE_j = \frac{1}{2} \dot{m} V_j^2$$

$$= \frac{1}{2} \rho A_j V_j \times V_j^2 = \frac{1}{2} \rho A_j V_j^3 \quad [j/s]$$

The jet power,

$$P_j = \frac{\frac{1}{2} \rho A_j V_j^3}{1000} \quad [\text{kW}]$$

The efficiency of nozzle,

$$\eta_n = \frac{\text{Jet power at outlet of nozzle}}{\text{Liquid power at inlet of nozzle}}$$

$$= \frac{\frac{1}{2 \times 1000} \rho A_j V_j^3}{\frac{\rho g \dot{Q} H}{1000}} = \frac{V_j^2}{2gH}$$

where H = total head of liquid at the inlet of nozzle.

The liquid jet exerts a force on a plate placed in front of it.

Solution $F_j = \rho \dot{Q} V_j = \rho A_j V_j V_j = \rho \frac{\pi}{4} D_j^2 V_j^2$

$$= 1000 \times \frac{\pi}{4}\left(\frac{75}{1000}\right)^2 \times (20)^2 = 1766.8 \text{ N}$$

$$= \mathbf{1.767\ kN}$$

2.11.2 Inclined Plate

A fluid jet with a velocity V_j strikes a fixed plate inclined at an angle θ with the direction of jet.

The components of jet velocity,

$$V_{jx'} = V_j \sin\theta$$
$$V_{jy'} = V_j \cos\theta = 0$$

The jet force,

$$F_j = -\rho \dot{Q} V_{jx'}$$
$$= -\rho \dot{Q} V_j \sin\theta.$$

The jet force has components in *X*-direction and *Y*-direction.

$$F_{jx} = F_j \sin\theta = -\rho Q V_j \sin^2\theta$$
$$F_{iy} = -\rho \dot{Q} V_j \sin\theta \cos\theta = 0$$

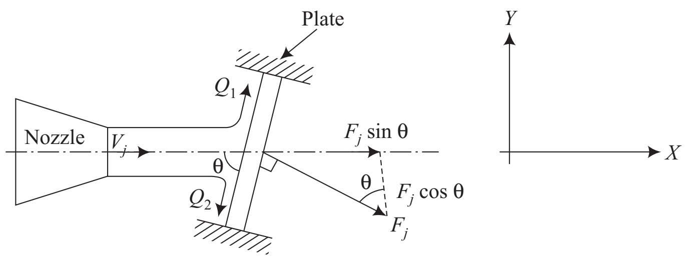


Fig. 2.11 Jet force on inclined fixed plate

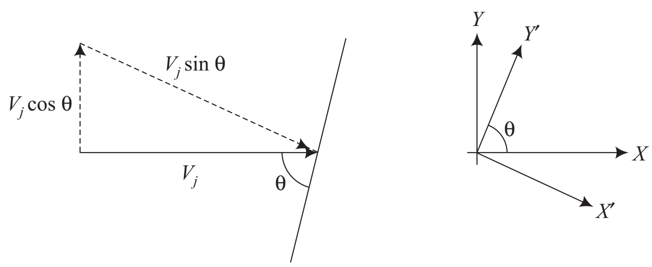


Fig. 2.12 Components of jet velocity

The volumetric discharge Q is divided into Q_1 and Q_2.

$$Q = Q_1 + Q_2$$

$$Q_1 = \frac{Q}{2}(1 + \cos\theta)$$

$$Q_2 = \frac{Q}{2}(1 - \cos\theta)$$

2.12 JET FORCE ON MOVING FLAT PLATE

The absolute velocity of jet issuing from the nozzle, $= V_j$

The velocity of moving plate in the direction of jet $= U$.

The jet force

$$F_j = \rho\dot{Q}(V_j - U)$$

The quantity of fluid mass striking the plate per second,

$$Q = A(V_j - U)$$

$$\therefore \quad F_j = \rho A(V_j - U)^2$$

The work done by the jet on the plate

$$W_j = F_j \cdot U = \rho\dot{Q}(V_j - U)U.$$

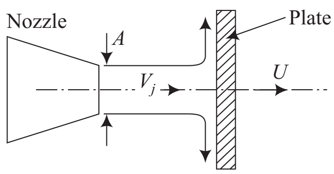


Fig. 2.13 Jet force on moving flat plate

The kinetic energy of jet

$$KE_j = \frac{1}{2}\dot{m}V_j^2 = \frac{1}{2}\rho\dot{Q}V_j^2 = \frac{1}{2}\rho A V_j^3$$

The efficiency of the system,

$$\eta = \frac{\text{Work done on the plate}}{\text{Kinetic energy of jet}} = \frac{W_j}{KE_j}$$

$$= \frac{\rho A(V_j - U)^2 . U}{\frac{1}{2}\rho A V_j^3} = \frac{2}{V_j^3}[V_j^2 U + U^3 - 2U_j U^2]$$

For a given V_j, maximum efficiency.

$$\frac{d\eta}{dU} = \frac{2}{V_j^3}(V_j^2 + 3U^2 - 4V_jU) = 0$$

$$\frac{2}{V_j^3} \neq 0$$

$$\eta_{max} = \frac{2\left(V_j - \frac{V_j}{2}\right)\frac{V_j}{2}}{V_j^2} = 0.5 \quad \text{or} \quad 50\%.$$

2.13 JET FORCE ON CURVED PLATE WHEN JET STRIKES TANGENTIALLY

2.13.1 Stationary Vane

The fluid jet enters the curved vane with absolute velocity V_1 glides along the smooth inner surface and leaves with absolute velocity, V_2, making angles α_1 and α_2 with the horizontal direction.

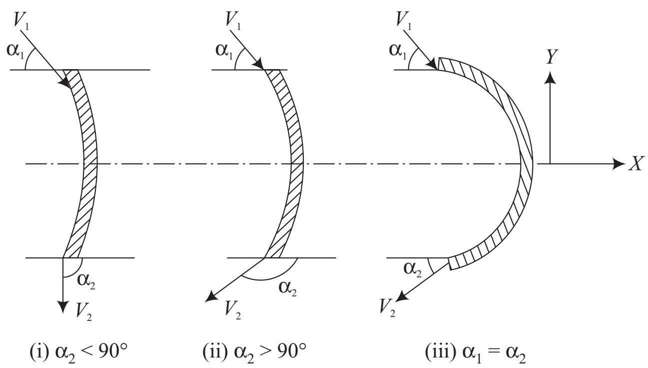


Fig. 2.15 Jet force on stationary vane

The velocity components in the X-direction are:

$$V_{X_1} = V_1 \cos \alpha_1$$
$$V_{X_2} = V_2 \cos \alpha_2.$$

The force exerted by the jet on the curved vane,

$$F_X = \rho Q(V_1 \cos \alpha_1 - V_2 \cos \alpha_2)$$

But $\quad Q = AV_1$

where A = cross-section area of jet.

$\therefore \quad F_X = \rho AV_1(V_1 \cos \alpha_1 - V_2 \cos \alpha_2)$

(i) If $\alpha_2 > 90°$

$$F_X = \rho AV_1(V_1 \cos \alpha_1 + V_2 \cos \alpha_2)$$

In order to get more power, $\alpha_2 > 90°$.

(ii) For symmetrical vane, $\alpha_1 = \alpha_2 = \alpha$ and $V_1 = V_2 = V$

$$F_X = \rho A V^2 (\cos\alpha + \cos\alpha) = 2\rho A V^2 \cos\alpha.$$

The hydraulic thrust in Y-direction

$$F_y = \rho A V^2 (\sin\alpha - \sin\alpha) = 0$$

(iii) For semicircular vane, $\alpha_1 = \alpha_2 = 0$.

$$\therefore \quad F_X = \rho A V^2 (\cos 0° + \cos 0°) = 2\rho A V^2$$

$$F_y = \rho A V^2 (\sin 0° - \sin 0°) = 0$$

Semicircular vanes give maximum hydraulic force and zero hydraulic thrust.

2.13.2 Moving Vane

A fluid jet of cross-sectional area A strikes a curved vane with an absolute velocity V_1. The curved vane moves with a velocity U in X-direction. The tip angles of vane are α_1 and α_2 at inlet and exit respectively. The velocity triangles at inlet and outlet of vane are drawn.

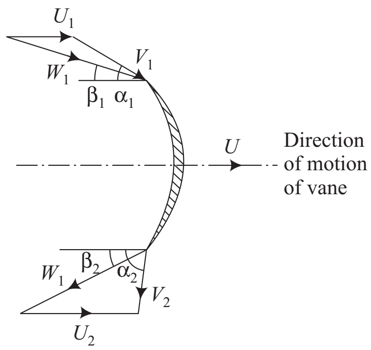


Fig. 2.16 Jet force on moving vane.

For frictionless vane, the relative velocity does not change,

$$W_1 = W_2$$

$$Q = A(V_1 - U)$$

$$F_X = \rho A (V_1 - U)(V_1 \cos\alpha_1 - V_2 \cos\alpha_2)$$

If $\alpha_2 > 90°$

$$F_X = \rho A (V_1 - U)(V_1 \cos\alpha_1 + V_2 \cos\alpha_2)$$

In actual hydraulic machines, a series of vanes is mounted on the periphery of a wheel.

The efficiency of the system,

$$\eta = \frac{W.D.}{E} = \frac{\rho Q(W-U).U}{\frac{1}{2}\rho Q W^2} = \frac{2(W-U)U}{W^2}$$

For maximum efficiency of the system

$$\frac{d\eta}{dU} = 0$$

$$\therefore \qquad \frac{d}{dU}\frac{2(WU-U^2)}{W^2} = 0$$

But $\qquad \dfrac{W^2}{2} \neq 0$

$$\therefore \qquad U = \frac{W}{2}$$

$$\eta_{max} = \frac{2\left(W-\frac{W}{2}\right)\frac{W}{2}}{W^2} = \frac{1}{2} = 50\%$$

QUESTION BANK NO. 2

1. How are the hydraulic machines classified?
2. Derive Euler's equation applied to fluid machines.
3. Derive the continuity equation in Cartesian coordinates.
4. Discuss the following aspects of hydraulic flow:
   (a) Closed and open systems.
   (b) Concept of continuum.
   (c) Lagrangian and Eulerian methods.
   (d) Streamline and stream tube
5. Derive momentum and angular momentum equation.
6. What are the applications of momentum and angular momentum equations in fluid machines?
7. Establish the Bernoulli's equation from Euler's equation of motion.
8. Explain water jet theory and how are the following principles established.
   (a) Impulse principle
   (b) Reaction principle
   (c) Propulsion principle
9. Derive equation of a jet force for moving flat plate and moving curved vane.

TUTORIAL SHEET NO. 2

1. A 25 cm diameter pipe carries lubrication oil of 0.9 specific gravity at a velocity of 3 m/s. Find the mass flow rate of oil. What will be the oil velocity at another section where diameter is reduced to 20 cm.

Solution Section 1:

$$D_1 = 25 \text{ cm} = 0.25 \text{ m}$$

$$A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.25)^2$$

$$= 0.049 \text{ m}^2.$$

$$V_1 = 3 \text{ m/s}.$$

$$\rho = 0.9 \times 1000 = 900 \text{ kg/m}^3.$$

$$\dot{m} = \rho A_1 V_1 = 900 \times 0.049 \times 3$$

$$= \mathbf{132.23\ kg/s} \quad \text{Ans.}$$

Section 2

$$D_2 = 20 \text{ cm} = 0.20 \text{ m}$$

$$A_2 = \frac{\pi}{4}(0.20)^2 = 0.314 \text{ m}^2$$

Applying equation of continuity at sections 1 and 2,

$$A_1 V_1 = A_2 V_2$$

$$\therefore \quad V_2 = \frac{A_1 V_1}{A_2} = \frac{0.049 \times 3}{0.0314} = \mathbf{4.68\ m/s} \quad \text{Ans.}$$

2. Water under a pressure of 29.43 N/cm^2 (g) and velocity of 2 m/s is flowing through a 5 cm diameter pipe. Find the total head of water at a section 5 m above datum line.

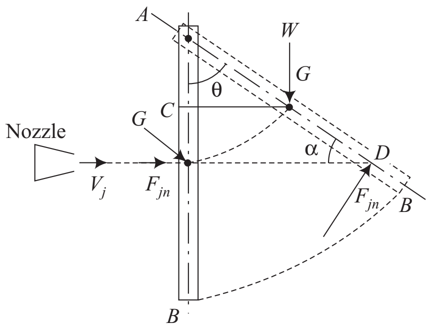
Solution Pressure head $= \dfrac{p}{\rho g} = \dfrac{29.43 \times 10^4}{1000 \times 9.81} = 30$ m.

$$\text{Kinetic head} = \frac{V^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204 \text{ m}$$

$$\text{Datum head} = Z = 5 \text{ m}.$$

$$\therefore \quad \text{Total head} = 30 + 0.204 + 5 = \mathbf{35.204\ m} \quad \text{Ans}$$

3. A jet of water 60 mm in diameter, having a velocity of 20 m/s, strikes a flat plate inclined at an angle of 30° to the axis of the jet. The plate moves at 5 m/s in the direction of the jet.



V_j. The plate swings through an angle θ. The plate experiences the following forces

1. Weight W of plate acting vertically through G.
2. Normal water jet force, F_{j_n},

$$F_{j_n} = \rho A V_j^2 \sin \alpha$$

where α = angle of jet with the centre line of plate.

$$\therefore \quad F_{j_n} = \rho A V_j^2 \sin (90° - \theta)$$

$$= \rho A V_j^2 \cos \theta.$$

The plate is in equilibrium under the two forces. Taking moments about point A.

$$F_{j_n}(AD) = W(CG)$$

$$AD = \frac{AG}{\cos\theta}$$

$$CG = AG \sin \theta.$$

$$\therefore \quad F_{j_n} \frac{(AG)}{\cos\theta} = W(AG) \sin \theta$$

$$\rho A V_j^2 \cos \theta \frac{(AG)}{\cos\theta} = W(AG) \sin \theta.$$

$$\therefore \quad \sin \theta = \frac{\rho A V_j^2}{W}$$ **Proved**.