2 CHAPTER Sequence and Series: Arithmetic Progression

2.1 SEQUENCE

A set of numbers arranged in definite order and according to some definite rule is called a sequence. $a_1, a_2 \ldots a_n$ formed a sequence an element of a sequence where $a_1, a_2 \ldots a_n$ elements of a sequence are called the terms of the sequence. There are two types of sequences:

(i) Finite Sequence: The number of terms are finite:

$$1, 3, 5, 7, 9 \quad \text{or} \quad 3, 6, 9, 12, 15, 18.$$

(ii) Infinite Sequence: The number of terms are infinite

$$b, b^2, b^3, b^4 \ldots b^n \ldots \text{ where } b \text{ is a real number and } 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}, \frac{1}{10} \ldots$$

A characteristic difference between finite and infinite sequences is that every finite sequence has a last term and an infinite sequence has no last term.

2.2 SERIES

A succession of quantities formed or arranged according to some definite rule is called a series or progression.

Each quantity of the series is called a term of the series.

Thus,

(i) 1, 3, 5, 7, 9, ...;

(ii) 1, 3, 9, 27, 81, ...;

(iii) 1, 4, 5, 9, 14, ...;

are examples of series. In (i) each term is formed by adding 2 to the preceding term; in (ii) each term is formed by multiplying the proceding term by 3; and in (iii) each term after the second is formed by adding the two preceding terms.

From these cases it follows that if the rule or law of formation of successive terms is recognised, we can write down as many terms of the series as we please.

Further, a series is said to be a finite series if the number of terms is finite and an infinite series if the number of terms is infinite.

Progressions: It is not necessary that the terms of a sequence always follow a certain part or they are described by some explicit formula for the n^{th} term. Those sequences which follow certain patterns are called progressions.

In this chapter, we shall study arithmetical progressions as defined below.

2.3 ARITHMETIC PROGRESSION

A series in which each term is formed by adding the same quantity (positive or negative) to the one preceding it is called an arithmetic series or arithmetic progression (A.P.). Thus, in an arithmetic progression, the difference between any two terms are chosen in the series. This constant quantity is called the common difference and is obtained by subtracting any term from the term which follows it.

Thus, the following series of terms are in A.P.

(i) 5, 9, 13, 17, 21, ...; common difference = 4.

(ii) 11, 6, 1, −4, −9, −14, ...; common difference = −5.

2.3.1 The General Term or n^{th} term of an A.P.

Let a be the first term and d the c.d. of an A.P., then the successive terms of the A.P. are:

First term	$a_1 = a = a + (1-1)d$
Second term	$a_2 = a + d = a + (2-1)d$
Third term	$a_3 = a + 2d = a + (3-1)d$
Fourth term	$a_4 = a + 3d = a + (4-1)d$
...	... ...
...	... ...

n^{th} term $a_n = a + (n-1)d$

Thus, n^{th} term a_n of A.P. = $\boldsymbol{a + (n-1)d}$.

Important Points

1. n^{th} term is also called the general term.
2. If a is the first term and d the c.d. of an A.P., then the A.P. is $a, a+d, a+2d, a+3d, \ldots$
3. If an A.P. has n terms, then n^{th} term is called the last term of A.P. and it is denoted by l.
4. If last term of A.P. be a_n and c.d. be d, then terms of A.P. from end are $a_n, a_n - d, a_n - 2d, \ldots$

EXAMPLE 4. How many terms are there in the following progression?

5, 8, 11, 14, 17, 20, 23...215.

Solution: The given progression an A.P. in which $a = 5$, $d = 8 - 5 = 3$. Let the number of terms be n. Then $T_n = 215$.

$$\therefore \quad T_n = a + (n-1)d.$$

$$215 = 5 + (n-1)3$$

$$210 = (n-1)3$$
$$(n-1) = 70$$
$$n = 71.$$ **Ans.**

EXAMPLE 5. Is 309 any term of the following sequences?

(i) 8, 15, 22, 29, … (ii) 3, 8, 13, 18, …

Solution: Let n^{th} term of this sequence be 309 then

$$T_n = a + (n-1)d$$
$$309 = 8 + (n-1)7$$
$$301 = (n-1)7$$
$$(n-1) = \frac{301}{7} = 43.$$

$\therefore$ $n = 44.$ **Ans.**

Hence, 309 is a term of the given sequence and the term is 43.

(ii) Here $a = 3$, $d = 5$

$$309 = 3 + (n-1)5$$
$$306 = (n-1)5$$

$\therefore$ $n - 1 = \frac{306}{5}$

$\therefore$ $n = \frac{306}{5} + 1 = \frac{311}{5}.$ **Ans.**

Thus, n is a fraction which is not possible as n is the number of terms hence, 309 is not any term of the given sequence.

EXAMPLE 6. The 5^{th} term of an arithmetic progression is 11 and its 9^{th} term is 7. Find its 15^{th} term.

Solution: Let a and d be the first term and common difference

Then, 5^{th} term $= a + 4d = 11$ …(i)

9^{th} term $= a + 8d = 7$ …(ii)

Subtracting (i) from (ii) we have $4d = -4 \Rightarrow d = -1$

$$a + 4(-1) = 11 \Rightarrow a = 15.$$

$\therefore$ 15^{th} term $= a + 14d = 15 + 14(-1) = 15 - 14 = 1$ **Ans.**

EXAMPLE 7. Show that the sequence $< a_n >$ defined by $a_n = 4n + 5$ is an A.P. Also, find its common difference.

Solution: We have, $a_n = 4n + 5$

Replacing n by $(n + 1)$, we get

EXAMPLE 12. How many terms are there in A.P. 10, 15, 20, 25, ...100?

Solution: Let the number of terms be n.

Given $a_n = 100, a = 10, d = 5$, we have to find n.

Now $a_n = a + (n - 1)d$

$\therefore$ $100 = 10 + (n - 1)5$

or $90 = (n - 1)5$

or $n - 1 = 18 \Rightarrow n = 19$ **Ans.**

EXAMPLE 13. The 11^{th} term of an A.P. is 86 and the 16^{th} term is 116. Find the 35^{th} term.

Solution: The n^{th} term of an A.P. whose first term is a and c.d. is d is given by

$$t_n = a + (n - 1)d$$

Given, $T_{11} = 86 \quad \therefore 86 = a + 10d$

and $T_{16} = 116 \quad \therefore 116 = a + 15d$

Subtracting (1) from (2), we get $30 = 5d \therefore d = 6$

Putting $d = 6$ in (1), we get $86 = a + 60 \therefore a = 26$.

$\therefore$ The 35^{th} term, $t_{35} = 26 + (35 - 1).\ 6 = 26 + 144 = 170$. **Ans.**

EXAMPLE 14. Is 70 a term of the sequence 4, 7, 10, 13, ...? If yes, where is it in the given sequence?

Solution: If possible let n^{th} term of the sequence be 70.

Now $T_n = a + (n - 1)d$. Here $T_n = 55,\ a = 4,\ d = 3$

$\therefore$ $70 = 4 + (n - 1)3$ or $(n - 1) = 22 \Rightarrow n = 23$.

Hence, 70 is 23^{rd} term of the given sequence. **Ans.**

EXAMPLE 15. If p^{th}, q^{th} and r^{th} terms of an A.P. are a, b, c respectively, then show that:

(a) $a(q - r) + b(r - p) + c(p - q) = 0$

(b) $(a - b)r + (b - c)p + (c - a)q = 0$

Solution: Let A be the first term and D be the common difference of the given A.P. Then,

$$a = p^{th} \text{ term} \Rightarrow a = A + (p - 1)D \quad \text{...(i)}$$

$$b = q^{th} \text{ term} \Rightarrow b = A + (q - 1)D \quad \text{...(ii)}$$

$$c = r^{th} \text{ term} \Rightarrow c = A + (r - 1)D \quad \text{...(iii)}$$

(i) Now,

$a(q - r) + b(r - p) + c(p - q)$

$= \{A + (p - 1)D\}(q - r) + \{A + (q - 1)D\}(r - p) + \{A + (r - 1)D\}(p - q)$

[Using (i), (ii) and (iii)]

$= A\{(q - r) + (r - p) + (p - q)\} + D\{(p - 1)(q - r) + (q - 1)(r - p) + (r - 1)(p - q)\}$

$= A.0 + D\{p(q-r) + q(r-p) + r(p-q) - (q-r) - (r-p) - (p-q)\}$

$= A.0 + D.0 = 0$

(B) On subtracting eq. (ii) from eq. (i), (iii) eq. from (ii) eq. and (i) eq. from (iii) eq. we get

$$a - b = (p-q) \times D\ (b-c) = (q-r)D\ c - a = (r-p)D$$

Now, $(a-b)r + (b-c)p + (c-a)q$

$= (p-a)Dr + (q-r)Dp + (r-p)Dq$

$= D[(p-q)r + (q-r)p + (r-p)q]$

$= D.O = 0$ **Ans.**

EXAMPLE 16. In an A.P. the p^{th} term is $\frac{1}{q}$ and the q^{th} term is $\frac{1}{p}$, find the $(pq)^{th}$ term.

Solution: The n^{th} term of an A.P. whose first term is a and c.d. is d is given by

$$T_n = a + (n-1)d.$$

Given, $T_p = \frac{1}{q} \quad \therefore \frac{1}{q} = a + (p-1)d$

and $T_q = \frac{1}{p} \quad \therefore \frac{1}{p} = a + (q-1)d.$

Subtracting (2) from (1), we get

$$\frac{1}{q} - \frac{1}{p} = (p-q)d$$

or $\frac{(p-q)}{pq} = (p-q)d \qquad \therefore d = \frac{1}{pq}$

Putting $d = \frac{1}{pq}$ in (1), we get $\frac{1}{q} = a + (p-1).\frac{1}{pq} \quad \Rightarrow \quad \therefore a = \frac{1}{pq}$

$\therefore \quad T_{pq} = \frac{1}{pq} + (pq-1).\frac{1}{pq} = 1 \qquad \left[\because a = \frac{1}{pa}, d = \frac{1}{pq}, n = pq\right]$

The $(pq)^{th}$ term of the A.P. is 1. **Ans.**

EXAMPLE 17. Show that the sum of $(m+n)^{th}$ and $(m-n)^{th}$ term of an A.P. is equal to twice of the m^{th} term.

Solution: Let a be the first term and d be the common difference of the A.P. then,

$a_{m+n} = (m+n)^{th}$ term $= a + (m+n-1)d$

and, $a_{m-n} = (m-n)^{th}$ term $= a + (m-n-1)d$

$\therefore \quad a_{m+n} + a_{m-n} = [a + (m+n-1)d] + [a + (m-n-1)d]$

$\Rightarrow \quad a_{m+n} + a_{m-n} = 2a + (m+n-1+m-n-1)d$

$\Rightarrow \quad a_{m+n} + a_{m-n} = 2a + 2(m-1)d$

$\Rightarrow \qquad a_{m+n} + a_{m-n} = 2\{a + (m-1)d\}$

$\Rightarrow \qquad a_{m+n} + a_{m-n} = 2a_m \qquad [\because a_m = a + (m-1)d]$

EXAMPLE 18. The 4^{th}, 42^{nd} and the last term of an A.P. are 0, –95, –125 respectively. Find the first term and the number of terms.

Solution: Let a, d and n be the first term, common difference and the number of terms of the A.P. respectively.

4^{th} term is $\qquad a + 3d = 0 \qquad$ ...(i)

42^{nd} term is $\qquad a + 41d = -95 \qquad$ ...(ii)

n^{th} term is $\qquad a + (n-1)d = -125 \qquad$ ...(iii)

From (i) and (ii) we have

$$(41-3)d = -95 \Rightarrow d = \frac{-95}{38} = -5/2$$

$$= a + 3d = 0 \Rightarrow a = 15/2$$

Putting the value of a and d in (iii), we get

$$\frac{15}{2} + (n-1)\left(\frac{-5}{2}\right) = -125$$

$$15 - 5(n-1) = -250$$

$$(n-1) = 50 + 3 = 53$$

$$n = 54.$$

$\therefore \qquad a = \frac{15}{2}, n = 54.$ **Ans.**

2.3.2 The sum of *n* terms of an A.P.

Let a, d, l and S_n be respectively the first term, common difference, last term and sum of n terms of an A.P.

Now, $\qquad S_n = a + (a + d) + (a + 2d) + \ldots + (t_n - 2d) + (t_n - d) + t_n \qquad$ ...(1)

Again, $\qquad S_n = t_n + (t_n - d) + (t_n - 2d) + \ldots + (a + 2d) + (a + d) + a \qquad$ ...(2)

Adding (1) and (2), we get

$$2S_n = (a + t_n) + (a + t_n) + (a + t_n) + \ldots + (a + t_n) + (a + t_n) + (a + t_n)$$

$$= (a + t_n) + (a + t_n) + (a + t_n) + \ldots \text{ to } n \text{ terms}$$

$$= n(a + t_n)$$

$\therefore \qquad 2S_n = n(a + t_n)$

$\therefore \qquad S_n = \frac{n}{2}[a + a + (n-1)d] \qquad [\because t_n = a + (n-1)d]$

$$= \frac{n}{2}[2a + (n-1)d]$$

Then, $\qquad$ **$S_n = \frac{n}{2}$ (1st term + last term).**

EXAMPLE 22. Find the sum to n terms of an A.P. whose n^{th} term is $T_n = 5 + 6n$, $n \in N$.

Solution: The given sequence is an A.P. with $T_n = 5 + 6n$, $n \in N$.

$\therefore$ The first term, $\quad T_1 = 5 + 6.1 = 11$

and the last term, i.e., the n^{th} term, $T_n = 5 + 6n$.
Now, sum to n terms of an A.P. is given by

$$S_n = \frac{n}{2}[T_1 + T_n] = \frac{n}{2}[11 + 5 + 6n] = n(8 + 3n) = 3n^2 + 8n.$$ **Ans.**

EXAMPLE 23. Find the sum of the series 99 + 95 + 91 + 87 + … to 20 terms

Solution: a, d and n are given and $S_n = ?$
The terms of the given series are in A.P. whose common difference $d = -4$ and the first term $a = 99$.
Now, sum of 20 terms of the series,

$$S_{20} = \frac{20}{2}[2.99 + (20 - 1)(-4)]$$
$$= 10(198 - 76) = 1220.$$ **Ans.**

EXAMPLE 24. Find the sum of the series 5 + 13 + 21 + … + 181.

Solution: $a = 5$, $d = 8$.

$$l = a_n = 181$$
$$\Rightarrow \quad a + (n - 1)d = 181$$
$$\Rightarrow \quad 5 + (n - 1) \times 8 = 181$$
$$\Rightarrow \quad 8n = 184 \quad \Rightarrow \quad n = 23$$

$\therefore$ Required sum $= \frac{n}{2}(a + l) = \frac{23}{2}(5 + 181) = 2139$ **Ans.**

EXAMPLE 25. Find the sum of all three-digit natural numbers, which are divisible by 7.

Solution: The smallest and the largest numbers of three digits, which are divisible by 7 are 105 and 994 respectively. So, the sequence of three-digit numbers which are divisible by 7 are 105, 112, 119, …, 994. Clearly, it is an A.P. with first term $a = 105$ and common difference $d = 7$.

Let there be n terms in this sequence. Then,

$$a_n = 994 \quad \Rightarrow \quad a + (n - 1)d = 994 \quad \Rightarrow \quad 105 + (n - 1) \times 7 = 994 \quad \Rightarrow \quad n = 128$$

Now, $\quad$ required sum $= \frac{n}{2}[2a + (n - 1)d]$

$$= \frac{128}{2}[2 \times 105 + (128 - 1) \times 7] = 70336$$ **Ans.**

EXAMPLE 26 The sum of how many terms of A.P. 17 + 15 + 13… is 72.

Solution: Given series is 17 + 15 + 13…

$$a = 17, \quad d = 15 - 17 = -2, \quad S_n = 72$$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$72 = \frac{n}{2}[2 \times 17 + (n-1)(-2)]$$

$$72 = n[17 - n + 1]$$

$$72 = 18n - n^2$$

$$n^2 - 18n + 72 = 0$$

$$n^2 - 12n - 6n + 72 = 0$$

$$n(n-12) - 6(n-12) = 0$$

$$(n-12)(n-6) = 0$$

$$n = 12, 6.$$

Ans.

EXAMPLE 27. Find the sum of all odd integers between 2 and 100 which are divisible by 3.

Solution: The odd integers between 2 and 100 which are divisible by 3 are 3, 9, 15, 21, …, 99. Clearly, it is an A.P. with first term $a = 3$ and common difference $d = 6$. Let there be n terms in this sequence. Then,

$$l = a_n = 99 \Rightarrow a + (n-1)d = 99 \Rightarrow 3 + (n-1) \times 6 = 99 \Rightarrow n = 17$$

$$\therefore \text{Required sum} = \frac{n}{2}[a + l] = \frac{17}{2}[3 + 99] = 867.$$

Ans.

EXAMPLE 28. If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, find the value of m.

Solution: Let S_n denote the sum of n terms and let the n^{th} term of the given A.P. Then,

$$S_n = 3n^2 + 5n$$

$$\Rightarrow \quad S_{n-1} = 3(n-1)^2 + 5(n-1) = 3n^2 - n - 2$$

[on replacing n by $(n - 1)$ in S_n]

Now, $a_n = S_n - S_{n-1}$

$$\Rightarrow \quad a_n = (3n^2 + 5n) - (3n^2 - n - 2)$$

$$\Rightarrow \quad a_n = 6n + 2$$

Now, $a_m = 164$ [Given]

$$\Rightarrow \quad 6m + 2 = 164$$

$$\Rightarrow \quad 6m = 162$$

$$\Rightarrow \quad m = 27$$

Ans.

EXAMPLE 29. Find the sum of first 20 terms of an A.P., in which 3^{rd} term is 7 and 7^{th} term is more than two thrice of its 3^{rd} term.

EXAMPLE 32. How many terms of the series 54 + 51 + 48 + 45 + ... must be added to make sum 513? Explain the double answer.

Solution: Let the sum of n terms of the given series be 513

$$\because \quad S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\therefore \quad 513 = \frac{n}{2}\{2.54 + (n-1)(-3)\} = \frac{n}{2}(111 - 3n)$$

or $\quad 1026 = 111n - 3n^2$ or $3n^2 - 111n + 1026 = 0$

or $\quad n^2 - 37n + 342 = 0$ or $n^2 - 18n - 19n + 342 = 0$

or $\quad (n-18)(n-19) = 0 \quad \therefore n = 18, 19.$

Explanation of double answer:

$$18^{th} \text{ term} = 54 + (18-1)(-3) = 3$$
$$19^{th} \text{ term} = 54 + (19-1)(-3) = 0$$

The sum of 18 terms and sum of 19 terms will be both 513 since 19^{th} term is zero.

EXAMPLE 33. If the m^{th} term of an A.P. is $\frac{1}{n}$ and the n^{th} term is $\frac{1}{m}$, show that the sum of the term mn is $\frac{1}{2}(mn + 1)$, where $m \neq n$.

Solution: Let a be the first term and d be the common difference of the given A.P. then,

$$a_m = \frac{1}{n} \Rightarrow a + (m-1)d = \frac{1}{n} \quad \text{...(i)}$$

and $$a_n = \frac{1}{m} \Rightarrow a + (n-1)d = \frac{1}{m} \quad \text{...(ii)}$$

Subtracting (ii) from (i), we get

$$(m-n)d = \frac{1}{n} - \frac{1}{m} \Rightarrow (m-n)d = \frac{m-n}{mn} \Rightarrow d = \frac{1}{mn}$$

Putting $d = \frac{1}{mn}$ in (i), we get

$$a + (m-1)\frac{1}{mn} = \frac{1}{n} \Rightarrow a + \frac{1}{n} - \frac{1}{mn} = \frac{1}{n} \Rightarrow a = \frac{1}{mn}$$

Now, $$S_{mn} = \frac{mn}{2}[2a + (mn-1)d]$$

$$\Rightarrow \quad S_{mn} = \frac{mn}{2}\left\{\frac{2}{mn} + (mn-1) \times \frac{1}{mn}\right\}$$

$$= \frac{mn}{2}\left[\frac{2}{mn}+1-\frac{1}{mn}\right]$$

$$= \frac{mn}{2}\left(\frac{2-1}{mn}+1\right) = \frac{mn}{2}\left[\frac{1+mn}{mn}\right]$$

$\Rightarrow$ $$S_{mn} = \frac{1}{2}\,(mn + 1).$$ **Ans.**

EXAMPLE 34. The sum of the 1st and 4th terms of an A.P. is 56. The sum of the last four terms is 112. If its 1st term is 11, then find the number of terms.

Solution: Let there be n terms in the A.P. with first term $a = 11$ and common difference d. Then,

Sum of first four terms = 56

$\Rightarrow \quad \frac{4}{2}\,[a_1 + a_4] = 56$

$\Rightarrow \quad 2\,[a + (a + 3d)] = 56$

$\Rightarrow \quad 2\,(2a + 3d) = 56$

$\Rightarrow \quad 2a + 3d = 28$

$\Rightarrow \quad 22 + 3d = 28$ $\quad [\because a = 11 \text{ (given)}]$

$\Rightarrow \quad 3d = 6 \Rightarrow d = 2$

We have,

Sum of the last four terms = 112

$\Rightarrow \quad a_n + a_{n-1} + a_{n-2} + a_{n-3} = 112$

$\Rightarrow \quad \frac{4}{2}\,[a_n + a_{n-3}] = 112$

$\Rightarrow \quad |a + (n-1)d| + |a + (n-4)d| = 56$

$\Rightarrow \quad 2a + (2n - 5)d = 56$

$\Rightarrow \quad 22 + (2n - 5) \times 2 = 56$ $\quad [\because a = 11, d = 2]$

$\Rightarrow \quad 4n - 10 = 34$

$\Rightarrow \quad 4n = 44 \Rightarrow n = 11$

Hence, there are 11 terms in the A.P. **Ans.**

EXAMPLE 35. The sum of the n terms of an arithmetic progression is 136, the common difference is 4 and the last term is 31. Find the number of terms.

Solution: Here, $d = 4$, $l = 31$ and $S_n = 136$

Let a be the first term and n the number of terms in the given A.P. Then,

$$a + (n-1) \times 4 = 31 \quad \text{...(i)}$$

$$\frac{n}{2}\,(a + 31) = 136$$

$a = 31 - 4(n-1) = 35 - 4n$ from equation (i)

Putting the value of a in (ii) we have

$$\frac{n}{2}[35 - 4n + 31) = 136.$$

$$n/2\,[66 - 4n] = 136.$$

$$n(33 - 2n) = 136 \Rightarrow 2n^2 - 33n + 136 = 0$$

$$2n^2 - 16n - 17n + 136 = 0 \Rightarrow 2n(n-8) - 17(n-8) = 0$$

$$(n-8)(2n-17) = 0 \Rightarrow n = 8,\ 17/2$$

Since the value of n cannot be a fraction, therefore $n = 8$. **Ans.**

EXAMPLE 36. If the sum of p terms of an A.P. is the same as the sum of q terms, show that the sum of $p + q$ is zero.

Solution: Let a be the first term and d the common difference of the A.P. Then,

$$S_p = \frac{p}{2}[2a + (p-1)d], \quad \text{and } S_q = \frac{q}{2}[2a + (q-1)d].$$

According to the question,

$$S_p = S_q$$

$$\therefore \quad \frac{p}{2}[2a + (p-1)d] = \frac{q}{2}[2a + (q-1)d]$$

$$\Rightarrow \quad p[2a + (p-1)d] = q[2a + (q-1)d]$$

$$\Rightarrow \quad 2a(p-q) = d[q(q-1) - p(p-1)]$$

$$\Rightarrow \quad 2a(p-q) = d[(q^2-p^2) - q(q-p)]$$

$$\Rightarrow \quad 2a(p-q) = d[(q-p)(q+p) - (q-p)]$$

$$\Rightarrow \quad 2a(p-q) = -d[(p-q)(p+q-1)]$$

$$\Rightarrow \quad 2a(p-q) = -d[p+q-1], \qquad (\because p \neq q)$$

$$\Rightarrow \quad 2a + (p+q-1)d = 0 \qquad \ldots\text{(i)}$$

$$\therefore \quad S_{p+q} = \frac{p+q}{2}[2a + (p+q-1)d]$$

$$= \frac{p+q}{2} \times 0, \qquad \text{[from (i)]}$$

$$\Rightarrow \quad S_{p+q} = 0.$$

Hence, proved.

EXAMPLE 37. The sum of n terms of two arithmetic series in the ratio $(2n+1):(2n-1)$. Prove that 10^{th} terms of both will be in the ratio 39 : 37.

Solution: Let a_1, a_2 be the first term and d_1, d_2 the common differences of the two given arithmetic series. Then, the sum of their n terms are

$$\frac{n}{2}[2a_1 + (n-1)d_1] \text{ and } \frac{n}{2}[2a_2 + (n-1)d_2]$$

respectively.

$$(a-3d)+(a-d)+(a+d)+(a+3d) = 20$$
$$4a = 20 \Rightarrow a = 5$$
$$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{2}{3}$$

Putting the value of a in (ii)

$$\frac{(5-3d)(5+3d)}{(5-d)(5+d)} = \frac{2}{3}$$
$$3(25-9d^2) = 2(25-d^2) \Rightarrow 75-27d^2 = 50-2d^2$$
$$-25d^2 = -25 \Rightarrow d = \pm 1$$

Hence, the required parts are $5-3d,\ 5-d,\ 5+d, 5+3d$.

i.e. $\quad 2, 4, 6, 8$ or $8, 6, 4, 2$ **Ans.**

EXAMPLE 40. The sum of three numbers in A.P. is 27 and the sum of their squares is 293. Find the numbers.

Solution: Let the three numbers in A.P. be $a-d, a, a+d$.

Given $\quad (a-d)+a+(a+d) = 27 \Rightarrow 3a = 27 \Rightarrow a = 9$

and, $\quad (a-d)^2 + a^2 + (a+d)^2 = 293$

$\Rightarrow \quad a^2 + d^2 - 2ad + a^2 + a^2 + d^2 + 2ad = 293$

$\Rightarrow \quad 3a^2 + 2d^2 = 293 \Rightarrow 3.9^2 + 2d^2 = 293$

$\Rightarrow \quad 2d^2 = 293 - 243 \Rightarrow 2d^2 = 50$

$\Rightarrow \quad d^2 = 25 \therefore d = \pm 5.$

If $d = -5$, the three numbers are 14, 9, 4.

If $d = 5$, the three numbers are 4, 9, 14. **Ans.**

EXAMPLE 41. The sum of four integers in A.P. is 24 and their product is 945. Find the numbers.

Solution: Let the four numbers be $a-3d, a-d, a+d, a+3d$.

Given, $\quad (a-3d)+(a-d)+(a+d)+(a+3d) = 24$

$\Rightarrow \quad 4a = 24 \Rightarrow a = 6$

and $\quad (a-3d)(a-d)(a+d)(a+3d) = 945$

$\Rightarrow \quad (a^2-9d^2)(a^2-d^2) = 945$

$\Rightarrow \quad (36-9d^2)(36-d^2) = 945$

$\Rightarrow \quad d^4 - 40d^2 + 144 = 105$

$\Rightarrow \quad d^4 - 40d^2 + 39 = 0$

$\Rightarrow \quad d^4 - 39d^2 - d^2 + 39 = 0$

$\Rightarrow \quad (d^2-1)(d^2-39) = 0$

$\Rightarrow \quad 25 + 5d^2 = 30 \quad [\because a = 5]$

$\Rightarrow \quad 5d^2 = 5 \quad \Rightarrow \quad d = \pm 1.$

If $d = 1$, then the numbers are 2, 4, 6, 8. If $d = -1$, then the numbers are 8, 6, 4, 2.
Thus, the numbers are 2, 4, 6, 8 *or* 8, 6, 4, 2. **Ans.**

UNSOLVED PROBLEMS

1. Write the three terms of the sequence defined by the following

   (i) $t_n = 3n + 1$ (ii) $t_n = n(n + 2)$ (iii) $t_n = \dfrac{n-3}{4}$ (iv) $t_n = \dfrac{n^2}{n+1}$

2. Write the next three terms of the following sequence
   (i) $t_2 = 2\ t_n = t_{n-1} + 1\ (n \geq 3)$
   (ii) $t_1 = t_2 = 2,\ t_n = t_{n-1} - 1,\ n > 2.$
3. If n^{th} term of a sequence is $4n^2 + 1$, find the sequence. Is this sequence of an A.P.
4. Find the 10^{th} term of the sequence 10, 5, 0, –5, –10.
5. In an A.P. if m^{th} term is n and n^{th} term is m where $m \neq n$ find the p^{th} term.
6. The p^{th}, q^{th}, and r^{th} term of an A.P. are a, b, c, respectively show that $(a - b)r + (b - c)p + (c - a)q = 0$.
7. If m times the m^{th} term of an A.P. is equal to n times the n^{th} term find the $(m + n)^{th}$ term.
8. The sum of three terms of an A.P. is 21 and the product of the first and the third terms exceeds the second term by 6, find the three terms.
9. Three numbers are in A.P. if the sum of these number is 27 and the product 648, find the numbers.
10. The angles of quadrilateral are in A.P. whose common difference is 10°. Find the angles.

ANSWERS

1. (i) 4, 7, 10 (ii) 3, 8, 15 (iii) –1/2, –1/4, 0 (iv) 1/2, 4/3, 9/4
2. (i) 25 (ii) 729
3. Sequence is 5, 17, 37, 65 ...(No)
4. –35
5. $m + n - p$
7. 0
8. 1, 7, 13
9. 6, 9, 12
10. 75°, 85°, 95°, 105°.