INDICATING INSTRUMENTS

Chapter 2

2.1 INTRODUCTION

There are various types of indicating instruments. Ammeters, voltmeters, wattmeters and power factor meters comes under this category.

2.1.1 Types of Ammeters and Voltmeters

The following are the types of ammeters/voltmeters.

1. Moving coil
   (a) Permanent magnet type (used on D.C. only)
   (b) Dynamometer (used on D.C. and A.C.)
2. Moving iron (used on A.C. and D.C.)
3. Hot wire type (used on A.C. and D.C.)
4. Electrostatic (used on A.C. and D.C.)
5. Induction type (used on A.C. only)

2.2 AMMETERS AND VOLTMETERS

Ammeters and voltmeters are classed together because the principle involved is same for both types of meters. Ammeter carries definite fraction of current to be measured, whereas voltmeter carries current proportional to the voltage. Ammeter is connected in series with the circuit in which current to be measured. Introduction of meter should not molest the quantity to be measured. Therefore, the resistance of the ammeter (r_a) should be very low. But as voltmeter is connected across the source to be measured and in an ideal case it should not draw any current. Therefore, the resistance of the voltmeter (r_v) must be high compared to the connected load. Usual convection is to have a

Because of the ampere turns of the coil, it produces magnetic field along the axis of the coil, when it carries current. A moving iron piece is attracted towards the coil due to magnetic field produced by the coil. For this reason it is called attraction type. The attraction of iron pieces is independent of the direction of the current in the coil and thus named "unpolarised instrument", because of magnetisation of iron piece it will be experienced by a torque. As the current increases deflection of the iron piece will also increase and T_D (deflection torque) is proportional to I^2 in this case. Let ϕ be the initial angle between the vertical axis and the axis of the iron piece when the coil carries no current.

Let I be the current in the coil and θ be the deflection of the axis of the iron piece when the magnetic field intensity $= H$

Let $m =$ pole strength of the iron piece
$H =$ pole strength proportional to the magnetic intensity along the axis of iron piece

$$\text{Therefore,} \quad m \; \alpha \; H \sin(\theta + \phi)$$

$F =$ force on the magnetised iron piece $\alpha \; mH$
$= \alpha H^2 \sin(\theta + \alpha)$
Let $l =$ distance between the centre of rotation and line of action of force
$T_D =$ deflecting torque

where

$T_D = F \times l \times H^2 \sin(\theta + \alpha) \cos(\theta + \alpha)$
$T_D \propto H^2 \sin(\theta + \alpha) \cos(\theta + \alpha)$
$H \propto I$
$T_D \propto I^2 \sin(\theta + \alpha) \cos(\theta + \alpha)$

$$T_D = K_1 I^2 \sin(\theta + \alpha) \cos(\theta + \alpha) \tag{2.1}$$

Assuming gravity control $T_C \propto \sin\theta$

$$T_D = K_1 I^2 \sin(\theta + \alpha) \cos(\theta + \alpha) \qquad T_C = K_2 \sin\theta$$

Under balanced conditions $T_D = T_C$

$$K_1 I^2 \sin(\theta + \alpha) \cos(\theta + \alpha) = K_2 \sin\theta$$

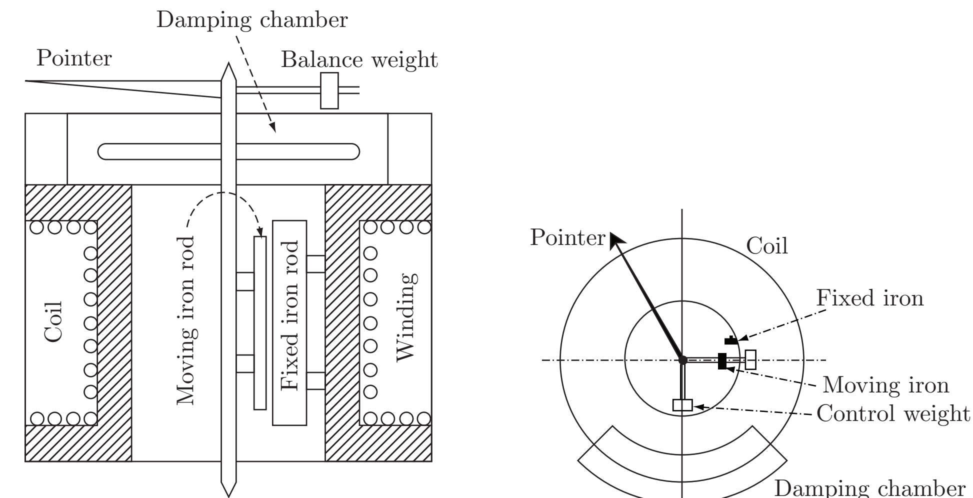
$$I^2 = K \frac{\sin\theta}{\sin(\theta + \varphi)\cos(\theta + \varphi)} = \frac{2K \sin\theta}{\sin 2(\theta + \varphi)} \tag{2.2}$$

$$I = \sqrt{\frac{2K \sin\theta}{\sin 2(\theta + \varphi)}} \text{ (Hence, scale is not uniform)}$$

If the instrument is to be used to measure the voltage, the coil will be connected across the supply after inserting suitable resistance in series with the coil. In this case, we can design the winding to carry full current to be measured because the winding is stationary. But the cost of instrument will be more due to extra copper.

2.3.2 Repulsion Type

Repulsion type of instrument shown in Fig. 2.3, also carries a coil which can carry current proportional to the voltage or current to be measured. It also consists of two round rods one is attached to the spindle and other is fixed to the former of the winding. We can assume that the distance between the rods is small compared to the length of the rods at all times.



(a) Repulsion type moving iron instrument (sectional plan)

(b) Elevation

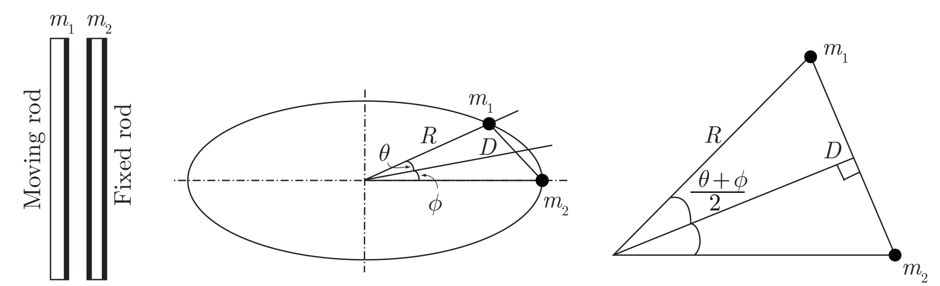
Fig. 2.3 Repulsion type moving iron instrument.

When the coil carries current, magnetic field will be established along the axis of the coil. Both the rods are magnetised with same polarity producing a force between like polarisations. This provides the deflecting torque because deflecting is obtained due to repulsion between the fixed rod and the movable rod and it is named as repulsion type. Control torque can be provided either by spring or gravity. But for gravity control the spindle must be horizontal. Air damping is employed in this case. Position of the rods after a deflection of 'θ' from the initial position is shown in Fig. 2.4.

Let R be the radius of rotation of the moving rod.
ϕ be the initial angular displacement between rods.
θ be the deflection when a current of I amperes flowing into the coil, and
D be the corresponding distance between the two rods.
m_1 and m_2 be the pole strengths of each rod.

$$\text{Force between the two rods } = 2\left[\frac{m_1 m_2}{D^2}\right] \tag{2.3}$$

$$D = 2R\sin\left(\frac{\theta+\phi}{2}\right)$$



(a) Iron rod (b) Motion of rod in the horizontal plane (c) Calculation of D

Fig. 2.4 Force between the rods.

From Fig. 2.4 (c)

$$\frac{D}{2} = R\sin\left(\frac{\theta+\phi}{2}\right)$$

$$D = 2R\sin\left(\frac{\theta+\phi}{2}\right)$$

$$\text{Deflecting torque} \quad T_D = F \times R\cos\left(\frac{\theta+\phi}{2}\right) \tag{2.4}$$

$$T_D = \frac{2m_1m_2}{D^2}R\cos\left(\frac{\theta+\phi}{2}\right)$$

$$T_D = \frac{m_1m_2\cos\left(\frac{\theta+\phi}{2}\right)}{2R\sin^2\left(\frac{\theta+\phi}{2}\right)} \tag{2.5}$$

As $m_1\ \alpha\ H$ and $m_2\ \alpha\ H$ but $H\ \alpha\ I$
Therefore, $H\ \alpha\ I^2$

$$T_D = \frac{KI^2\cos\left(\frac{\theta+\phi}{2}\right)}{\sin^2\left(\frac{\theta+\phi}{2}\right)} \quad \text{where } K \text{ is a constant.}$$

If gravity control is used $T_C = K_1\sin\theta$
If spring control is used $T_C = K_1\theta$
Under balanced condition $T_D = T_C$

$$\frac{KI^2 \cos\left(\dfrac{\theta+\phi}{2}\right)}{\sin^2\left(\dfrac{\theta+\phi}{2}\right)} = K_1 \sin\theta \tag{2.6}$$

$$I^2 = \frac{C \sin\theta \sin^2\left(\dfrac{\theta+\phi}{2}\right)}{\cos\left(\dfrac{\theta+\phi}{2}\right)} \tag{2.7}$$

$$\text{For gravity control } I = \sqrt{\frac{C \sin\theta \sin^2\left(\dfrac{\theta+\phi}{2}\right)}{\cos\left(\dfrac{\theta+\phi}{2}\right)}}$$

$$\text{For spring control } I = \sqrt{\frac{C' \theta \sin^2\left(\dfrac{\theta+\phi}{2}\right)}{\cos\left(\dfrac{\theta+\phi}{2}\right)}} \tag{2.8}$$

Scale is not uniform but it is cramped. The reading of the meter is proportional to r.m.s. values of current or voltage to be measured.

2.3.3 Relation between Torque and Inductance

Considering the attraction type of moving iron instrument, the inductance of the coil will change with position of the iron piece. Let L be the inductance of the coil for any position of iron piece whose axis is at an angle $(\theta + \phi)$ at that instant.

Let I be the current through the coil

$$E = \text{Energy stored} = \frac{1}{2}LI^2$$

$$\frac{\partial E}{\partial(\theta+\phi)} = \frac{1}{2}I^2 \frac{dL}{d(\theta+\phi)} \qquad \text{Let } \theta + \phi = \beta$$

$$\text{Change in energy/rod} = \frac{\partial E}{\partial(\beta)} = \frac{1}{2}I^2 \frac{dL}{d(\beta)}$$

If T_D is the deflecting torque, then work done by the iron piece in moving through the small angle $\partial\beta$.

$$T_D \times \partial\beta = \text{change in energy stored in the coil.}$$

$$T_D \times \partial\beta = \frac{1}{2}I^2 \partial L$$

Electrical power supplied when I is changed to $I + dI$

$$\text{Elementary power } = dp = dVI = -eI = LI\frac{dI}{dt} + I^2\frac{dL}{dt}$$

$$\begin{aligned}\text{Electrical energy supplied} &= dE = dp \times dt \\ &= LIdI + I^2 dL \end{aligned} \tag{2.11}$$

For energy balance,

Electrical energy supplied = change in energy stored in system + work done by the system.

$$LI\ dI + I^2 dL = \frac{1}{2}I^2 dL + LI\ dI + T_D \times d\beta$$

$$T_D = \frac{1}{2}I^2\frac{dL}{d\beta} \text{ J/radian} \tag{2.12}$$

I is the current in r.m.s value. β is measured in radians.

Example 2.1 *The full-scale torque of a 5 A moving iron ammeter is* 9.8×10^{-6} *Nw.m. Estimate in* μH *per radian the rate of change of self-inductance of the instrument at full scale.*

Solution

$$\text{Torque} = \frac{1}{2}I^2\left(\frac{dL}{d\theta}\right) \text{ Nw.m}$$

Given: $I = 5$ A $\quad T = 9.8 \times 10^{-6}$ Nw.m

$$\frac{dL}{d\theta} = \frac{2T}{I^2}$$

$$= \frac{2 \times 9.8 \times 10^{-6}}{25} = 0.785 \text{ µH/radian}$$

$$\frac{dL}{d\theta} = 0.785 \text{ µH/radian}$$

Example 2.2 *The relationship between the inductance of a 2 amps, moving iron ammeter, the current and the position of the movement is as follows.*

Ammeter reading(Amp)	0.8	1.0	1.2	1.4	1.6	1.8	2.0
Deflection of pointer (Degree)	16	26	36.5	49.5	61.5	74.5	86.5
Inductance in mill (H)	573.2	574.2	575.2	576.2	577.2	578.2	579.2

Deduce an expression for the deflecting torque in terms of the ratio of change of the inductance with position of the movement; calculate the deflecting torque at 1 amp and 2 amps.

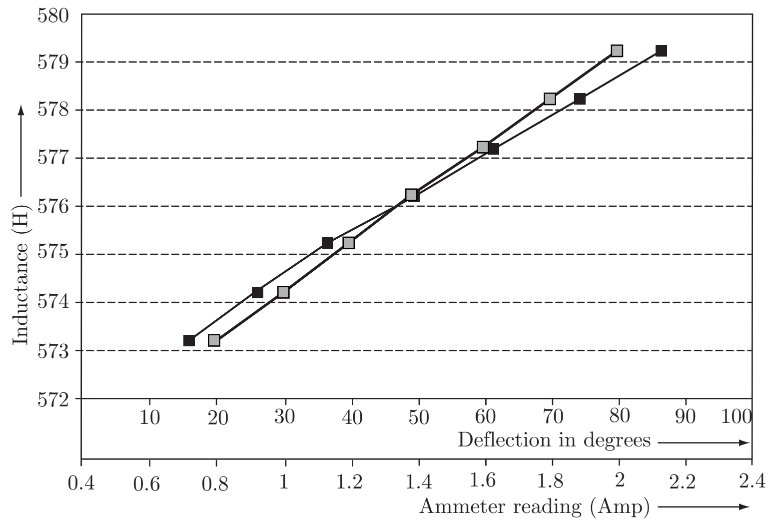


Fig. 2.5(a) Relation between the deflection, ammeter reading in amps and the inductance.

Solution

$$E = \text{Energy stored} = \frac{1}{2}LI^2$$

$$\delta E = \frac{1}{2}I^2\delta L$$

$$T \times \delta\theta = \delta E = \frac{1}{2}I^2\delta L$$

$d\theta$ is measured in radians

(Amp) *I*	0.8	1	1.4	1.6
***dL/dθ* mill/degree**	0.100	0.100	0.1	0.0875

$$\text{Torque} = \frac{1}{2}I^2\left(\frac{dL}{d\theta}\right) \text{ Nw.m}$$

$I = 1$

$$\frac{dL}{d\theta} = 0.1 \times 10^{-3} \text{ H}/0°$$

$$= 0.1 \times 10^{-3} \times \frac{180}{\pi} \text{ H/radian}$$

$$= \frac{1}{2} \times 0.1 \times 10^{-3} \times \frac{180}{\pi}$$

$$= \frac{18}{2\pi} \times 10^{-3} \text{ Nw.m}$$

$$= \frac{9}{\pi} \times 10^{-3} \text{ Nw.m} = 2.37 \times 10^{-3} \text{ Nw.m}$$

$$= 2.87 \times 10.2 = 29.2 \text{ gm.cm}$$

Torque experience when it is carrying a current of 1 amp = 2.87×10^{-3} Nw.m

2.3.5 Errors in the Moving Instrument

1. **Friction error:** This is a common error in every instrument; and can be minimised with reduction of weight of the moving system. A vertical spindle will have free friction than horizontal spindle.

2. **Temperature error:** Since the inductance and resistance of the coil is small, a series resistance is generally added to the instrument that limits the current through the coil for full-scale deflection. In case of voltmeter, by changing the resistance, the range of the instrument can be varied. This added resistance anyhow causes additional loss in the instrument and thus raises the temperature of the system. Due to change in temperature, the resistance of the coil circuit changes and at high temperature the meter will read slightly less than the actual value. The material used for resistance must have low temperature coefficient. Manganese and nichrome are used.

3. **Hysteresis error:** This is a serious source of error in moving iron instrument owing to hysteresis in the iron parts of the moving system, the reading will have descending values of current voltage to be measured than for ascending values of current. The error can be minimised by employing small piece of iron at low flux densities. Another effect of hysteresis is to cause an error due to change in the position of poles as the moving iron occupies different positions. This error is very small.

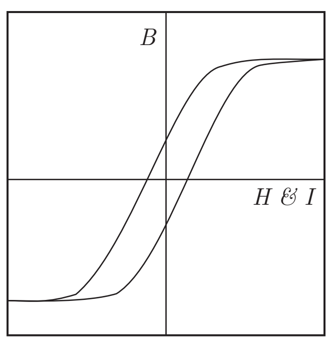


Fig. 2.6 Hysteresis loop.

4. **Stray magnetic field:** Since iron is used in the construction of the meter, external magnetic fields will change the meter readily due to extraneous magnetisation. This error can be eliminated by screening the meter with a casing of magnetic materials.

5. **Frequency error:** This error is introduced whenever there is a change of supply frequency resulting change in the reactance of the coil. Small error is also eliminated due to change in eddy currents because of change in reactance of the disc. At high frequencies the meter rods have low values. If the error due to variation of frequency is to be eliminated, the impedance of the coil must be independent of frequency. For this, a capacitance is shunted across the swamping resistance of the coil. Let L and R be the inductance and resistance of the coil and r be the swamping resistance in series with the coil.

Let C be the value of the capacitance to be shunted so that impedance remains constant. Total impedance must be independent of frequency.

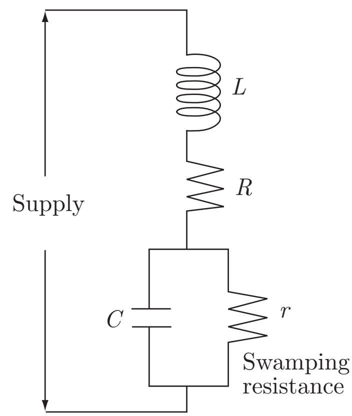


Fig. 2.7 Inductance compensation.

$$Z = R + j\omega L + \frac{r}{1 + j\omega cr} \tag{2.13}$$

$$Z = R + j\omega L + \frac{r}{1 + (\omega cr)^2} - j\frac{(\omega cr)^2}{1 + (\omega cr)^2}$$

$$Z = R + \frac{r}{1 + (\omega cr)^2} + j\left(\omega L - \frac{(\omega cr)^2}{1 + (\omega cr)^2}\right)$$

$$R + r = R + \frac{r}{1 + (\omega cr)^2} + j\left(\omega L - \frac{(\omega cr)^2}{1 + (\omega cr)^2}\right)$$

$$R + r = R + \frac{r}{1 + (\omega cr)^2} \quad \text{(which is true if } (\omega cr)^2 << 1)$$

$$\left(\omega L - \frac{(\omega cr)^2}{1 + (\omega cr)^2}\right) = 0 \qquad [(\omega cr)^2 << 1]$$

$$\omega cr^2 = \omega L$$

$$C = L/r^2 \tag{2.14}$$

Second method: If the condenser is not employed $Z = R + r + j\omega L$. If the value of L is kept constant to a very small value composed to $R + r$, Z will be independent of frequency approximately. If $L/R + r < 0.0005$ the whole thing behaves like a non-inductive circuit.

Example 2.4 *The coil of a moving iron voltmeter has a resistance of 300 Ω and an inductance of 1.0 H. The swamp resistor is 2000 Ω. The meter reads 250 V when a direct voltage of 250 V is*

Example 2.5 *An alternating current voltmeter with a maximum scale reading of 50 V has a resistance of 500 Ω and an inductance of 0.09 H. The magnetising coil is wound with 50 Ω of copper wire and the remainder of the circuit is a non-inductive resistance in series. With it what additional apparatus is needed to make this instrument need correctly both on direct current and alternating current circuits at 50 Hz?*

Solution

For frequency compensation

$$c = \frac{L}{r^2} = \frac{0.09}{25 \times 10^4}$$

$$= 0.36\ \mu F$$

$$R + r = R + \frac{r}{1 + (\omega c R)^2}$$

$$\omega L = \frac{\omega c r^2}{1 + (\omega c R)^2}$$

$$\therefore c = \frac{L}{r^2}$$

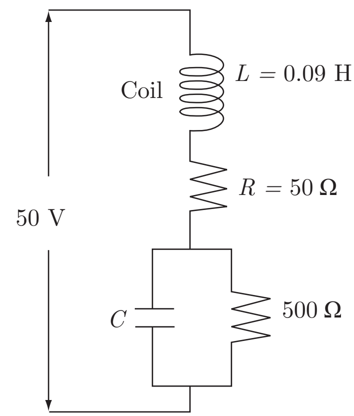


Fig. 2.7(b) Inductance compensation.

Example 2.6 *A permanent magnet moving coil ammeter indicates 20 mA when connected across two opposite corners of a bridge rectifier, the other two corners of which are connected in series with a condenser to a 100 kV, 50 c/s supply. Calculate the capacitance of the condenser.*

Solution

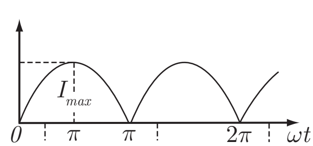
Let $i = I_{\max} \sin \theta$

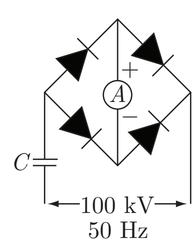
Hence,

$$\text{average value} = \frac{1}{2\pi}\left[2\int_0^{\pi} I_{\max} \sin\theta\, d\theta\right]$$

$$= \frac{2 I_{\max}}{\pi} = 0.02 \text{ A}$$

$$I_{\max} = 0.1 \times \pi \text{ A}$$





Let X_c be the capacitive reactance and assuming zero impedance of the rectifier during conduction.

$$I_{\max} = \frac{100 \times 10^3 \times \sqrt{2}}{X_c}$$

$$X_c = \frac{100 \times 10^3 \times \sqrt{2}}{I_{\max}}$$

$$\frac{1}{\omega c} = \frac{10^5 \times \sqrt{2}}{I_{\max}}$$

$$C = \frac{I_{\max}}{\omega \times \sqrt{2}} \times 10^{-5}$$

$$C = \frac{10^{-2} \times \pi}{2\pi f \times \sqrt{2}} \times 10^{-5}$$

$$= \frac{1}{\sqrt{2}} \times 10^{-5}$$

$$= 0.707 \times 10^{-9} \text{ F} = 707 \text{ μμF}$$

Example 2.7 *How are the temperature errors of a switch board type D.C. ammeter compensated for D? In a particular case, the P.D. between the potential of the shunt of 1000 A instrument is 0.03 V. The connecting leads to the instrument are of copper and have a resistance of 0.12 Ω and the moving coil has a resistance of 1.2 Ω and requires a current of 15 mill amps to deflect it to the 1000 A point of the scale. The temperature coefficient of the alloy used for the shunt is 0.00001 and that of copper 0.004 per ℃ in terms of the resistance at 15 ℃. To what extent could this instrument be compensated if it is accurately adjusted at 15 ℃? What error would be expected when all the parts are at 35 ℃.*

Solution

Current through the instrument $= 15$ mA

Voltage across the instrument $= 0.03$ V

Resistance of the instrument $= \dfrac{0.03}{0.015} = 2\ \Omega$

But the resistance of the coil + connecting loads

$$= 1.2 + 1.2$$

$$= 1.32\ \Omega$$

Hence, swamping resistance $= 2 - 1.320 = 0.68\ \Omega$

Current through the shunt $= 1000 - 0.015 = 999.985$ A

Shunt resistance $= \dfrac{0.03}{999.985}$

$$= 0.03 \times 10^{-3} = 3 \times 10^{-5}\ \Omega$$

Shunt resistance at 35 °C $= 3 \times 10^{-5}[1 + 20 \times 0.00001]$

$$= 3 \times 10^{-5}[1.0002]$$

$$= 3.0006 \times 10^{-5}\Omega$$

Instrument resistance at 35 °C $= 1.32[1 + 0.004 \times 20]$

$$= 1.08 \times 1.32 = 1.425\ \Omega$$

Hence, the total resistance

$$P = 1.425 + 0.68 = 2.105\ \Omega$$

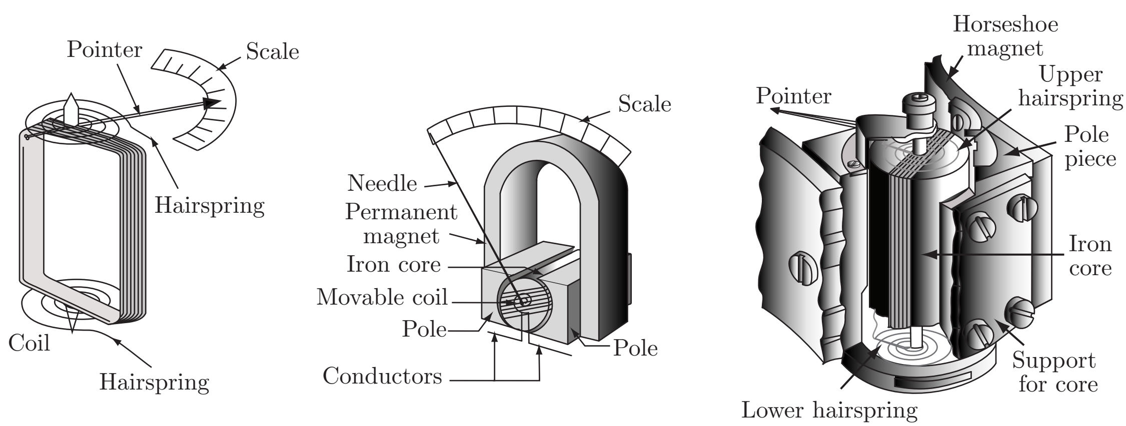
Hence, instrument current at 35 °C

$$\frac{1000 \times 3.0006 \times 10^{-5}}{2.105 + 3.0006 \times 10^{-5}} = 14.25\ \text{mA}$$

Hence, instrument will read $= \dfrac{1000}{15} \times 14.250 = 951\ \text{A}$

Hence, error $= \dfrac{49}{1000} \times 100 = 4.9\%$

2.4 MOVING COIL PERMANENT MAGNET TYPE (PMT)



(a) Moving coil (b) Plan (c) Sectional view

Fig. 2.8 Moving coil permanent magnet type instrument.

$$= 2b \times B2lIN$$
$$= BIN[2b \times 2l] \qquad (2.16)$$

Let A be cross-sectional area of the coil $= 2b \times 2l$

$$T_D = BINA = (B \times A)IN = \phi IN \text{ N.m} \qquad (2.17)$$

where T_D = Deflecting torque $\propto$ current
T_c = Control torque $\propto \theta$
$I \propto \theta$

Hence, total torque on the coil at any position is the product of the flux linking with the coil and the ampere turns of that coil.

In this case graduated scale will be uniform and the maximum deflection of the coil is 90 Ω. The resistances used in voltmeter for extension of range are called multipliers.

Let r = resistance of the moving coil
V = supply voltage
v = drop across the moving coil.

$$v = \frac{V}{R+r} \times r$$
$$\frac{V}{v} = \frac{R+r}{r} = 1 + \left(\frac{R}{r}\right) = N$$

where N is the multiplying power of the multiplier

$$\frac{V}{v} = 1 + \left(\frac{R}{r}\right) \qquad (2.18)$$

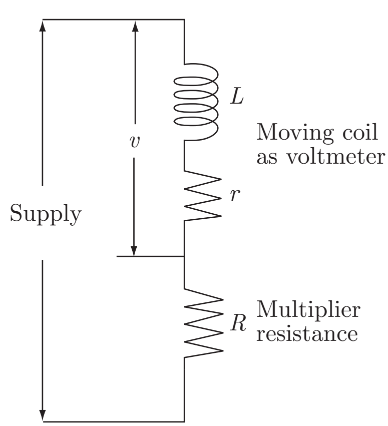


Fig. 2.10 Moving coil as voltmeter.

Let

I be the total current
I_c be the current through moving coil
R_s be the resistance of the shunt
R_c be the resistance of the coil

$$\text{Multiplying power of shunt } N = \frac{I}{I_c} \tag{2.19}$$

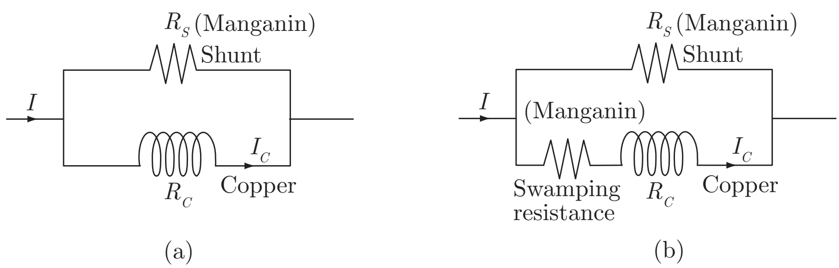


Fig. 2.11 Moving coil as an ammeter.

The ratio $\frac{I}{I_c} = \frac{R_c + R_s}{R_s}$ must be constant for all temperatures in order the meter should read correctly. But the temperature coefficient of copper is higher than that of manganin. The result is that it reads less than the actual value under high temperatures. To avoid this type of error, the following modification is made according to B.S.S.

The temperature error introduced in this connection (b) is less than the error introduced in connection (a).

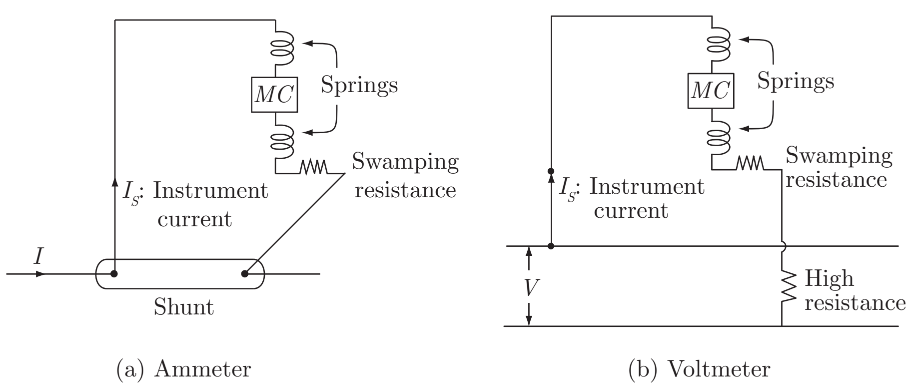


Fig. 2.12 Connection of moving coil instrument as voltmeter and ammeter.

The connecting leads must run from the instrument only. But, on the other hand, if the leads are run from the shunt the total resistance in the shunt circuit will get effected and the meter reads erroneous results.

Advantages of moving coil (permanent magnet type)

(a) Power consumption is very small.
(b) Torque/weight rate is very high.
(c) It has got uniformly calibrated scale and has possibility of a very long scale (3000) cirscale type.
(d) A single instrument can be used with suitable shunts and multipliers either as an ammeter or as voltmeter.
(e) It is free from errors due to extraneous magnetic fields since the strength of the working field is high.
(f) Unlike in moving iron instrument no hysteresis error is present.
(g) Damping is perfect because of eddy currents induced in the former of the moving coil.

2.4.3 Cirscale Type of Moving Coil Permanent Magnet Type

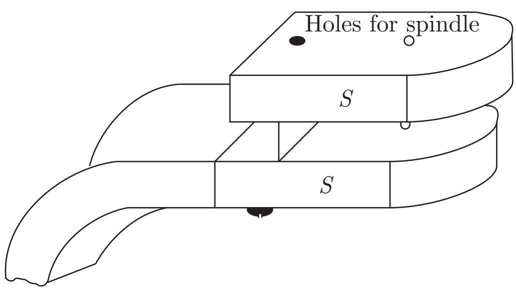


Fig. 2.13(a) Arrangement of magnet.

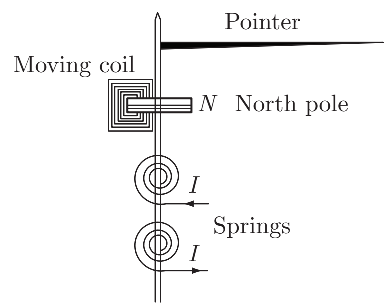


Fig. 2.13(b) Arrangement of moving coil range.

Cirscale type of instrument can read through a complete range of 300°. The arrangement is shown above in Fig. 2.13 where the north pole in the circular spiral shape is filled into the gap between two half south poles. The working current is led into the coil through the springs. The advantage of this type of instrument is that it can be used for large ranges with good accuracy.

The current carrying capacity of moving coil instrument cannot be increased owing to increase in weight of moving system and this reducing the torque/weight ratio. To limit the current in the coil, they are used in conjunction with shunts which are made of manganin strips with a very low resistance temperature coefficient. The moving coil also must have low temperature coefficient so that the ratio instrument current/total current remains constant for all the temperatures.

Extension of instrument ranges

(a) As an ammeter:

Let, I be the current in the circuit,
R_e be the resistance of the coil,
R_s be the resistance of the shunt used.

Example 2.9 *Two ammeters are joined in series in a circuit carrying 10 A. Ammeter A has a resistance of 1000 A and is shunted by 0.02 Ω. The corresponding value for ammeter B are 1500 Ω and 0.01 Ω. What will the instruments read if the shunts are interchanged?*

Solution:

Circuit current is 10 A

$$\text{Current through the ammeter A} = \frac{10 \times 0.02}{1000 + 0.02} = 0.02 \times 10^{-2} = 0.2 \text{ mA}$$

$$\text{Current through the ammeter B} = \frac{10 \times 0.01}{1500 + 0.01} = 0.00667 \times 10^{-2} = 0.06 \text{ mA}$$

When shunts are interchanged.

$$\text{Current through the ammeter A} = \frac{10 \times 0.01}{1000 + 0.01} \cong 0.1 \text{ mA}$$

$$\text{Current through the ammeter B} = \frac{10 \times 0.02}{1500 + 0.02} = 0.12 \text{ mA}$$

0.2 mA corresponds to 10 A deflection

$$0.1 \text{ mA corresponds to } \frac{10}{0.02} \times 0.1 = 5 \text{ mA}$$

Ammeter A will read only 5 A

0.06 mA corresponds to a deflection of 10 A

$$0.12 \text{ mA corresponds } \frac{10}{0.06} \times 0.12 = 20 \text{ mA}$$

Ammeter A will read only 20 A. Ammeter B will read only 20 A.

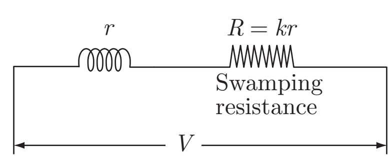
Example 2.10 *A voltmeter has a working coil of copper in series with an invariable swamp resistance. What must be the ratio of swamp resistance to coil resistance if the error introduced by a temperature rise from 15 °C to 25 °C is not to exceed 1% of the indication?*

Solution:

Current drawn by the meter at 25 °C $i = \dfrac{v}{r + kr}$

Change in resistance in r due to temperature

$= r[1 + 32.0 \times 10^{-4} \times 10] = 1.0320r \qquad \alpha = 39 \times 10^{-4}$



Hence, current drawn by the meter at 25 °C

$i_1 = \dfrac{v}{r + 1.0320r}$

Voltmeter reading corresponding to i_1

$$v^1 = \frac{v}{i} \times i_1 = v\left[\frac{r + kr}{kr + 1.0370r}\right]$$

$$\text{Error} = \frac{v - v^1}{v} < 0.01$$

$$1 - \frac{v^1}{v} < 0.1$$

$$1 - \frac{r + kr}{kr + 1.0370r} < 0.1$$

$$1 - \frac{1 + k}{k + 1.0370} < 0.1$$

$$k \geq 2.861 \qquad k = 3$$

$$\frac{\text{Swamping resistance}}{\text{Coil resistance}} = 3$$

2.5 DYNAMOMETER TYPE (ELECTRODYNAMIC) MOVING COIL

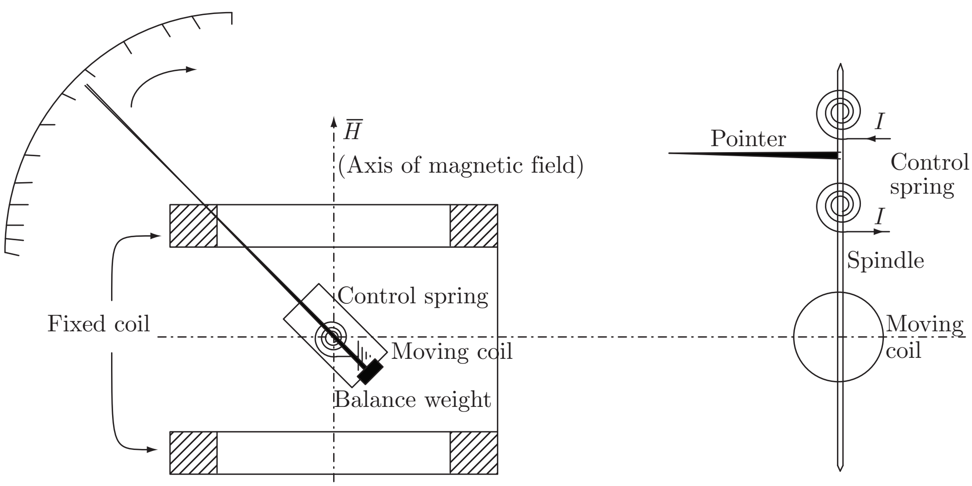


Fig. 2.15(a) Dynamometer type moving coil Instrument. **Fig: 2.15(b)** Elevation.

Dynamometer type moving coil instrument can be used on A.C. and D.C. It consists of two fixed coils and one moving coil and are usually air core. In some cases to increase the magnetic field strength Nickel-Iron core is employed. This slightly improves the torque versus weight ratio. Control torque is provided with springs.

Damping employed is generally air piston or enclosed vane type. The two fixed coils are connected in series such that the magnetic fields produced by them will aid each other. Torque on the moving coil is produced due to interaction of magnetic fields produced by fixed coil and the moving coil. Control springs are used as leads (current) for the moving coil to pass current.

Average torque is proportional to $I^2_{r.m.s.}$

We cannot get uniformly graduated scale but it is cramped.

If N_2 is the number of turns of the moving coil ϕ_{12} is the fraction of flux linking with moving coil

$$M = \frac{(N_2\phi_{12})}{I_1}$$

$$T_D = i_1 i_2 \frac{d(N_2\phi_{12})/i_1}{d\theta} = i_2 N_2 \left(\frac{d\phi_{12}}{d\theta}\right) \tag{2.23}$$

$$= \text{Change in the energy stored/radian}$$

Generally, the current carrying capacity of moving coil is small which reduces the weight of the system. Therefore, small torque is sufficient to deflect the moving system. The current that can be passed through the moving coil is restricted because springs are used as leads for the moving coil. The current carrying capacity of the control springs is small. Arrangements to regulate current in the moving coil as voltmeters and ammeters are shown in Fig. 2.16.

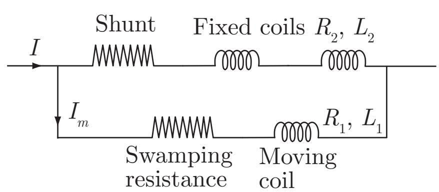


Fig. 2.16(a) Connections as ammeter.

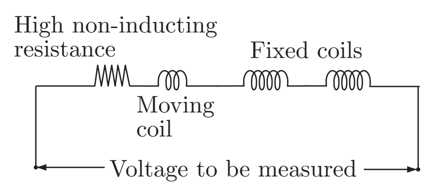


Fig. 2.16(b) Connections as voltmeter.

The impedance of the moving coil and swamping resistor is very high compared to the resistance of the shunt and fixed coils. Therefore, most of the current is taken by fixed coils. The current through moving coil (I_m) is given by

$$I_m = \frac{I\sqrt{R_2^2 + (L_2\omega)^2}}{\sqrt{(R_1 + R_2)^2 + [(L_1 + L_2)\omega]^2}} = \frac{IZ_2}{Z_1 + Z_2} \tag{2.24}$$

If the meter is to read correctly for all frequencies the ratio IM/I must be constant. This remains constant provided

$$\frac{L_1}{R_1} = \frac{L_2}{R_2} \tag{2.25}$$

which means the time constant of both the circuits must be same. Under these conditions

$$\frac{I_m}{I} = \frac{R_2}{R_1 + R_2} \quad \text{(as an ammeter)} \tag{2.26}$$

As voltmeter:

For using it as voltmeter, all the three coils must be connected in series with a high non-inductive resistance to limit the current to the normal value to give full-scale deflection. Frequency errors can be made small in this case when L is made small (total inductance of the circuit) and $H.R. \gg L$.

2.5.2 Disadvantages

1. Even though the instrument can be used for precise measurements on A.C. it is not favourable to use it on D.C. measurements because of small torque/weight ratio and therefore frictional errors tend to be serious along with internal heating errors.

2. Because of the absence of iron core, magnetic field strength is small which obviously reduces the magnitude of the deflecting torque. If the deflecting torque to be increased, ammeters on the moving coil should be increased and therefore weight of the system increases which in turn increases the friction and reduces the torque vs weight ratio.

3. Since the deflecting torque is governed by square law, the calibrated scale will not be uniform but it is cramped.

4. Cost of instrument is high when compared to D.C. meters (permanent magnet type).

For the above reasons generally this type of instrument is not used for measurement of current and voltage but most commonly used as dynamometer wattmeter.

2.6 THERMOCOUPLE INSTRUMENTS

Thermocouple is a name given to the combination of two wires of dissimilar materials that have the following property:

If the two junctions formed by the combination of dissimilar materials are kept, one at (hot junction) higher temperature and another at lower temperature, a current flow is observed in the circuit formed by two dissimilar metals. This effect is called *Seebeck's effect*.

Peltier effect:
In a circuit formed by two dissimilar metals, current flow is maintained then the two junctions formed by the dissimilar metals will attain different temperatures.

Kelvin effect:
Thermal e.m.f will be developed in a conductor when there is a difference of temperature gradient along the conductor.

Materials for thermocouple: Platinum-iridium, gold palladium.

2.6.1 Thermocouple Ammeter

Thermocouple instrument is shown in Fig. 2.17. Kelvin effect is used for measurement of current by placing "hot junction" in thermal contact (but not necessarily electrical contact) with a heater carrying the current to be measured. The cold junction is placed in conjunction with a milliammeter.

2.6.2 Compensated Type

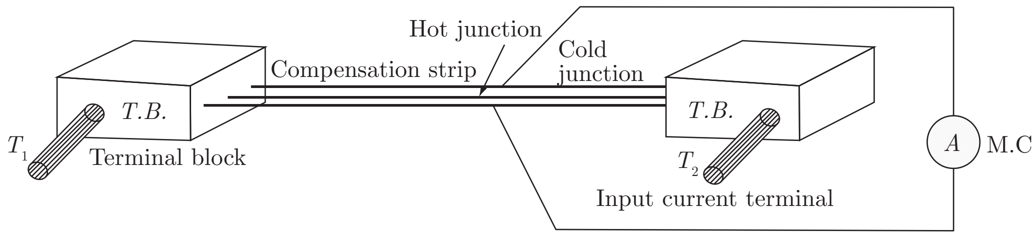


Fig. 2.18 Compensated thermocouple ammeter.

A compensated type of thermocouple is shown in Fig. 2.18. It consists of two large terminal blocks of copper with heater element between them. The thermocouple junction is welded or hard soldered to the midpoint of the heater. The other ends of the thermocouple wires are connected one to each of two copper strips that bridge the space between the terminal blocks. These strips are called compensation strips and are fastened to the blocks mechanically but are separated from the blocks by a thin sheet of mica to give electrical insulation while permitting effective thermal contact. The instrument wires are connected to the two compensating strips. By this means the cold junction is kept effectively at the average temperature of the two end blocks. It can be used from 1 to 50 amps. This can be used up to 50 MHz with an accuracy of 1%.

Voltmeters may be constructed up to 500 V with a series resistance. They are available with sensitivities of 100 to 500 Ω/V. But frequency range is limited upto 15 kHz.

Effect of frequency:
As the frequency is increased beyond several MHz the current distribution with the heater element becomes non-uniform and its resistance increases and consequently the voltage required to circulate a given current in the heater wire will be more. Another result of high frequency is the high impedance associated with the effective series inductance and shunt capacitance of the heater wire. In a 5 A heater, this may amount 60 to 70 Ω at 200 MHz and requiring a voltage drop of 300 V or more. To minimise this tabular type of heater element is used.

2.6.3 Bridge Type Thermocouples

Bridge type of thermocouple is shown in Fig. 2.19. This is used for higher ranges. No separate heater is used instead the current to be measured passes directly through the thermocouples and raises their temperature in proportion to the I^2r. The cold junctions are at the pins that are embedded in the common bakelite base and hot junctions at splices midway between the pins. The couples are oriented as shown in the Fig. 2.19 so that the resultant thermal voltages give rise to a D.C. difference of potential from A to B. The A.C. potential between these points is zero because of the balanced resistances in the four arms, so no alternating current flows through the instrument. This construction gives greater output voltage than the single couple used in the vacuum mounting and is more rugged in withstanding overload. It can be constructed conveniently to measure greater currents. It can be used from 0.1 to 1 amp.

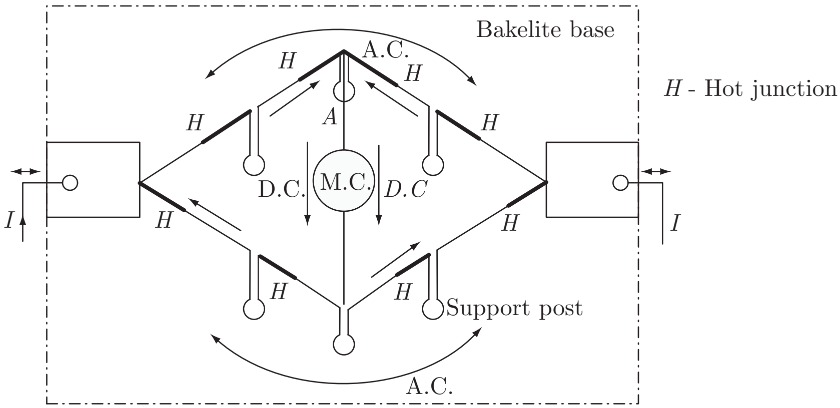


Fig. 2.19 Bridge type of thermocouple.

Mathematical proof:

Law of thermal conduction states $q = ak\dfrac{d\theta}{dx}$

where q is the heat dissipated or lost by conduction in watts/sec by strip of length x metres
a is the c.s. of the heater element in sq.mts
k is the thermal conductivity of the strip material (heater) in watts/degree/metre
$\dfrac{d\theta}{dx}$ is temperature gradient in degree/m. at a distance x from the terminal blocks

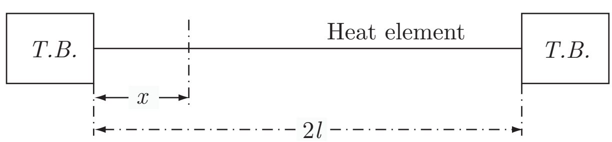


Fig. 2.20 Model of thermocouple instrument.

Let W be the rate of heat generation per unit length of the heater element.

$$W = +\frac{dq}{dx} \quad \text{(at steady temperature)}$$

If K and a are assumed to be constant

$$\int W dx = +\int dq$$

$$Wx = +q \quad \text{[heat generated = heat dissipated]}$$

$$Wx = ak\frac{d\theta}{dx}$$

$$T_M = \alpha \frac{1}{2\pi} I_{\max}\phi_{\max}\frac{2\pi\cos\alpha}{2}$$

$$T_M = \left(\frac{I_{\max}}{\sqrt{2}}\right)\left(\frac{\phi_{\max}}{\sqrt{2}}\right)\cos\alpha \tag{2.31}$$

$$T_M \propto I_{r.m.s.}\phi_{r.m.s.}\cos\alpha$$

$$T_M = KI_{r.m.s.}\phi_{r.m.s.}\cos\alpha \tag{2.32}$$

From the above expression, it can be seen that the phase angle between ϕ and i must be less than 90°. These are two methods of achieving phase differences.

One method is by splitting the winding of the electromagnet in which the flux exists into two portions, one of which is high inductive and the other is non-inductive. Ferrari's type makes use of this method.

Second method is by splitting the phase of the working flux by a copper band placed around a portion of the poles of the electromagnet. This type is called shaded pole type.

2.7.1 Ferrari's Type

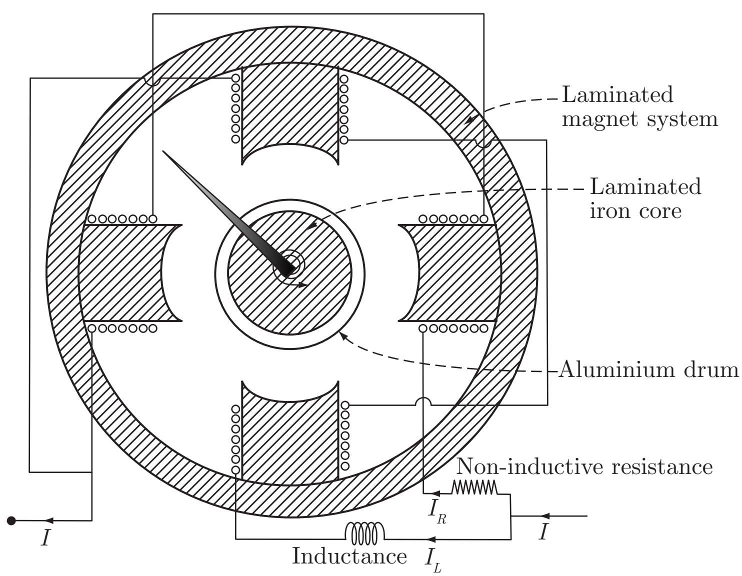


Fig. 2.21 Induction ferrari's type instrument.

Ferrari's type of ammeter/voltmeter is shown in Fig. 2.21. This instrument operates on the same principle as an induction motor. Rotating field is produced by two pairs of coils wound on laminated magnet system. The pairs of coils excited by the same source, but a phase displacement approximately 90° is produced between the currents in them by connecting inductance in series with one pair and a non-inductive resistance in series with the other pair. The rotating field produced by them induces eddy currents in the aluminium drum and causes the drum to follow the field but because of control spring, the drum will rotate until $T_C = T_D$. The drum and the moving system is carried by a spindle whose ends fit in jewelled bearings. Inside the drum is a cylindrical laminated iron core to strengthen the magnetic field cutting the drum. The spindle also carries an aluminium damping disc the edge of which moves in the air gap of permanent magnet.

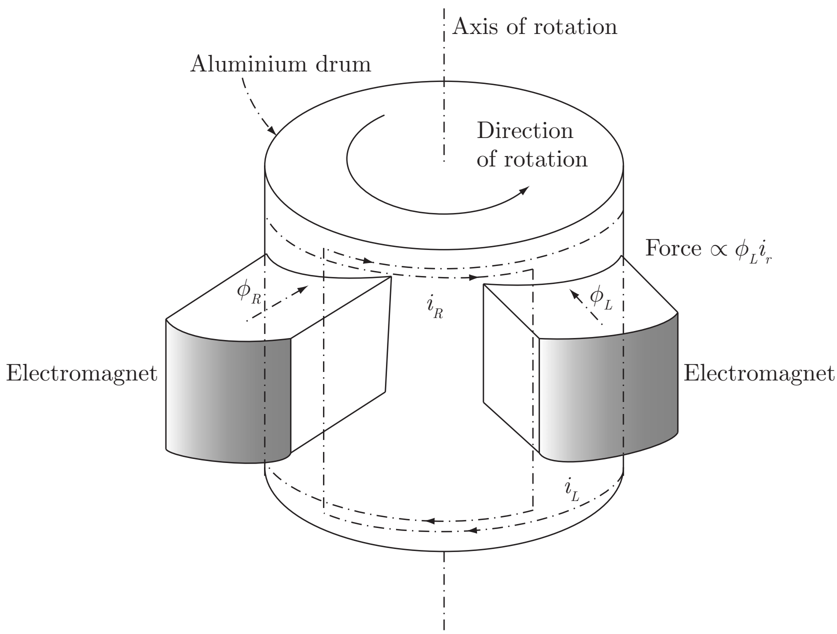


Fig. 2.22 Eddy currents and torque on the aluminium drum.

2.7.2 Torque

I_R and I_L are the currents in the two pairs of coils with phase angle difference of β.

E_R and E_L back emfs induced in the drum and each is lagging the corresponding flux producing it by 90°.

i_R and i_L are the corresponding eddy current in the drum due to e_R and e_L respectively. ϕ_R and ϕ_L and i_R and i_L are the r.m.s. values

$$T_{\text{Mean}} = T_{\text{average}} = [-\phi_R i_L \cos(90 + \alpha + \beta) + \phi_L i_R \cos(90 + \alpha - \beta)] \tag{2.33}$$

$90 + \alpha + \beta$ = phase angle between ϕ_R and i_L

$90 + \alpha - \beta$ = phase angle between ϕ_L and i_R

As regards the torque of the instrument it can be seen from the figure that there will be two components one proportional to $\phi_L\ i_R$ in the direction of rotation of the rotating field and the other proportional to $\phi_R\ i_L$ in opposite direction. In the Fig. 2.23 both the fluxes ϕ_R and ϕ_L are assumed

Error due to change in frequency is more serious than in case of induction type of voltmeters. Frequency errors can be compensated by shunting the ammeter by a non-inductive shunt. When frequency is increased, impedance of the branches 1 and 2 will increase thus diverting the more current through non-inductive shunt, I_L and I_R are reduced and the product will remain constant resulting the same torque.

2.7.4 Temperature Errors

$$T_M \propto \frac{I_R I_L f}{Z} \cos\alpha \sin\beta \qquad (2.34a)$$

The magnitude of Z is dependent of temperature. When α is small Z will be pure resistance; with increase of temperature the magnitude of Z will be increased. The temperature errors can be eliminated by shunting with a resistance with high temperature coefficient.

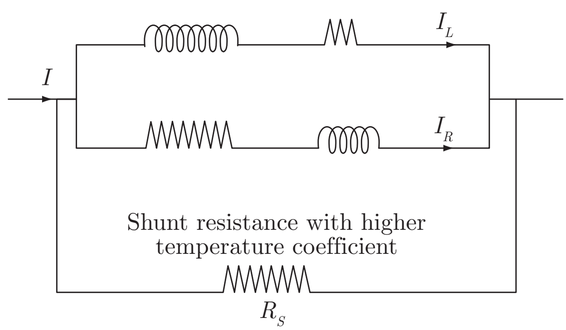


Fig. 2.25 Compensation for temperature variation.

An increase in temperature will increase R_s thus forcing more current to flow through 1 and 2. i.e., I_L and I_R will increase to maintain the magnitude of the torque as constant. It is clear a suitable shunt will compensate both temperature and frequency errors. But frequency errors cannot be compensated satisfactorily and therefore induction type instruments are used as switchboard purpose when frequency changes are small.

Advantages:

1. We can have a very long scale giving deflection up to 300 °C.
2. Stray magnetic fields don't have much effect on the instrument readings.
3. Perfect damping is employed due to eddy currents.

Disadvantages:

1. It can be used on A.C. only.
2. A large deflection results fatigue (stress) in the control spring.
3. Errors due to change of frequency and temperature are present.
4. Fairly high power consumption.
5. Cost of instrument is high.

2.7.5 Shaded Pole Type

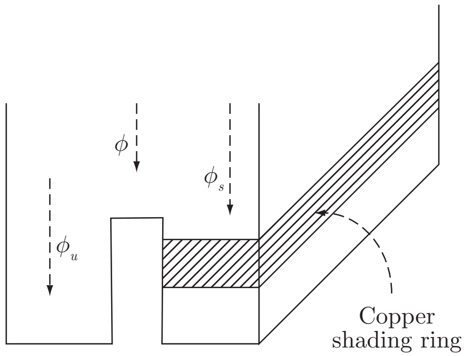


Fig. 2.26(a) Shaded pole.

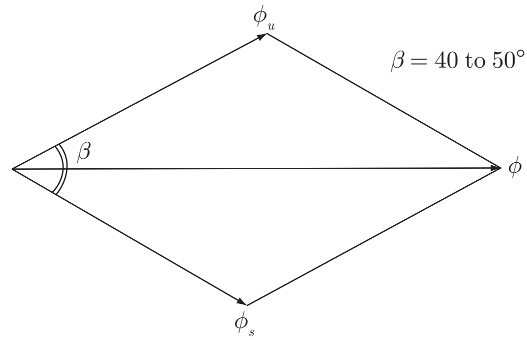


Fig. 2.26(b) Phasor diagram.

When a pole of an electromagnet is split into parts and one part is shaded by copper band to act as a secondary, then there will be phase displacement between the two flux entering shaded and unshaded portions of the magnet. When there is a change of flux, emf will be induced in the copper band which drives some current through it. This current reacts with the main flux, resulting a flux ϕ_S which lags ϕ_U about 40° to 50°. But the resultant of ϕ_S and ϕ_U will always give ϕ. This principle is also used for starting of a single phase induction method.

2.7.6 Construction

A thin aluminium disc is mounted on a spindle which is supported by jewelled bearings. The spindle carries a pointer and control torque is provided by a spring attached to it.

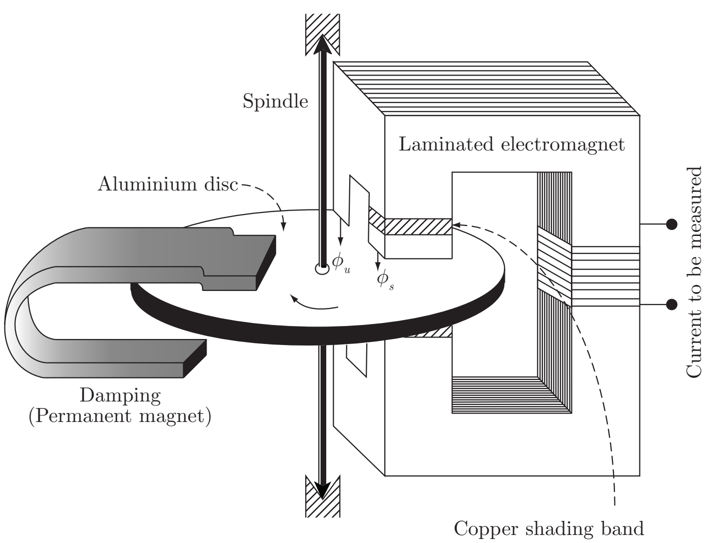


Fig. 2.27 Shaded pole type of ammeter/voltmeter.

$$T_M = k\frac{\phi_u \phi_s f}{Z}[\sin(\alpha+\beta) - \sin(\beta-\alpha)]$$

$$T_M \propto \frac{\phi_u \phi_s f}{Z}\sin\beta\cos\alpha \qquad (2.37)$$

For a given frequency

$$T_M \propto I^2 \qquad (2.37a)$$

Here T_M is proportional to frequency and inversely proportional to impedance of the eddy current path. Therefore, change in frequency and temperature will introduce error in the instrument. Compensation for both frequency and temperature variation can be provided by shunting the exciting coil with non-inductive resistance as discussed in Ferrari's type.

2.8 HOT-WIRE INSTRUMENTS

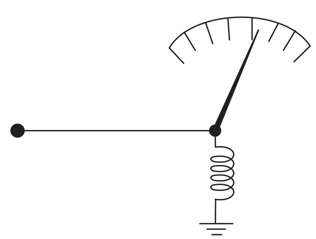


Fig. 2.28(a) Principle of hot-wire instrument.

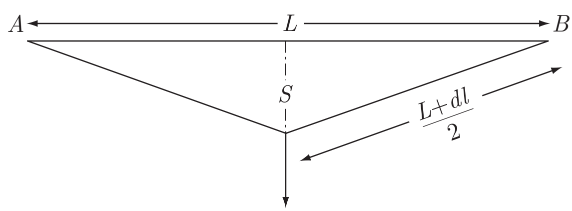


Fig. 2.28(b) Single sag instrument.

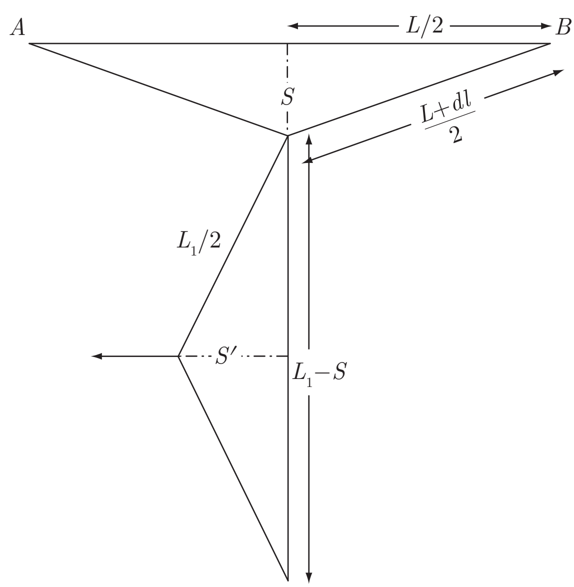


Fig. 2.28(c) Double sag instrument.

Principle of hot-wire instrument is shown in Fig. 2.28. This type of instruments can be used for both A.C. and D.C. In these instruments the current to be measured or a definite fraction of it, is passed through a fine wire, which expands due to the heating effect of the currents. If the resistance and coefficient of expansion of the wire are constant, the heating and expansion are both directly proportional to the square of the current. This expansion of the fine wire is used to produce deflection of a pointer which are proportional to the square of the current. Normal expansion of 17 cm length of free wire will be 2 mm.

2.8.1 Magnification of the Expansion

In commercial instruments, the hot-wire is too short for the expansion to be used directly for the production of a deflection. A magnifying device is used and the theory of which follows below. Let AB be the hot-wire of length L and dL be its expansion after attaining the steady temperature. Let S be the sag obtained when tension is applied at the centre of the wire.

$$S^2 = \frac{(L+dL)^2}{2} - \left(\frac{L}{2}\right)^2 = \left(\frac{dL}{2}\right)^2 + \frac{LdL}{2} \tag{2.38}$$

$$S = dL\sqrt{\frac{1}{4} + \left(\frac{L}{2dL}\right)} \cong dL\sqrt{\frac{L}{2dL}} \cong \sqrt{\frac{LdL}{2}} \tag{2.39}$$

$$\text{Magnification} = \frac{S}{dL} = \frac{\sqrt{LdL/2}}{dL} = \sqrt{\frac{L}{2dL}} \tag{2.40}$$

(a) But further magnification is possible as in case.

(b) Here sag of the downward wire is utilised. Let L_1 be the length of the downward wire and S_1 be the corresponding sag.

$$S_1{}^2 = \left(\frac{L_1}{2}\right)^2 - \frac{(L_1 - S)^2}{4} = \frac{2SL_1 - S^2}{4} \tag{2.41}$$

$S_1{}^2 = \dfrac{SL_1}{2}$ since $S_1{}^2$ is small compared to SL_1

$$S = dL\sqrt{\frac{1}{4} + \left(\frac{L}{2dL}\right)} \cong dL\sqrt{\frac{L}{2dL}} \cong \sqrt{\frac{LdL}{2}}$$

$$S_1{}^2 \cong \frac{2SL_1}{4} = \frac{SL_1}{2}$$

$$S_1 = \sqrt{\frac{SL_1}{2}} \tag{2.42}$$

$$\text{but } S = \sqrt{\frac{LdL}{2}} \qquad S_1 = \sqrt{\frac{L_1}{2\sqrt{2}}}[LdL]^{1/4}$$

$$S_1 \propto [dL]^{1/4} \propto [I^2]^{1/4} \propto \sqrt{I} \tag{2.43}$$

2.8.4 Disadvantages

1. Sluggishness is indication owing to the time taken for the wire to attain steady temperature.
2. Power consumption is high.
3. Overload capacity of the instrument is nil because of the fine wire used as heating element which quickly fuses when overloaded.
4. There will be change in zero position of instrument due to change in temperature external to the instrument. (it requires frequent zero adjustment)
5. It is a fragile instrument.

The greatest advantage of hot-wire instruments is ability to read even on radio frequencies because the principle of instrument does not depend upon frequency. This is generally used for measurements with high frequency. The other type which we can use on RF is thermocouple type.

Example 2.11 *The working wire of a single sag, hot-wire instrument is 15 cm long, and is made of platinum silver with a coefficient of linear expansion of 16 × 10^{-6}. The temperature rise of the wire is 85℃ and the sag is taken up at the centre. Fin the magnification (a) With no initial sag (b) With an initial sag of 1 mm.*

Solution

(a) With no initial sag from the equations 2.39a and 2.42 and referring to Figs. 2.37(b) and 2.37(c)

$$
\begin{aligned}
S &= \sqrt{\frac{LdL}{2}} \\
S^1 &= \sqrt{\frac{L^1 S}{2}} \\
dL &= L \times \alpha\ t \\
&= 15 \times 16 \times 10^{-6} \times 85 = 1.5 \times 1.6 \times 8.5 \times 10^{-3} \\
&= 20.4 \times 10^{-3} = 0.0204 \text{ cm} \\
S = Sag &= \sqrt{\frac{15 \times 0.204}{2}} = \sqrt{15 \times 0.0102} = \sqrt{0.153} \\
S &= 0.391 \text{ cm} \\
\frac{S}{dL} &= \frac{0.391}{0.0200} = \frac{39.1}{2.04} = 19.2
\end{aligned}
$$

$$\text{Hence, magnification} = \frac{S}{dL} = 19.2$$

(b) With an initial sag S = 0.1 cm

$$
\begin{aligned}
S^2 &= \frac{LdL}{2} \\
\text{Initial expansion } dL &= \frac{2\text{S}^2}{L} = \frac{2 \times 0.01}{15} \\
&= 0.001335 \text{ cm}
\end{aligned}
$$

But increase in length due to temperature = 0.0204 cm

$\therefore \quad \text{Total } dL = 0.001335 + 0.0204 = 0.021735$

Sag under new conditions $dL = 0.021735$ cm

$$S = \sqrt{\frac{LdL}{2}} = \sqrt{7.5 \times 0.21735} = \sqrt{0.783} = 0.403 \text{ cm}$$

$$S = 0.403 \text{ cm}$$

Hence, actual change is due to expansion = 0.403 − 0.10 = 0.303 cm

$$\text{Hence, magnification} = \frac{0.303}{0.0204} = 14.85$$

2.9 ELECTROSTATIC TYPE

These type of instruments are mainly used as voltmeter and indirectly as ammeters and watt-hour meters.

Advantages:

1. Power consumption is extremely small.
2. All the errors due to hysteresis and eddy currents will not be present because of the absence of the iron parts.
3. It reads correctly both on A.C. and D.C.
4. The action of the instrument is independent of variation of waveform and frequency.
5. This type of instrument can be designed from low voltage range (50 to 500 kV).

Disadvantages:

1. Operating forces are very small for low voltages below 500 V.
2. Torque/wt ratio is low compared to other instruments.
3. Formation of corona at high voltages is one of the limitations.
4. At very high frequencies, the capacitive reactance of the instrument may have the same value as load circuit impedance in which case there will be an error in the reading.

2.9.1 Types of Instruments

The electrostatic type of the instruments can be broadly classified as

1. Repulsion type
2. Symmetrical type
3. Attraction type

2.9.4 Symmetrical Type

There are basically two types in the symmetrical type of electrostatic measuring instruments. They are:

1. String electrometer
2. Quadrant electrometer.

2.9.4.1 String electrometer

Metal plates are connected to the battery shown which charges the plates. A metal coated quartz fibre is suspended in between the charged plates and it is kept under tension by the help of a string. When the voltage to be measured is applied to the fibre and the midpoint of the battery (ground), the fibre will be charged, and this charged fibre will experience force due to both negatively and positively charges plates. The deflection of the fibre is directly proportional to (voltage)^2.

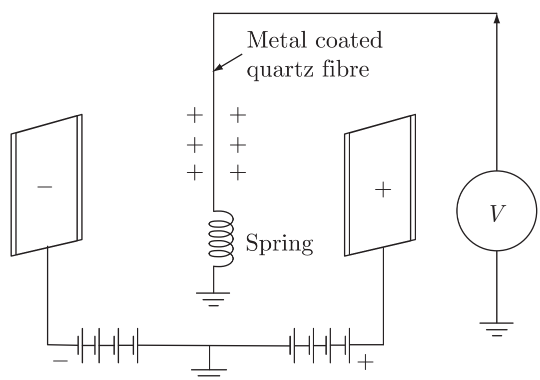


Fig. 2.32 String electrometer.

2.9.4.2 Quadrant electrometer

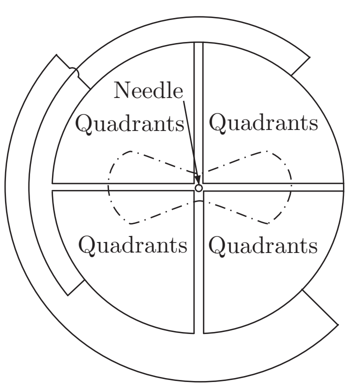


Fig. 2.32(a) Quadrant electrometer.

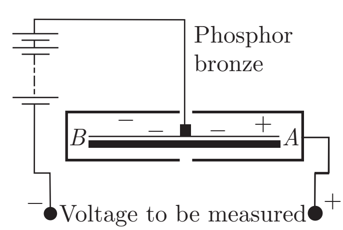


Fig. 2.32(b) Hetrostatic.

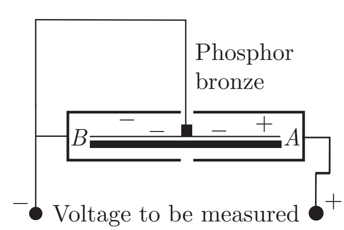


Fig. 2.32(c) Idiostatic.

It consists of four fixed metal double quadrants arranged to form shallow circular box with short air gaps between the quadrants. In this, a thin metal sector type needle is suspended by a thread of phosphor-bronze and is symmetrically placed inside the circular box. There are two methods of connecting the quadrants and the needle. In hetrostatic case, a H.T. battery is used to charge the moving vane. The battery voltage is considerably higher than the voltage to be measured and it is placed between the needle and the quadrant to which the negative of the voltage to be measured is connected. The moving vane is charged to higher potential when compared to the potential of B. But in idiostatic connection needle is directly connected to the quadrant which is charged to the negative voltage. This type of connection is generally used in commercial instruments.

The forces of attraction and repulsion are not purely rotational, but have components in the direction perpendicular to the needle, one upward and other downward at each end of the needle cancelling each other leaving only the rotational components.

2.9.4.3 Theory

Considering only half of the moving vane, the vane will be repelled from quadrant(A) and attracted by V_2 quadrant(B).

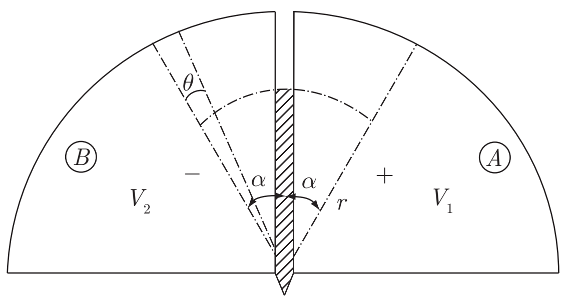


Fig. 2.33 Torque on the vane.

Let V_3 be the potential of the moving vane
V_2 and V_1 are the corresponding potentials of the quadrants
2α is the angle subtended by the sector at a radius r
w be energy stored in the electric field
θ be the small deflection in needle corresponding to w.

$$\text{Hence, torque } T = \frac{dw}{d\theta} \tag{2.44}$$

$$W = \frac{1}{2}[c_1(v_3 - v_1)^2 + c_2(v_3 - v_2)^2] \tag{2.45}$$

$$\text{where } c_1 = \frac{KA}{d} = \frac{2k\frac{1}{2}r^2(\alpha - \theta)}{d} \text{ taking both sides of moving vane.} \tag{2.46}$$

$$c_2 = \frac{2k\frac{1}{2}r^2(\alpha + \theta)}{d}$$

2.9.4.4 Attraction type

From the parallel plate capacitor shown in Fig. 2.34 the capacitance 'c' is given by

$$c = \frac{Ak}{d} \tag{2.51}$$

$$\text{Stored energy} = \frac{1}{2}cv^2 = \frac{Ak}{2d}v^2 = \frac{Akv^2d}{2d^2} \tag{2.52}$$

where A is the area of the plates.

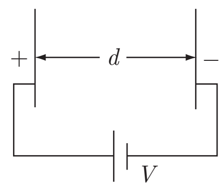


Fig. 2.34 Parallel plate capacitor.

$$\text{Stored energy/unit volume} = \frac{kv^2}{2d^2}$$

Let F be the force acting between the plates. Work done in moving plates through 'dx'

$$F \times dx = \text{change in energy stored.}$$

$$F dx = \frac{Akv^2dx}{2d^2}$$

$$F = \frac{Ak}{2d^2}v^2 = \frac{Ak}{2}^{2}\overline{E^2} \tag{2.53}$$

where $\quad E = \dfrac{v}{d}$

$$\text{Force between parallel condenser} = \frac{Ak}{2d^2}v^2$$

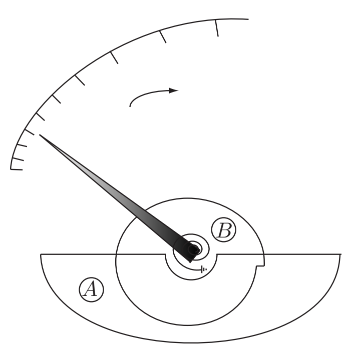


Fig. 2.35 Attraction type electrostatic voltmeter.

Electrostatic voltmeter is shown in Fig. 2.35 which is based on the principle of attraction, A is a fixed plate and B is a movable plate. Voltage is applied between them. Deflection is proportional to v^2.

Second type:
Modified and commercial version of the above instrument is shown in Fig. 2.36. Let c be the capacitance between the fixed plate and moving vane. Capacitance varies between two plates with deflection.

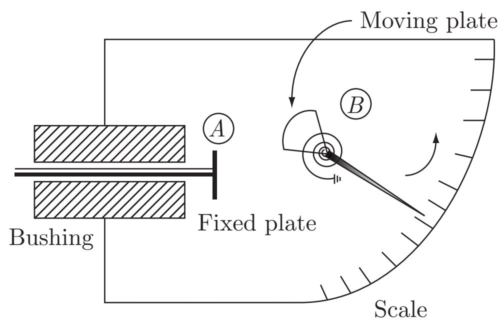


Fig. 2.36 Modified electrometer.

$$\text{W} = \text{energy stored} = \frac{1}{2}cv^2$$

$$T_D = \text{deflecting torque} = \frac{dw}{d\theta} = \frac{1}{2}v^2\frac{dc}{d\theta} \tag{2.54}$$

$$T_c = k\theta$$

$$\frac{1}{2}v^2\frac{dc}{d\theta} = k\theta$$

$$v^2 \propto \theta \tag{2.55}$$

It is a square law scale. But $dc/d\theta$ can be made proportional to $1/v$ by modifying the shape of the plates; in which case we can get uniform scale. This type of instrument can be used upto 50 kV.

2.9.5 Kelvin Absolute Electrometer

This was one of the earliest instruments employing attracted disc principle. The moving disc is carried by a spring and is suspended from a micrometer head. Surrounding the moving disc a guard ring is used to make the field between moving and fixed plates uniform. Guard ring is electrically connected to the moving disc. Effective area of the moving disc = actual area of the disc + half area of the air gap. A fine cross hair is carried by the moving disc which is used in conjunction with lenses and two finely pointed rods for zero setting of the disc more accurately. When potential is applied between the two plates, moving disc is attracted downwards. It is brought back to its zero position by means of micrometer head. The spring and micrometer head are calibrated to give the voltage applied directly.

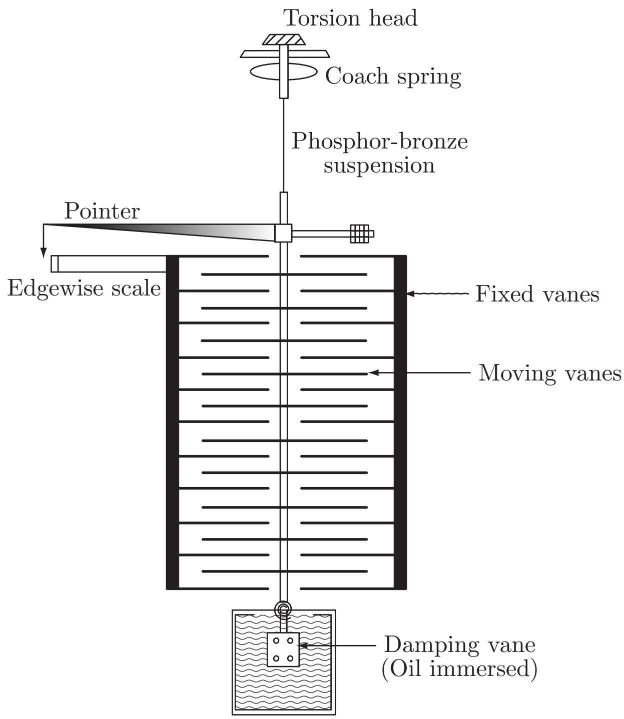


Fig. 2.38 Kelvin multicellular voltmeter.

2.9.7.1 Resistance potential divider

(a) If the capacitance of the voltmeter is neglected $V/v = R/r$ = multiplying power of the divider.
(b) The resistance R is highly non-inductive and usually oil immersed.
(c) If the capacitance of the voltmeter is not neglected. Impedance of the parallel combination r and $c = Z = \dfrac{r}{1+j\omega cr}$. Total impedance Z is

$$(R-r)+\frac{r}{1+j\omega cr}=\frac{R+j\omega cr(R-r)}{1+j\omega cr}$$

$$\frac{V}{v}=\frac{Z_r}{Z}=\frac{R+j\omega cr(R-r)}{r}=\frac{R}{r}+j\omega c(R-r)$$

$$\frac{V}{v}=\sqrt{\left(\frac{R}{r}\right)^2+[\omega c(R-r)]^2}$$

$$\frac{V}{v}=\frac{R}{r}\sqrt{1+\left[\frac{\omega cr}{R}\right]^2(R-r)^2}$$

$$\frac{V}{v}=\frac{R}{r}\text{ if }[\omega cr]^2\ll 1 \tag{2.56}$$

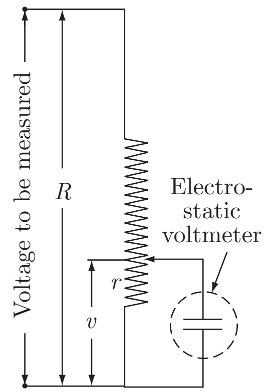


Fig. 2.39 Extension of range of electrostatic voltmeter by potential divider.

Disadvantages:

1. It is useful only for few thousand volts.
2. At high voltages power loss in the resistor will be more.
3. It is difficult to design the resistor for high voltage with no inductance.
4. Extra apparatus for oil immersion is necessary.

For above reasons capacitor multipliers are used for higher voltages.

2.9.7.2 Capacitor multipliers

$$N = \frac{V}{v} = \frac{c_v(c_v + c)}{c \times c_v} = 1 + \frac{c_v}{c} \tag{2.57}$$

Multiplying power $= 1 + \frac{c_v}{c}$ $(c_v >> c)$

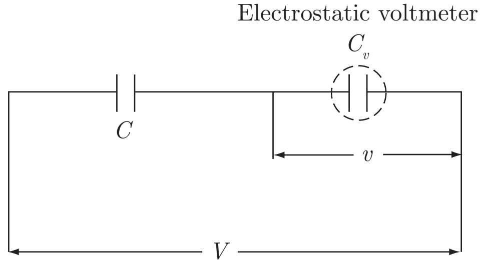


Fig. 2.40 Extension of range of instrument using capacitor multiplier.

The value of c_v will be changing with the deflection of the voltmeter. If it reads correctly on full range, it cannot read correctly on some other range because of change of c_v to limit errors due to change in multiplying power of the capacitor the capacitance c connected external to the voltmeter must be small compared to c_v.

I_1^*, I_2^*, I_0^* corresponding conjugate phasor, then

$$P + jQ = 3E_1I_1^* + 3E_2I_2^* + 3E_0I_0^* \tag{2.59}$$
$$= 3[(E_1I_1\cos\theta_1 + E_2I_2\cos\theta_2 + 3E_0I_0\cos\theta_0) + j[(E_1I_1\sin\theta_1 + E_2I_2\sin\theta_2 + 3E_0I_0\sin\theta_0)]$$

$$\text{Vector power factor} = \frac{P}{\sqrt{P^2+Q^2}} = \frac{\text{Actual power}}{\text{Vector power}} \tag{2.60}$$

Power factor meters are classified under two types

1. Dynamometer type
2. Moving iron type.

2.10.2 Principle of Power Factor Meter

2.10.2.1 Force between two magnets

Before actually studying the principle of dynamometer type it is better to understand the behaviour of two magnets placed in the horizontal plane as shown in the Fig. 2.42. Let us assume that the magnet 1 produces uniform magnetic field intensity (H). Let the pole strength of a small magnet (2) is m. Then the force acting on the two poles is $F = mH$, when the axis of the magnet 1 is at an angle θ with the axis of the main magnet. The force F can be resolved along and perpendicular to the axis of the second magnet. The components $F\cos\theta$ gets cancelled and other component $F\sin\theta$ constitute a couple.

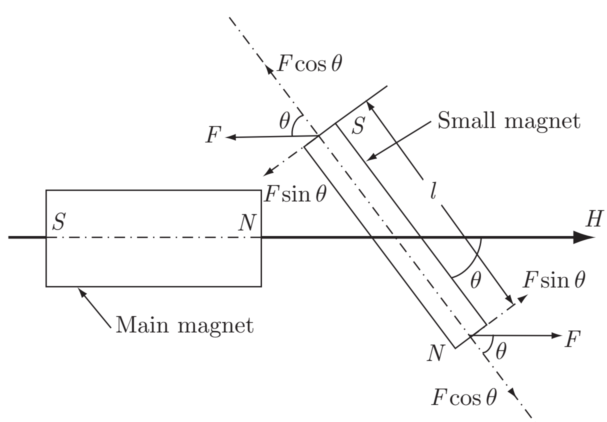


Fig. 2.42 Force between two magnets.

Hence, couple acting on the small magnet $= F\sin\theta \times l = mH\sin\theta \times l$. It can be seen that couple acting on the magnet is proportional to $\sin\theta$ and it is free to rotate, it will rotate such that the torque acting on the small magnet is zero. This is true only when the axis of the two magnets coincide each other (i.e. $\theta = 0$).

2.10.2.2 Force between two coils carrying currents

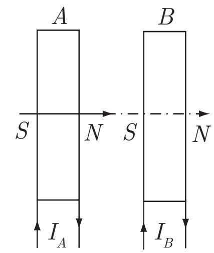


Fig. 2.43 Coils carrying currents.

Now let us consider two coils A, B carrying currents I_A and I_B. The two coils are placed in the same horizontal plane such that they are coaxial as shown in Fig. 2.43. If the two coils are energised represent two magnets because the two coils establish their own magnetic field along the axis of the coil. The force between the two coils is proportional to the product of the two current. If ϕ be the phase angle difference between I_A and I_B, then the instantaneous force on the coils $= kI_A \times I_B$, where $i_A = I_{A\,\max} \sin\alpha$ and $i_B = I_{B\,\max} \sin(\alpha - \phi)$

$$F(t) = KI_{A\,\max} \sin\alpha \times I_{B\,\max} \sin(\alpha - \phi)$$

$$\text{The average force over a cycle} = \frac{KI_{A\,\max} I_{B\,\max}}{2\pi} \int_0^{2\pi} \sin\alpha \times \sin(\alpha - \phi) d\alpha$$

$$= \frac{KI_{A\,\max} I_{B\,\max}}{2\pi} \times \frac{1}{2} \times [\cos\phi \times 2\pi]$$

$$F_{av} = KI_{A\text{rms}} I_{B\text{rms}} \cos\phi \tag{2.61}$$

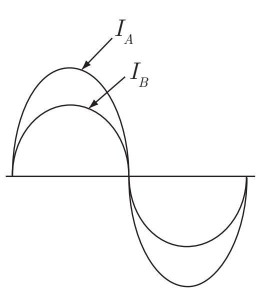


Fig. 2.44(a) Currents are in phase (force is non-zero).

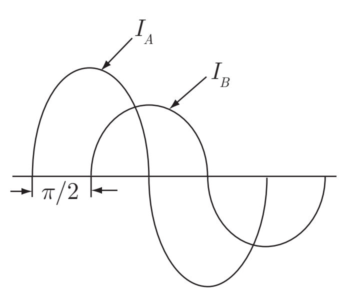


Fig. 2.44(b) Currents are out of phase by 90° (force is zero).

It can be seen from the expression that the force between the two coils is maximum when the currents are in phase and the force is zero when the two currents are out of phase by 90°. If the coil B is free to rotate and the axis is making angle θ with the axis of the coil A. Then the torque acting on the coil will be proportional to $I_A I_B \sin\theta \cos\phi$. The torque acting on the coil B is zero when the axis of the coil B coincides with the axis of the coil A.

2.10.3 Single Phase Dynamometer Type Power Factor Meter

In a dynamometer 1-ϕ type two coils A, A carry the current in the circuit under test. The two coils are arranged in such a way the magnetic field established by the two coils are aiding each other as shown in Fig. 2.45. In between these two coils a third coil composed of two coils L and R is pivoted and the two coils L and R are rigidly fixed at 90° apart. The two coils are energised by the supply voltage. In series with the two coils L and R an inductance and a resistance are connected respectively.

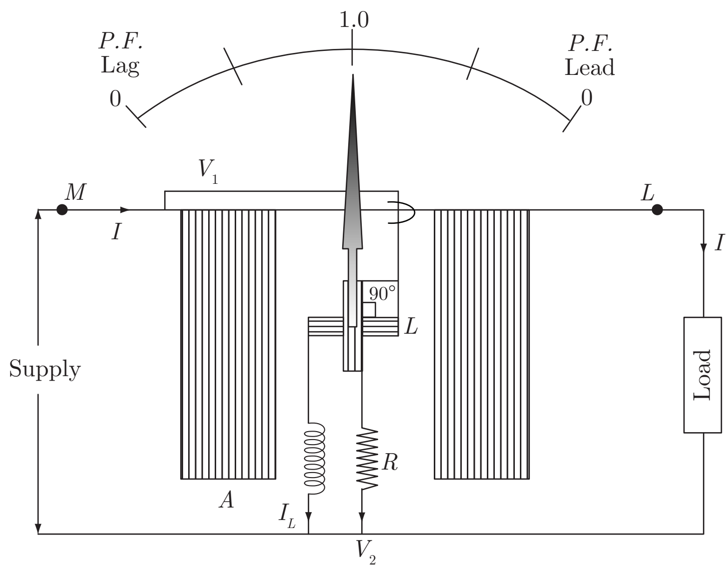


Fig. 2.45 Single phase dynamometer type *P.F.* meter.

The values of L and R are adjusted in such a way that the magnitude of the current I_L and I_R flowing through the coils is equal in value. In this meter no controlling torque is necessary because the coil will occupy such a position depending upon *P.F.* that the average torque on the coil is zero.

Case-1:

Phasor diagram is shown in Fig. 2.46 when the *P.F.* of the load is U.P.F.

I : Load current

V : Supply voltage

I_L : Current through coil L

I_R : Current through R

$$|I_R| = |I_L|$$

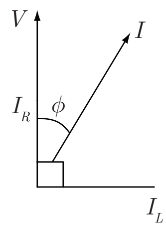


Fig. 2.48(a) Phasor diagram when *P.F.* is $\cos\phi$.

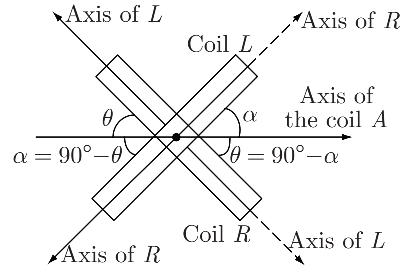


Fig. 2.48(b) Position of the coils when *P.F.* is $\cos\phi$.

Torque acting on the coil R due to A

$$= K \times I \times I_R \cos\phi \sin\theta \qquad (\theta = \alpha)$$
$$= K \times I \times I_R \cos\phi \sin\alpha \tag{2.64}$$

Torque acting on the coil L due to A

$$= -K\ I\ I_L \cos(90^\circ - \alpha)\ \sin(90^\circ - \alpha)$$
$$= -K\ I\ I_L \cos\alpha\ \sin\phi$$
$$= -K\ I\ I_L \sin\phi\ \sin(90^\circ - \alpha) \tag{2.64a}$$

The negative sign in the above expression is due to the negative angle $(90^\circ - \alpha)$. Under these conditions total torque on the system is zero.

Adding equations 2.64 and 2.64a.

$$K\ I\ I_R \cos\phi\ \sin\alpha - K\ I\ I_L \sin\phi\ \cos\alpha = 0$$
$$K\ I\ I_R \cos\phi\ \sin\alpha = K\ I\ I_L \sin\phi\ \cos\alpha$$
$$\tan\phi = \tan\alpha \tag{2.65}$$

This is true when $\phi = \theta$

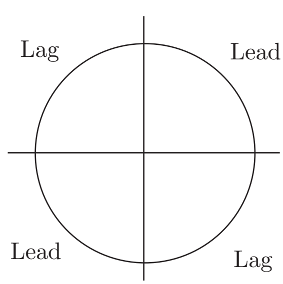


Fig. 2.49 Scale of four quadrant *P.F.* meter.

From this, we can conclude that the angle through which the system moves is equal to the *P.F.* angle and the instrument can be directly calibrated in terms of *P.F.* of the load. The voltage range of the *P.F.* meter can be increased by changing the values of L and R suitable. In actual practice the phase angle between I_L and I_R will not be 90°. In case say the phase angle is 87°, the two coils L and R must be set so that axis of the two are at 87° (mechanical). In certain meters 360° are used for *P.F.* calibration as shown in Fig. 2.49. In this case, we get double range for reading *P.F.*'s. When the current or voltage coils is reversed, the meters still can read on the other two quadrants and thus avoiding the risk of reconnecting the system when the meter reading is negative.

2.10.3.1 Disadvantages

1. This is sensitive to frequency variations and results error in the reading with change of frequency because the phase angle between I_L and I_R will be changed.
2. Ligaments are used as leads in directing the current through the moving coils. The use of ligaments prevents the scale to cover 360° and also exert a little constraint on the moving system.
3. The scale covers usually less than two quadrants and represents a limited range of *P.F.* for lead and log for only one direction of current flow (connection to the current coil to be reverse connected for reading on the dial if the current is reversed in direction).

2.10.4 Three-phase *P.F.* Meters: Dynamometer Type for Balanced Loads

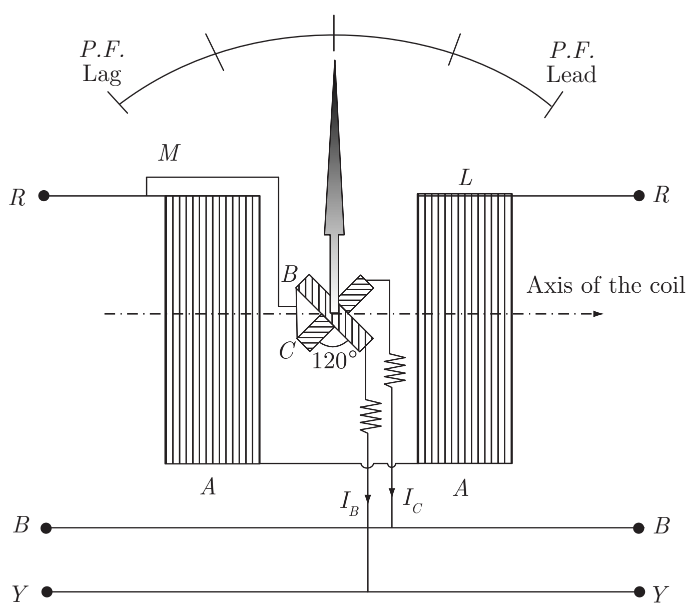


Fig. 2.50 Dynamometer type three-phase *P.F.* meter.

Three-phase *P.F.* meter is shown in Fig. 2.50. In this instrument, the two moving coils are fixed with their axis at 120° apart and are connected across two different phases of the supply circuit, the fixed coils A. A are connected in series in the third phase R and carry line current. There is

This portion is already shown in the Fig. 2.51(b) corresponding to the vertical position of the pointer, i.e., the angle BOC is bisected by the axis of the coil A.

Case (b): Phasor diagram is shown in Fig. 2.52(a), when the $P.F.$ is 90° lag and the position of the coil in Fig. 2.52(b).

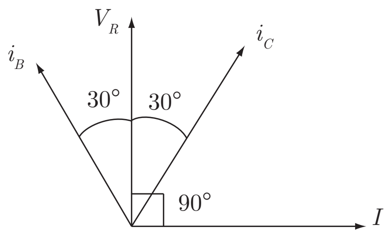


Fig. 2.52(a) Phasor diagram.

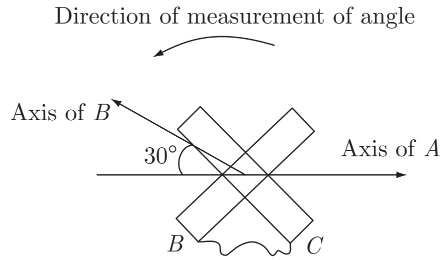


Fig. 2.52(b) Position of the moving coil.

Torque on the coil B due to A

$$= K\ I\ I_B \sin\theta\ \cos 120 \tag{2.68}$$

Torque on the coil C due to A

$$= K\ I\ I_C \sin -(120\theta)\ \ \cos(60) \tag{2.69}$$

Total torque on the system

$$-K\ I\ I_B \cos 60\ \sin\theta - K\ I\ I_C \cos 60 \sin(120\theta) = 0$$

$$\sin\theta + \sin(120\theta) = 0 \tag{2.70}$$

This equation is satisfied for values of

$$\theta = 150°, -30°$$

$\theta = 150°$ corresponds to $P.F.$ 90 lag
$\theta = -30°$ corresponds to $P.F.$ 90 lead.

The position of the coils for $P.F.$ 90° lag is shown in the figure when the axis of the coil B makes angle of 150° with the axis of coil A.

2.10.5 Three-Phase Dynamometer $P.F.$ Meter

We know a rotating magnetic field can be produced by displacing three coils in the same plane spaced at 120° and carrying balanced $3-\phi$ currents. $P.F.$ meters can be constructed utilising this principle for balanced 3-phase loads and such instruments may have three current coils and a single voltage coil or three voltage coil and a single current coil.

But for unbalanced loads $3-\phi$ dynamometer type can be constructed by using 6 coils as shown in Fig. 2.53. In this instrument three fixed coils carry the line currents and three moving coils star

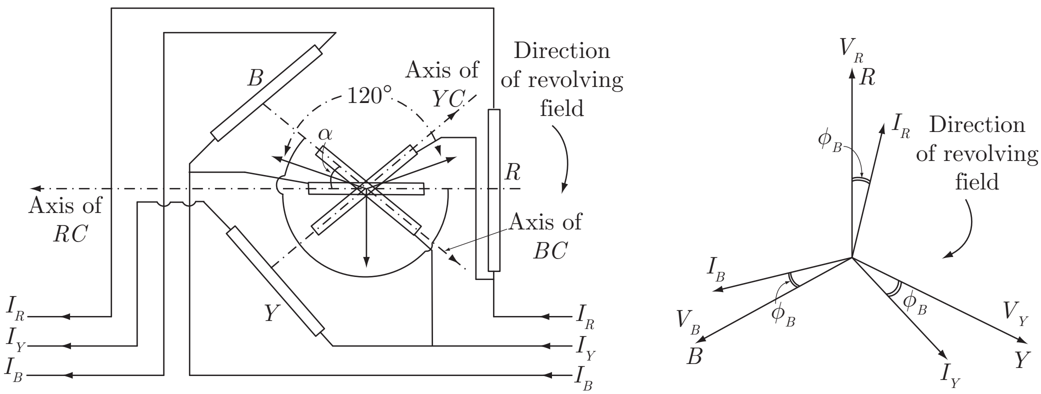


Fig. 2.53 Three-phase dynamometer type *P.F.* meter.

connected on one side through non-inductive resistances are energised by voltages across the lines. Assuming balanced voltage system (for simplicity of analysis) the currents drawn by the voltage coils (moving system) will produce a rotating field.

Assuming currents are not balanced, the resultant field produced by them will not be a uniformly rotating one, but can be expressed in terms of positive sequence and negative sequence components. With proper connection to current coils we can adjust such that the field produced by positive sequence currents will rotate in the same direction as the field produced by voltage coils and the field due to negative sequence of currents will obviously rotate in the opposite sense to the field produced by the voltage coils. The field due to negative sequence cannot produce torque with the interaction of the field produced by voltage coils. But the field due to positive sequence currents will produce torque on the moving system with the interaction of the field due to voltage coils. It can be proved that the moving system will move in the direction of the positive sequence field due to voltage coils. It can be proved that the moving system will move in the direction of the positive sequence field and takes a mean position given by ϕ_0, where

$$\tan\phi_0 = \frac{\sum VI\sin\phi}{\sum VI\cos\phi} \tag{2.71}$$

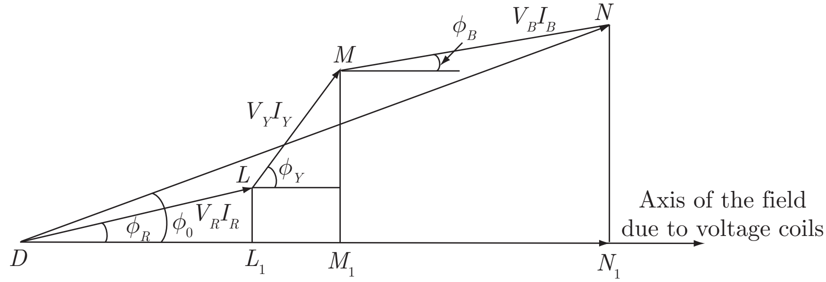


Fig. 2.54 Phasor diagram.

Rearranging the terms

$$
\begin{aligned}
&= \frac{3}{2}[I_R \sin(\phi_R + \alpha) + I_B \sin(\phi_B + \alpha) + I_y \sin(\phi_y + \alpha)] \\
&\quad - \frac{I_R}{2}[\sin(\phi_R - \alpha) + \sin(240 + \phi_R - \alpha) + \sin(480 + \phi_R - \alpha)] \\
&\quad - \frac{I_y}{2}[\sin(\phi_y - \alpha) + \sin(240 + \phi_y - \alpha) + \sin(480 + \phi_y - \alpha)] \\
&\quad - \frac{I_B}{2}[\sin(\phi_B - \alpha) + \sin(240 + \phi_B - \alpha) + \sin(480 + \phi_B - \alpha)] = 0 \\
&\equiv I_R \sin(\phi_R + \alpha) + I_B \sin(\phi_R + \alpha) + I_y \sin(\phi_y + \alpha) = 0
\end{aligned}
$$

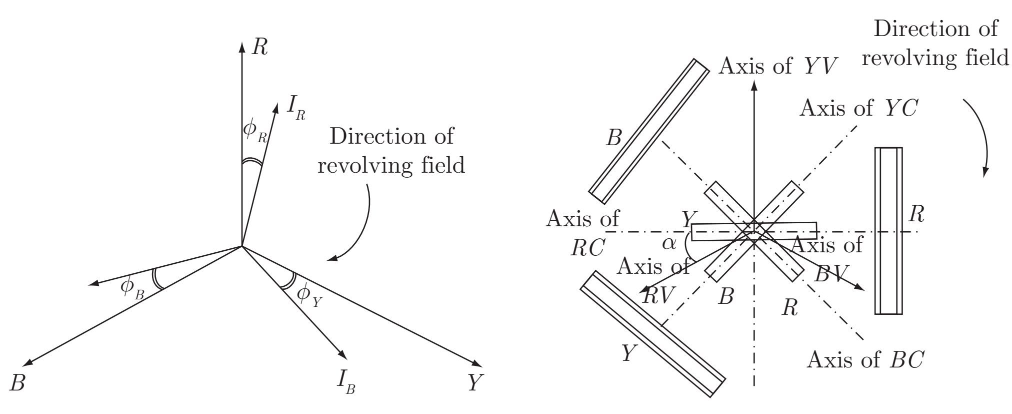
$$
\cos\alpha[I_R \sin\phi_R + I_B \sin\phi_B + I_y \sin\phi_y] + \sin\alpha[I_R \cos\phi_R + I_B \cos\phi_B + I_y \cos\phi_y] = 0
$$

$$
-\tan\alpha = \frac{\sum I_R \sin\phi_R}{\sum I_R \cos\phi_R} = \frac{\text{sum of reactive volt amperes}}{\text{sum of active volt amperes}} \tag{2.75}
$$

$$
-\tan\alpha = \tan\phi_0
$$

where ϕ_0 is the vector power factor. The significance of negative sign is that the making system should deflect in the opposite direction to that it is assumed in the Fig. 2.53.

Case 2: When the currents are unbalanced



(a) Phasor diagram (b) Position of the moving coil

Fig. 2.55 Phasor diagram and position of moving coils when the currents are unbalanced.

Assuming the moving system has turned anticlockwise by an angle α and taking measurement of angle as positive in the anticlockwise direction, the torque on the voltage coil R is

$$
V[I_R \cos\phi_R \sin(-\alpha) + I_y \cos(120 + \phi_y) \sin -(120 + \alpha) + I_B \cos(240 + \phi_B) \sin -(240 + \alpha) \tag{2.76}
$$

Similarly, torque on coil Y is

$$V[I_y \cos\phi_y \sin(-\alpha) + I_B \cos(120+\phi_B)\sin -(120+\alpha) + I_R \cos(240+\phi_R)\sin -(240+\alpha)] \quad (2.77)$$

Similarly, torque on the coil B is

$$V[I_B \cos\phi_B \sin(-\alpha) + I_R \cos(120+\phi_R)\sin -(120+\alpha) + I_y \cos(240+\phi_y)\sin -(240+\alpha)] \quad (2.78)$$

If the system is stable total torque on the moving system must be zero. The total torque expression is

$$\begin{aligned} &I_R \cos\phi_R \sin\alpha + I_y \cos(120+\phi_y)\sin(120+\alpha) + I_B \cos(240+\phi_B)\sin(240+\alpha)+\\ &I_y \cos\phi_y \sin\alpha + I_B \cos(240+\phi_B)\sin(120+\alpha) + I_R \cos(240+\phi_R)\sin(240+\alpha)+\\ &I_B \cos\phi_B \sin\alpha + I_R \cos(120+\phi_R)\sin(120+\alpha) + I_y \cos(240+\phi_y)\sin(240+\alpha) \equiv 0 \end{aligned} \quad (2.79)$$

Rewriting the terms as

$$\begin{aligned} &\frac{I_R}{2}[\sin(\alpha-\phi_R)+\sin(\phi_R+\alpha)] + \frac{I_y}{2}[\sin(\alpha-\phi_y)+\sin(240+\phi_y+\alpha)]+\\ &\frac{I_B}{2}[\sin(\alpha-\phi_B)+\sin(\alpha+\phi_B)] + \frac{I_B}{2}[\sin(\alpha-\phi_B)+\sin(480+\phi_B+\alpha)]+\\ &\frac{I_R}{2}[\sin(\alpha-\phi_R)+\sin(240+\phi_R+\alpha)] + \frac{I_v}{2}[\sin(\alpha-\phi_y)+\sin(480+\phi_y+\alpha)]+\\ &\frac{I_y}{2}[\sin(\alpha-\phi_y)+\sin(\alpha+\phi_y)] + \frac{I_B}{2}[\sin(\alpha-\phi_B)+\sin(240+\alpha+\phi_B)]+\\ &\frac{I_R}{2}[\sin(\alpha-\phi_R)+\sin(480+\phi_R+\alpha)] \equiv 0 \end{aligned} \quad (2.80)$$

Rearranging the terms

$$\begin{aligned} &\frac{1}{2}[I_R\sin(\alpha-\phi_R)+I_B\sin(\alpha-\phi_B)+I_y\sin(\alpha-\phi_y)]+\\ &\quad\frac{I_R}{2}[\sin(\alpha+\phi_R)+\sin(240+\phi_R+\alpha)+\sin(480+\phi_R+\alpha)]+\\ &\quad\frac{I_y}{2}[\sin(\alpha+\phi_y)+\sin(240+\phi_y+\alpha)+\sin(480+\phi_y+\alpha)]+\\ &\quad\frac{I_B}{2}[\sin(\alpha+\phi_B)+\sin(240+\phi_B+\alpha)+\sin(480+\phi_B+\alpha)]\\ &\equiv I_R\sin(\alpha-\phi_R)+I_B\sin(\alpha-\phi_B)+I_y\sin(\alpha-\phi_y)=0 \end{aligned}$$

$$\cos\alpha[I_R\sin\phi_R+I_B\sin\phi_B+I_y\sin\phi_y] = \sin\alpha[I_R\cos\phi_R+I_B\cos\phi_B+I_y\cos\phi_y]=0$$

$$\tan\alpha = \frac{\sum I_R\sin\phi_R}{\sum I_R\cos\phi_R} = \frac{KVAR}{Kw} = \tan\phi_0 \quad (2.81)$$

When the axis of the voltage coil coincides with the axis of corresponding current coil, then for lagging power factor the resultant phasor representing rotating field due to voltage coils lead coils A, B and C. The spindle of the system carries a pair of moving vanes V_1 and V_2 and are magnetised by the stationary coil D. When the three coils are energised a rotating field will be setup which deflects the moving vane. The deflection of the moving system is approximately equal to the angle of phase displacement between current and voltage in the $3-\phi$ circuit.

Theoretically the moving system reaches equilibrium when the axis of the rotating field coincides with the axis of the moving vanes at the instant of maximum magnetisation (when the current in the stationary coil attains maximum value). In practice, there is a small additional torque set up by the voltage system alone due to hysteretic drag which makes the rotor try to follow the rotating field. The error introduced from this cause is greatest at low current values when the torque due to hysteretic may represent a large fraction of the whole. This effect can be reduced considerably by constructing the moving vanes from nickel-iron low loss alloy and by suitable shaping of vanes. This gives satisfactory performance when the load current is above 20% of the normal rating.

Mathematical analysis:

Let the supply voltage be $V = V_m \sin \alpha$.

$$\text{Rotating field will be proportional to } V_m \sin(\alpha - \beta) \tag{2.82}$$

where β is the phase angle between the line voltage and rotating field.

The field seen by the moving vane is

$$B_1 = K_1 V_m \sin(\alpha - \beta - \theta) \tag{2.83}$$

where θ is the angle between the moving element and the reference position. The magnitude of the field of the moving element at any instant is given by

$$B_2 = K_2 I_m \sin(\alpha + \phi) = K V_m \sin(\alpha + \phi) \tag{2.84}$$

T_i = Instantaneous torque on the vane is

$$T_i = K E_m I_m \sin(\alpha - \beta - \phi) \sin(\alpha + \phi) \tag{2.85}$$

$$T_{av} = \frac{K E_m I_m}{2\pi} \int_0^{2\pi} \sin(\alpha - \beta - \phi) \sin(\alpha + \phi)\ d\alpha \tag{2.86}$$

$$= \frac{K E_m I_m}{2\pi} \times \frac{1}{2} \left[\int_0^{2\pi} \cos(\beta + \theta + \phi)\ d\alpha - \int_0^{2\pi} \cos(2\alpha - \beta - \theta + \phi)\ d\alpha \right]$$

$$T_{av} = \frac{K E_m I_m}{2\pi} \times \frac{1}{2} \cos(\beta + \theta + \phi) \times 2\pi = 0 \tag{2.87}$$

$$i.e.,\ \beta + \theta + \phi = 90^0$$

$$\therefore \theta = \left(90^0 - \beta\right) - \phi$$

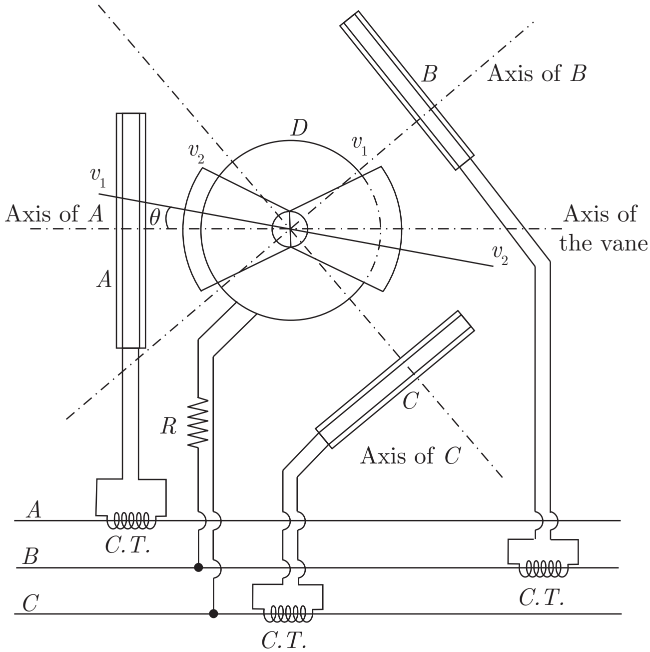


Fig. 2.57(a) Arrangement of connection to the coils.

2.10.6.2 Mathematical proof

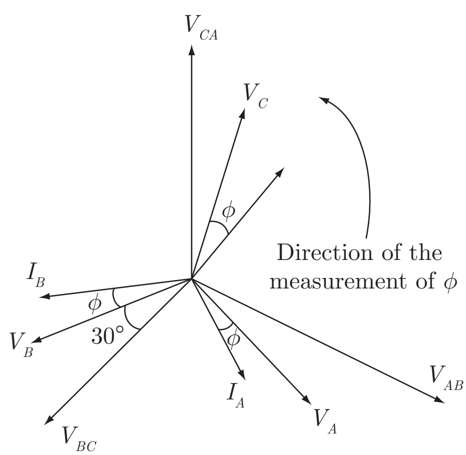


Fig. 2.57(b) Phasor diagram.

The flux produced by 'D' can be reduced into the components one along and other perpendicular to axis of the spindle. The former one cannot produce torque and the latter one is adjusted to coincide with the axis of the moving vane.

Advantages and disadvantages of moving iron type instrument:

1. It has large working force.
2. Absence of ligaments (current lead is spirals) eliminate the influence of small control torque
3. A scale extending over 3600 may be used and is advantageous in applications where energy may flow in either direction.
4. But it is less accurate than dynamometer type.

2.10.6.3 Nalder and Lipman *P.F.* meter (for balanced loads):

This instrument carries three moving vanes which are mounted on a spindle and are separated by non magnetic pieces as shown in the Fig. 2.58. The three vanes are displaced by 120° apart. Each iron vane is magnetised by the coil it carries and is connected to supply in series with a suitable resistors. The current coil A is split into two parts which establish magnetic field in the same direction. The entire moving mechanism with damping vanes is symmetrically placed between the two parts of the current oil. No controlling torque is provided because the system will be stable and hence the total torque is zero. The current coil is connected in series with one of the lines.

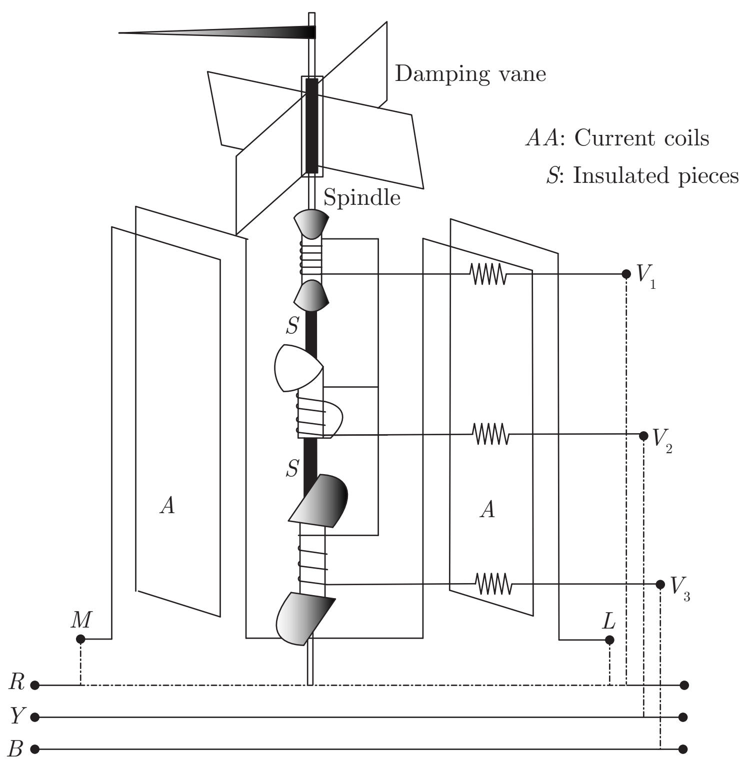


Fig. 2.58 Nalder and Lipman *P.F.* meter.

This instrument can be used for balanced loads and for unbalanced loads it can be modified. When the voltage coils are energised there can be no rotating field because the coils are not in the same plane. The moving system is equivalent to three coils on the spindle separated by 120° in the same plane but without producing rotating field. When the instrument is used for measurement of *P.F*'s the moving system turns to such a position that the mean torque on one of the iron pieces is neutralised by other two torques so that the resultant torque is zero. In the study position, the deflection of the iron piece which is magnetised by the same phase as the current coil is equal to the phase angle between the currents and voltages of the $3-\phi$ circuit, provided the effect of iron loss and inductance of the pressure coils are neglected.

The meter is free from the tendency of the moving system to rotate continuously following the drag of rotating field as in case of Westing house type.

The meter is not affected appreciably due to variations of frequency voltage and waveform. For unbalanced loads three current coils are arranged in tiers at the levels of the moving vanes.

2.11 SYNCHROSCOPES

Synchroscopes are the instruments used for synchronising two alternators. Before two alternators are synchronised, they should satisfy the following conditions.

1. Voltages of the two alternators must be equal in magnitude.
2. Polarity of the voltages must be same.
3. Frequency of the two voltages must be same.
4. Phase difference between the two voltages must be zero.

If the conditions are satisfied, synchronising switch can be closed without imposing transient conditions. The first condition is verified by a voltmeter connected to the switchboard. The function of the synchroscopes is to indicate the phase difference and therefore the principle of any *P.F.* meter may be applied to the design of a synchroscope.

2.11.1 Type of Synchroscopes

1. Dynamometer type
2. Moving iron type.

2.11.2 Linwin Type Synchroscope (Dynamometer Type)

The diagram of a dynamometer type synchroscope is shown in Fig. 2.68. It consists of a fixed coil C energised by the incoming source E_2 through a high non-inductive resistor. The moving coil consists of two fixed coils A and B rigidly fixed at right angle. The moving system carries the spindle with three slip rings. A resistance is connected in series with the coil B. Both the coils are energised by the source E_2. E_1 and E_2 are the supply voltages from the two alternators which are to be synchronised. The value of R and L are choose such that I_R and I_L are equal in magnitude but differ in phase by 90°. For correct synchronisation $E_1 = E_2$ and they must be in phase.

Let θ be the angle between the axis of the coil A with the axis of the electromagnet when E_1 and E_2 are in phase.

T_{DB} = Torque on the coil B

$$T_{DB} \propto I\ I_L \cos 90 \sin(90° - \theta) \tag{2.94}$$

Let us consider at any instant phase diagram is
T_{DB} = Torque on the coil B

$$T_{DB} = K\ I\ I_L \cos - (90° - \phi)\ \ \sin(90° - \theta) \tag{2.96}$$

T_{DA} = Torque on the coil A

$$T_{DA} = K\ I\ I_L \cos\phi\ \ \sin\theta \tag{2.97}$$

For steady readings $T_{DB} + T_{DB} = 0$

$$-\cos(90° - \phi)\sin(90° - \theta) + \cos\phi \sin\theta = 0 \tag{2.98}$$

$$\left.\begin{aligned} &-\cos\theta\ \ \sin\phi + \sin\theta\ \ \cos\phi = 0 \\ &\tan\phi = \tan\theta \\ &\phi = \theta \end{aligned}\right\} \tag{2.99}$$

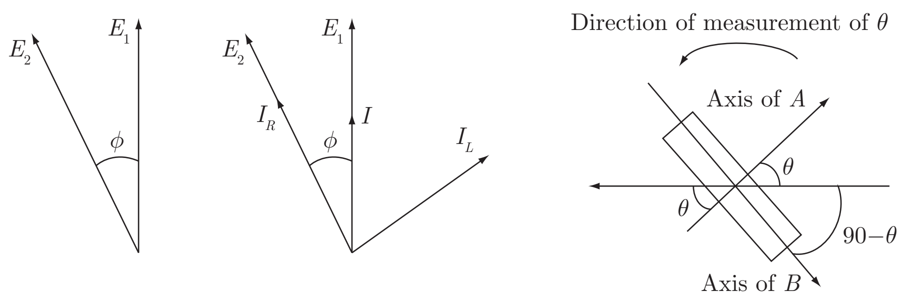


Fig. 2.61 Phasor diagram and position of moving coil.

If the incoming alternator is faster the pointer will rotate in one direction but if the incoming alternator is slower then the pointer will rotate in opposite direction. But, on the other hand, if the pointer is steady and is not vertical, it indicates the frequencies are the same but there is a phase difference. When the pointer is passing through the vertical position, synchronising switch can be closed. This instrument cannot take into account the magnitude of E_1 and E_2, therefore, fails to indicate whether they are equal or not.

For $3 - \phi$ alternator we can use the same meter but corresponding voltages from a single line with proper sequence must be taken.

2.11.3 Westing House Type

The principle is same as Lincoln type but moving coils are made to be stationary and fixed coils are free to rotate.

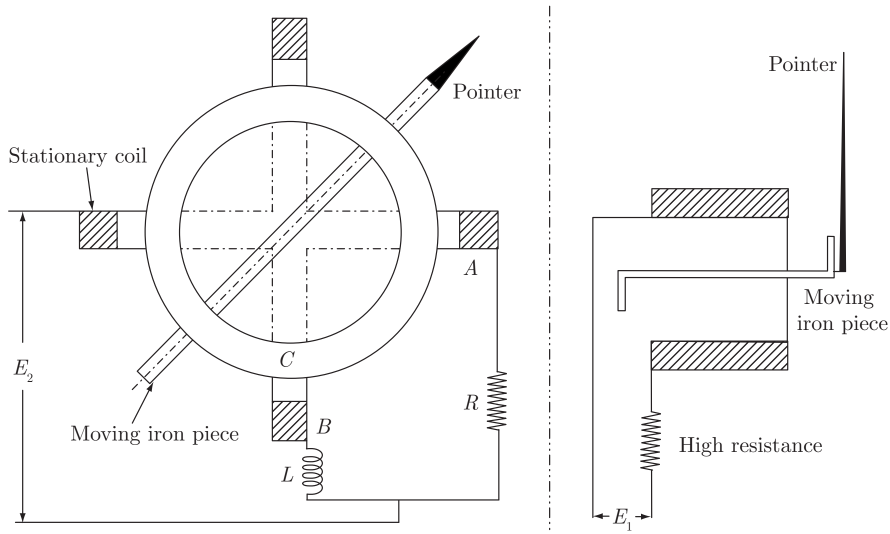


Fig. 2.62 Westing house synchroscope.

2.11.4 Weston Type

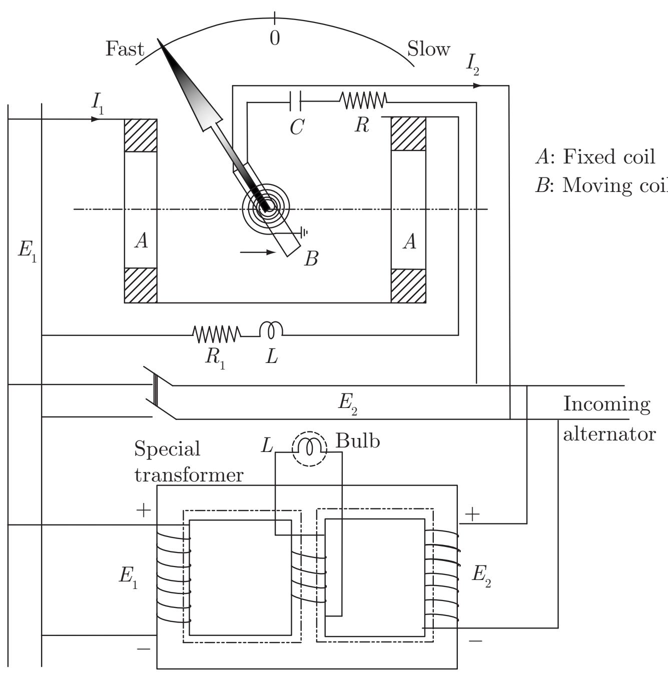


Fig. 2.63 Weston synchroscope.

$$\text{The force on the coil } B \propto I_1 I_2 \cos(90^\circ + \phi) - K I_1 I_2 \sin\phi \qquad (2.101)$$

Assuming E_2 is slower than E_1; *i.e.*, $f_2 < f_1$
Angle between I_1 and $I_2 = 90^\circ - \alpha - \phi + \alpha = 90 - \phi$

$$\text{The force on the coil } B \propto I_1 I_2 \cos(90^\circ - \phi) = K I_1 I_2 \sin\phi \quad (2.102)$$

It can be seen that the force on the coil will change its sign depending upon E_2 is faster or slower than E_1. Therefore, the dial can be marked faster and slower on the two sides. If E_1 and E_2 are out of phase still the pointer will be in the vertical position but it is the worst instant for synchronisation. But the bulb will be dark corresponding to this instant. Therefore, the correct time for closing the switch is the pointer should be in the vertical position and simultaneously the bulb will glow with a maximum brightness.

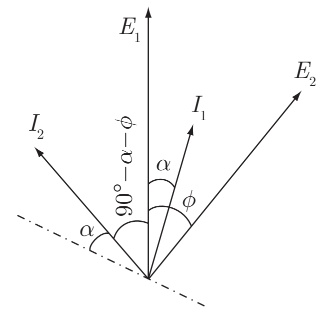


Fig. 2.64(b) Phasor diagram when $f_2 < f_1$.

The shadow of the point is thrown on an open glass dial and is therefore visible when the lamp is bright.

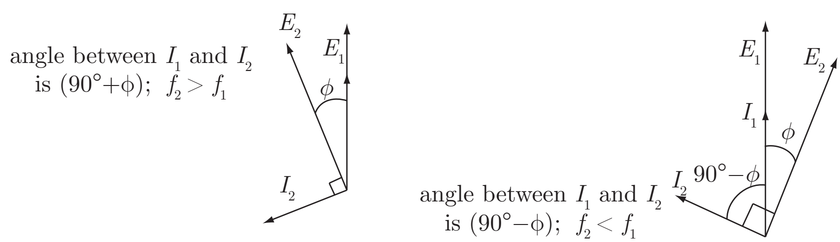


Fig. 2.64(c) Approximate phasor diagram.

2.12 MEASUREMENT OF FREQUENCY

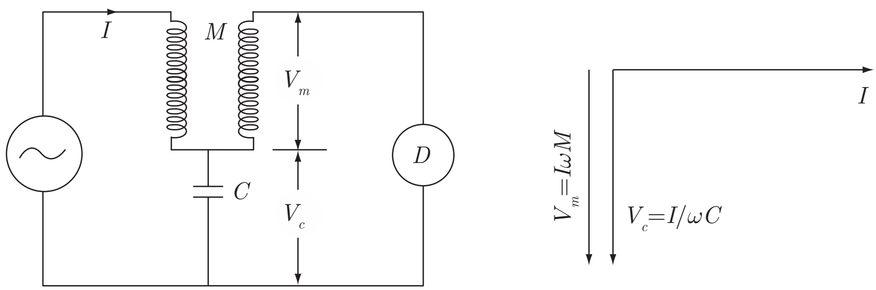
Fundamental standard for frequency is the rotation of earth. The frequency of earth rotation can be determined very accurately by astronomical methods. All the meters which are intended for frequency measurement are calibrated against the frequency of earth rotation and are checked periodically.

But the quartz frequency standard has reached a degree of precision that is far ahead of any other electrical standard. Therefore, quartz oscillator operating at 100 kHz with a low temperature coefficient of frequency with thermostatic control of temperature has been considered as primary standard. We can obtain an accuracy part in 102 over a period of several months.

Very stable oscillators used as secondary standards and are checked against the primary standards. Measurement of frequency can be done by:

1. Bridge methods
2. Frequency meters.

2.12.1 Campbell's Frequency Bridge



(a) Campbell's frequency bridge

(b) Phasor diagram

Fig. 2.65 Frequency meter.

The bridge which is commonly used has the advantage of simplicity and of a fairly large range of frequency provided the capacitor employed is loss free and the mutual inductance is free from impurity. The connections are shown in the figure and the detector (D) may be either telephones or a vibrating galvanometer according to the frequency under test. Assuming no impurities in the inductor and capacitor the current in the detector can be made zero by varying mutual inductance (M). This can be achieved by reverse connecting the secondary of the mutual inductance. Under perfect balance conditions

$$j\omega MI - \frac{-jI}{\omega c} = 0$$

$$\omega M = \frac{1}{\omega c}$$

$$\omega^2 = \frac{1}{Mc}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{Mc}} \tag{2.103}$$

Perfect balance is possible only when M and C are free from impurity and also the waveform shall be free from harmonics. In this bridge mica condenser is used. For low frequencies below 250 Hz large values of M and C are required. If M and C are not free from impurity, either the impurities must be known and taken into account, or modification to the simple circuit as shown in the figure must be made.

A second form of frequency meter also due to Campbell can be made to cover a range of frequencies down to power frequencies and avoid the necessity for low condensers. It requires a second mutual inductance which must be known but need not be variable. The emf in the detector circuit is the sum of following

1. The p.d. across 'r' due to I_1
2. The p.d. across 's' due to I_2

2.12.2 Frequency Meters

These meters indicate the supply frequency of a circuit directly and are thus more convenient for most practical purposes than bridge methods. These meters use one of the following principles:

1. Mechanical resonance
2. Electrical resonance
3. Variation of impedance of an inductive circuit with frequency.

2.12.2.1 Vibrating reed frequency meter

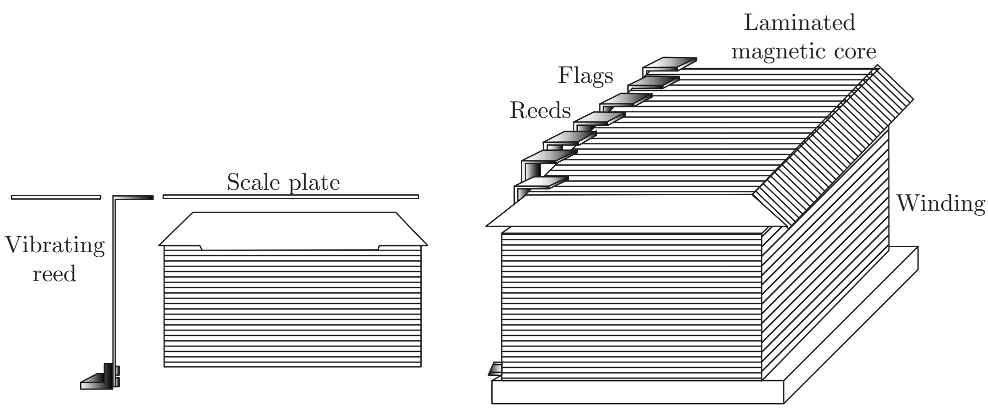


Fig. 2.67 Vibration reed frequency meter.

The principle of this meter depends upon mechanical resonance. It consists of number of thin steel strips or reeds arranged both side and close to an electromagnet. The electromagnet is laminated and its winding is connected in series with a resistance across the supply whose frequency is to be measured. The terminals of the meter can be connected across the supply as a voltmeter. The reeds, which are about 4 mm wide and 1/2 mm thick are not exactly similar to each other but are either slightly different lengths or breadths or carry slightly different loads or flags at their upper ends. These differences cause the natural frequencies of vibration of the reeds to differ and they are arranged in ascending order of natural frequency. The natural frequency of a reed can be determined from the formula

$$f = \frac{10^{-3}}{2\pi l^2}\sqrt{\frac{E_k}{\Delta_k(1+4.1k)}} \tag{2.111}$$

where l = length of the reed in metres
x = breadth of the reed in metres
E_k = modulus of elasticity of the material of the reed in Nw/m^2
k = density of the material in kg/m^3
K = ratio of mass of load or flag to the mass of the reed itself

$$E_k = 1.98 \times 1011 \text{ Nw/m}^2$$

$$\Delta k = 7.8 \times 103 \text{ kg/m}^3$$

When the frequency meter is in use the poles of the electromagnet alternate with the frequency of the supply and exerts an attractive force upon the reeds once in every half cycle. All the reeds tend to vibrate but only the one whose natural frequency is double that of the supply will vibrate appreciably due to mechanical resonance.

The flags at the top of the reeds are painted white and the frequency is reed directly from the instrument by observing the scale mark opposite to the reed which is vibrating. Most of the supply frequency lies halfway between the natural frequencies of two adjacent reeds, both of these vibrate equally. The range of this instrument is 47 to 53 Hz.

The range of such an instrument may be doubled by polarising the reeds. This is done by superimposing D.C. over A.C. whose frequency is to be measured. The reeds are then attracted to the core only once per cycle instead of twice. For example, a 50 cycle reed now has 100 attractive impulses per second when the supply frequency is 100 Hz and therefore vibrates.

Advantages:

Indications are independent of the applied voltage (magnitude) and waveform provided the voltage is not so low.

2.12.3 B.T.H. Resonance Frequency Meter

The principle of this meter depends upon electrical resonance. A magnetising coil is connected across the supply whose frequency is to be measured. It is wound on a laminated iron core. A moving coil is also wound on the laminated iron core but pivoted as shown in Fig. 2.68 but a pointer attached to it. The ends of the moving coil is connected across a capacitor C. The principle of operation can be understood from the three phasor diagrams given in Fig. 2.69.

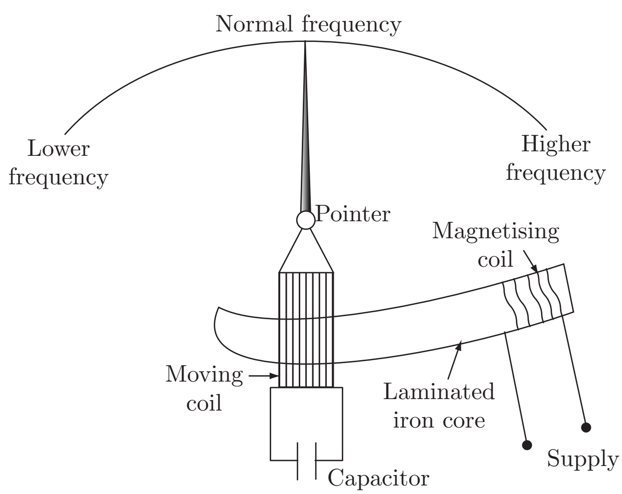


Fig. 2.68 Resonance frequency meter.

Two coils A and B are arranged so that the magnetic axes are perpendicular to each other as shown in Fig. 2.69. At the centre of the coils, a pivoted soft iron needle is placed which is long and thin. The spindle bearing the needle also carries a pointer and damping vanes but there is no controlling device.

Coil A is connected in series with an inductance L_A, across a non-inductive resistance R_A. But the coil B is connected in series with a resistance R_{B+}. The inductance L is for purpose of damping out harmonics in the waveform of the current through the instrument and so eliminating errors due to such harmonics.

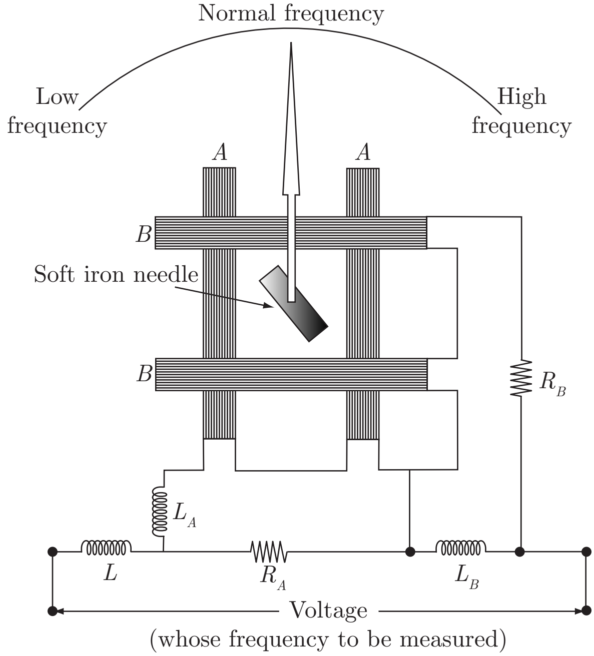


Fig. 2.70 Weston frequency meter.

The soft iron needle takes up a definite position which is dependent upon the currents through the coils A and B. If the frequency is increased the current through the coil A is reduced, since the reactance of L_A increases current in the coil B increases leading to increased voltage drop across L_B. The net result is the pivoted needle orients itself more nearly parallel to the axis of the coil B. But decrease of frequency makes the needle to orient more nearly parallel to the axis of coil A. Therefore, variations in frequency are followed by the pointer.

SOLVED EXAMPLES

Example 2.12 *An electrostatic voltmeter reading up to 1000 V is controlled by a spring with a torsion constant of* 9.807×10^{-8} *Nw.m per degree, and has a full-scale deflection of 80°. The capacitance at zero is* 10 $\mu\mu F$. *What is the capacitance when the pointer indicates 1000 V?*

Solution

Work done in moving the pointer = Change in energy stored

$$9.807 \times 10^{-8} \times 80 \times \frac{80}{180} \times \pi = \frac{1}{2}CV^2$$
$$C = 9.807 \times \frac{128}{1.8} \times \pi \times 10^{-14} = \frac{9.807 \times \pi \times 1.28}{1.8} \text{ µµF} = 2.19 \text{ µµF}$$

Example 2.13 *A moving coil voltmeter with a resistance of 10 Ω gives full-scale deflection with a potential difference of 45 mV. The coil has 100 turns, an effective depth of 3 cm and width of 2.5 cm. The controlling torque exerted by the spring is 4.9×10^{-5} Nw.m for full-scale deflection. Calculate the flux density in the air gap.*

Solution

$$\text{Torque} = 2bB_dI \times N \text{ Nw.m}$$

$b = 2.5$ cm $d = 3$ cm $N = 100$ $T = 4.9 \times 10^{-5}$ Nw.m $I = 4.5$ m.A

$$4.9 \times 10^{-5} = \frac{2.5}{100} \times \frac{3}{100} \times B \times 4.9 \times 10^{-3} \times 100$$
$$\frac{4.9}{4.5 \times 25 \times 3} = B = \frac{4.9}{4.5 \times 7.5} = 0.145 \text{ web/m}^2$$

Example 2.14 *A sinusoidal alternating voltage of amplitude 100 V is applied to a circuit containing a rectifier device which entirely prevents current flowing in one direction and offers a non-inductive resistance of 10 Ω to the flow of current in the other direction. Find the readings on (a) hot-wire (b) a moving coil ammeter in the circuit.*

Solution

Hot-wire instruments reads r.m.s. value

$$i = 10 \sin\theta$$
$$I_{r.m.s.} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} 10^2 \sin^2\theta \, d\theta} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} 100 \sin^2\theta \, d\theta + \int_\pi^{2\pi} 0}$$
$$I_{r.m.s.} = \sqrt{\frac{1}{2\pi} \times 100 \times 2 \times \frac{1}{2} \times \frac{\pi}{2}} = 5 \text{ A}$$

Hot-wire instrument reads 5 A

Impedance of the meter on A.C. on 100 Hz

$$= 3000 + j100 \times 2\pi \times 0.75 = 3000 + j471$$
$$|Z| = 1000\sqrt{9 + 0.222} = 3040\ \Omega$$

Hence, current drawn by the meter $= \dfrac{150}{3040} = 0.494$ A

0.05 A corresponds to 150 V

0.0494 A corresponds to $150 \times \dfrac{0.0494}{0.05} = 148.25$ V

Hence, percentage error $= \dfrac{1.75}{150} \times 100 = 1.170\%$

Frequency compensation can be achieved by shunting the swamping resistance 2600 A. By a capacitor C

$$\omega L = \frac{\omega cr^2}{1 + (\omega cR)^2} \qquad \omega cr^2 \ll 1$$

$$c = \frac{L}{r^2} = \frac{0.75}{(2600)^2} = \frac{0.75}{6.76} \times 10^{-6} = 0.111\ \mu\text{F}$$

Taking an error of 0.1% for change in temperature

$$\alpha + \frac{R_c}{R_{sp} + R_c} = \frac{0.001}{0.999} \qquad t = 1$$
$$\alpha = \frac{0.001}{0.999} \times \frac{3000}{400} = 0.001 \times 7.5 = 0.0075$$

Hence, temperature coefficient = 0.0075.

Example 2.17 *A soft iron voltmeter for a maximum reading of 120 V has an inductance of 0.6 H and a total resistance of 2400 Ω. It is calibrated to read correctly at 60 cHz. What series resistance would be necessary to increase the range to 600 V?*

Solution

Impedance of the meter on 60 Hz

$$Z = 2400 + j \times 6 \times 2\pi \times 60 = 2400 + j72 \times \pi$$
$$= 2400 + j226 = 1000\sqrt{(2.4)^2 + (2.26)^2}$$
$$|Z| = 100\sqrt{5.75 + 0.051} = 2410\ \Omega$$

Hence, current drawn by the meter for full-scale deflection $= \dfrac{120}{2410}$ A

When it is connected on 600 V it requires $\dfrac{120}{2410}$ A for full-scale deflection.

Hence, impedance $= \dfrac{600}{120} \times 2410 = 12{,}050\ \Omega$

Hence, resistance of the meter $= \sqrt{(12{,}050)^2 - (226)^2} = 12{,}040$

Hence, the resistance to be inserted to increase the range to 600 V $= 12{,}040 - 2400 = 9640\ \Omega$

Example 2.18 *The moving coil of a galvanometer has 60 turns a width of 2 cm and a depth of 3 cm. It hangs in a uniform radial field of 50 m web/m^2. Find the turning moment of the coil when it is carrying a current 1 mA.*

Solution

Force $= B\ l\ I \times N$ Nw.m

Torque $= 2r\ B\ l\ I \times N$ Nw.m

$L = 3$ cm

$2r = 2$ cm

$B = 50$ m web/m^2

$I = 1 \times 10^{-3}$ A

$$\begin{aligned}\text{Torque} &= \frac{2}{100} \times 50 \times 10^{-3} \times \frac{3}{100} \times 10^{-3} \times 60 \\ &= 1.8 \times 10^{-6}\ \text{Nw.m} \\ &= 1.8\ \ \mu\text{Nw.m}\end{aligned}$$

Example 2.19 *A moving coil voltmeter with a resistance of 10 Ω gives full-scale deflection with a potential difference of 45 mV. The coil has 100 turns, an effective depth of 3 cm and width of 2.5 cm. The controlling torque exerted by the spring is 4.9 × 10^{-5} Nw.m for full-scale deflection. Calculate the flux density in the air gap.*

Solution

$$\text{Torque} = 2b\ B_d\ I \times N\ \text{Nw.m}$$

$b = 2.5$ cm $\quad d = 3$ cm $\quad N = 100 \quad T = 4.9 \times 10^{-5}$ Nw.m $\quad I = 4.5$ m.A

$$4.9 \times 10^{-5} = \frac{2.5}{100} \times \frac{3}{100} \times B \times\ 4.9 \times 10^{-3} \times 100$$

$$\frac{4.9}{4.5 \times 25 \times 3} = B = \frac{4.9}{4.5 \times 7.5}$$

$$= 0.145\ \text{web/m}^2$$

Example 2.20 *A voltmeter gives a full-scale reading of 300 mV. The moving coils wound with 20 turns of copper wire each 0.274 mm diameter. The mean length per turn is 13 cm and the mean depth of the coil is 4 cm. There is a resistance of 4.24 Ω in series with the coil and the gap density is 0.15 wb/m^2. Calculate the controlling torque in Nw.m exerted by the springs when the reading is 300 mV.*

$$I_F = \frac{V}{4.82} \text{ amp}$$

$$T_{D_1} \propto \frac{V^2}{31.8 \times 4.82}$$

$\frac{V^2}{90}$ corresponds to V

$\frac{V^2}{31.8 \times 4.82}$ corresponds to $V \times \left[\frac{90}{31.4 \times 4.82}\right] = 0.624 \times \frac{30}{31.4} = 0.597\ V$

Hence, error in reading $= V - 0.597V = 0.403$

Hence, percentage error $= 40.3\%$ low.

b) When a resistance of 3 Ω is connected in series with fixed coil on A.C.

$$I_c = \frac{V}{31.4}\text{A}$$

$$|Z_F| = 6 + j \times 3.7 = 7.06\ \Omega$$

$$I_F = \frac{V}{7.06}$$

$$T_{D_1} \propto \frac{V}{31.4 \times 7.06}$$

$$= \frac{V \times 6 \times 30}{31.4 \times 7.06} = 0.812V$$

Hence, error $= V - 0.812V = 0.188V$

Hence, percentage error $= 18.8\%$ low

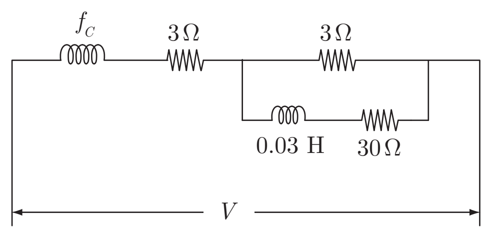


Fig. 2.71(b)

Case(b):
When the moving coil is shunted across a resistance 3 Ω
When D.C. supplied the circuit will be reduced

$$R = 3 + \frac{3v30}{33} = 3 + 2.72 = 5.72\ \Omega$$

$$I_F = \frac{V}{5.72}$$

$$I_C = \frac{V}{5.72} \times \frac{3}{33} = \frac{V}{11 \times 5.72}$$

Hence, T_D in D.C. case $\propto I_F \times I_C$

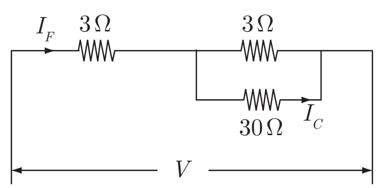


Fig. 2.71(c)

$$T_D \propto \frac{V^2}{(5.72)^2 \times 11}$$

On A.C./D.C. supply:

Impedance of the fixed coil $= 3 + j37.7$

Impedance of the moving coil $= 30 + j9.42$
Hence, impedance of the parallel combination

$$= \frac{3[30 + j9.42]}{33 + j9.42}$$

$$= \frac{[90 + j28.26](33 - j9.42)}{33^2 + 9.42^2}$$

$$= \frac{[90 + j28.26](33 - j9.42)}{11.79}$$

$$= [32.36 + j1.0]\frac{1}{11.79}$$

$$= 2.75 + j.0848$$

Hence, total impedance $= 3 + j3.77 + 2.75 + j0.08 + 8$

$$|Z| = [5.75 + j3.855]$$

$$|Z| = \sqrt{(5.75)^2 + (3.855)^2}$$

$$= \sqrt{33 + 14.8} = \sqrt{47.8} = 6.92$$

$$I_F \propto \frac{V}{6.92}$$

$$I_C = \frac{3}{33 + j9.42} I_F = \frac{3}{|3.42|}\frac{V}{6.92}$$

$T_D =$ Deflecting torque $\propto I_F I_C$

$$T_D \propto \frac{V^2}{(6.92)^2} \times \frac{3}{34.2}$$

$$\text{Hence, reading on A.C.} = V \times \frac{(5.72)^2 \times 11 \times 3}{34.2 \times (6.92)^2}$$

$$= \frac{33}{34.2}\left(\frac{5.72}{6.92}\right)^2 V$$

$$= 0.965 \times 0.69V = 0.665V$$

Hence, error in reading $= V - 0.665V = 0.335V$

Hence, percentage error $= 33.5$

Example 2.22 *The following data refer to a moving coil voltmeter resistance 10,000 Ω, dimensions of the coil 3 cm × 3 cm, number of turns on the coil 100; flux density in gap 80 m wb/m^2; Spring control, 30 × 10^{-7} Nw.m/degree. Find the deflection produced by 200 V.*

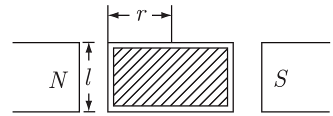


Fig. 2.72

Total torque developed = $0.106 \times 10^{-5} \times 90$ Nw.m

Assuming 9 °C corresponds to full scale deflection

Deflecting torque = $[B\ l\ I \times N \times 2r]$ Nw.m

Assuming the coil is symmetrical $B\ l\ I \times 2r = \phi$

When current is changed to $+I$ to $-I$, the flux linkages measured will be $2N\phi$. But when it is switched on change in flux linkages $N\phi$.

$$\phi N = \frac{3}{2} \times 10^{-3}$$

$$I = \frac{0.196 \times 90 \times 10^{-5} \text{ Nw.m}}{\frac{3}{2} \times 10^{-3}} = \frac{0.196 \times 9}{\frac{3}{2}} \times 10^{-1}$$

$$= 2 \times 0.3 \times 0.196 = 0.0588 \times 2 \text{ A} = 0.1175$$

$$R = \text{Resistance in the circuit} = \frac{0.075}{2 \times 0.568} = \frac{0.1275}{2}\ \Omega = 0.0637\ \Omega$$

$$\text{Temperature error} = \frac{R}{R_{sp} + R} dt = 0.001$$

Taking an error 0.1% for a change of ± 10 °C

R is the resistance of the coil.

$$t = 1$$

$$\frac{0.1}{0.637}\alpha = 0.001$$

$$\alpha = 0.00637$$

Let I be the current at normal temperature and i_1 be the current at new temperature.

$$\frac{i_1}{i} = \frac{R}{R^1}$$

$$\frac{i}{i^1} = \frac{R^1}{R}$$

$$\frac{i}{R^1} = \frac{i_1}{R} = K = \frac{i - i^1}{R^1 - R}$$

$$(i - i^1) = (R^1 - R)$$

$$\frac{i - i^1}{i} = \frac{K(R^1 - R)}{R^1 K} = \left[1 - \frac{R}{R^1}\right]$$

$\left[1 - \frac{R}{R^1}\right] = \epsilon;\quad \epsilon$ is the error

$$1 - \epsilon = \frac{R}{R^1}$$

$$\frac{R^1}{R} = \left[\frac{1}{1-\epsilon}\right]$$

$$\frac{R_{sp}(\alpha t + 1)R_C}{R} = \frac{1}{1-\epsilon}$$

R_{sp} = Swamp resistance $\quad$ R_C = Coil resistance.

$$R = R_{sp} + R_C$$

$$1 + \frac{R_C}{R}\alpha t = \left[\frac{1}{1-\epsilon}\right]$$

$$1 - \left[\frac{1}{1-\epsilon}\right] = \frac{R_C}{R_{sp} + R_C}\alpha t$$

$$\frac{-\epsilon}{1-\epsilon} = \frac{R_C}{R_{sp} + R_C}\alpha t$$

for $\pm\, 1^\circ$ error = 0.001

$$\frac{0.001}{0.999} = \frac{0.1}{0.637}\alpha$$

$$\alpha = 0.0063$$

Hence, temperature coefficient = 0.00639.

Example 2.25 *A moving coil permanent magnet, indicating instrument for use with external shunts is to comply with the following conditions: full-scale voltage drop across shunt, 7.5 mV flux density in the air gap (unshunted) 0.15 wb/m*^{2}*, resistance of the instrument between ends of shunt leads, 5 Ω, resistance of moving coil only (copper wire) 0.75 Ω, length and breath of moving coil 2.5 × 2.2 cm; torque/wt ratio 147 ×* 10^{-7} *Nw.m/*10^{-3} *kg. (147 ×* 10^{-4}*) Nw.m/kg; total weight of movement; 2.5 times copper coil. Calculate the air gap flux density to be diverted through the magnetic shunt to give the maximum (whole) number of turns, the coil section area and the full-scale torque.*

Solution

Mean length of each turn = 5 + 4.4 = 9.4 cm

l = length of the coil = $9.4 \times N$ cm.

Let a be the c.s. of the wire in mm^2

R_C = Resistance of the coil = $\dfrac{9.4N \times 1.74 \times 10^{-6}}{a}\ \Omega$

$$\begin{aligned}\text{Weight of the coil} &= \text{volume} \times \text{sp. gravity}\\ &= (l \times a \times 10^{-2}\,\text{cm}^3)8.9 \text{ gm}\\ &= 8.9 \times 9.4 \times a \times 10^{-5} \text{ kgm}\\ \text{Hence, weight of system} &= 8.9 \times 2.5 \times 9.4 \times a \times 10^{-5} \text{ kgm}\end{aligned}$$

$$\begin{aligned}\text{Total torque} &= 8.9 \times 1.47 \times 10^{-4} \times 2.5 \times 9.4(N \times a)10^{-5} \text{ Nw.m}\\ &= 8.9 \times 1.47 \times 2.5 \times 9.4(N \times a)10^{-9} \text{ Nw.m}\end{aligned}$$

$$\text{Current through the coil} = \frac{0.075}{5.75} = 0.01305 \text{ A}$$

$$\begin{aligned}\text{Torque developed (without magnetic shunt)} &= \text{flux } I\ N \text{ Nw.m}\\ &= 0.15 \times \frac{2.2 \times 2.5}{10^4} \times 0.01305 \times N\\ &= 8.9 \times 147 \times 2.5 \times 9.4(N \times a)10^{-9}\\ &= 1.5 \times 2.5 \times 2.2 \times 1.305N \times 10^{-7}\end{aligned}$$

$$\begin{aligned}a &= \frac{1.5 \times 2.2 \times 1.305}{1.47 \times 9.4 \times 8.9} = \frac{4.3}{123} = 0.035 \text{ m.m}^2\\ R_C &= \frac{9.4 \times 1.73 \times N \times 10^{-6}}{a} = 0.75\ \Omega\\ N &= \frac{0.035 \times 7.5}{9.4 \times 1.73} \times 10^6\\ &= \frac{3.5 \times 7.5}{9.4 \times 1.73} \times 10^3\end{aligned}$$

Taking $N = 1800$ as whole number

$$\begin{aligned}0.75 &= \frac{9.4 \times 1.73 \times N}{a} \times 10^{-6}\\ a &= \frac{9.4 \times 1.73 \times 1800}{0.75} \times 10^{-6} = 0.039\end{aligned}$$

c.s. of the coil corresponding to $N = 1800 = 0.039 \text{ m.m}^2$

$$\begin{aligned}\text{Weight of the coil} &= 8.9 \times 9.4 \times Na \times 10^{-5} \text{ kgm}\\ &= 8.9 \times 9.4 \times 1.8 \times 0.39 \times 10^{-3} \text{ kgm}\\ &= 58.7 \times 10^{-3}\end{aligned}$$

$$\text{Total weight of the moving system} = 2.5 \times 5.87 = 0.1465 \text{ kgm}$$

$$\partial M = \frac{10^{-7} \times 110}{10^2} \times \partial\theta$$

$$\partial\theta = 110 \times \frac{\pi}{180}$$

$$\partial M = \frac{10^{-7} \times 110}{10^2} \times \frac{\pi}{180}$$

$$= 10^{-7} \times \frac{121}{180}\pi = 2.11 \times 10^{-7}$$

$$\partial M = 0.211 \quad \mu\text{F}$$

Final inductance $- L + \partial M = 2.211\mu\text{F}$

Example 2.28 *The inductance of a 25 A electrodynamic ammeter changes uniformly at the rate of 0.0035μ H per degree. The torsion constant of the controlling spring is 10^{-6} Nw.m/per degree. Determine the angular deflection for full scale.*

Solution

Let α be the deflection in degree

$$T_C = 10^{-6} \times \alpha$$

$T_C = T_D =$ when meter reading is steady

$$T_D = i_1 i_2 \frac{\partial M}{\partial\theta} \quad i_1 = i_2 = 25 \text{ A}$$

$$\frac{\partial M}{\partial\theta} = 0.0035 \times 10^{-6} \times \frac{180}{\pi} \text{ h/radian}$$

$$\alpha \times 10^{-6} = 0.0035 \times 10^{-6} \times 25^2 \times \frac{180}{\pi}$$

$$\alpha = 0.0035 \times \frac{625 \times 180}{\pi} = 125.4^\circ$$

Example 2.29 *In a torsion head type electro dynamometer, a current of 25 A requires a deflection of 90° in the head to give balance (a) Find the range of the instrument for maximum deflection of 360° (b) For what value of current will the deflection be 180°. (c) What will be the deflection for a current of 20 A (d) Find the constant of instrument in amp-degree units.*

Solution

$$T_D \propto i^2 \propto T_C \propto \theta$$

Assuming both the coils are connected in series

$$25^2 \propto 90$$

$$50^2 \propto \theta$$

$$\left(\frac{50}{20}\right)^2 = \frac{\theta}{90}$$

$$\theta = 4 \times 90 = 360$$

Hence, 50 A corresponds to the full range.

(b)

$$25^2 \propto 90$$

$$i^2 \propto 180$$

$$\left(\frac{i}{25}\right)^2 = 2$$

$$i = 25\sqrt{2} = 25 \times 1.415 = 35.4 \text{ A}$$

(c) Deflection corresponding to 20 A

$$20^2 \propto \alpha$$

$$25^2 \propto \ 90$$

$$\frac{\alpha}{90} = \left(\frac{20}{25}\right)^2$$

$$\alpha = 90 \times \frac{1}{1.56} = 57.6^\circ$$

(d) Constant of the meter

$$I^2 \propto \theta$$

$$I^2 = K\theta \quad I = 25 \quad \theta = 90$$

$$K = \frac{I^2}{\theta} = \frac{625}{90} = 6.84 \text{ am}^2/\text{degree}$$

$$C^2 = K \Rightarrow C = \sqrt{K} \text{ am/degree}$$

$$C = \sqrt{6.84} = 2.62 \text{ am/degree}$$

Example 2.30 *The inductance of a 50 V dynamometer voltmeter increases uniformly over the whole scale of 90°. The initial inductance is 0.25 H, and the torque for full-scale deflection is 3.92×10^{-5} Nw.m, the corresponding D.C. being 0.05 A. Determine the difference between D.C. and 50 c/s readings at (a) 50 V (b) 25 V.*

Solution

$$T = i_p i_c \frac{dM}{d\theta} \text{ Nw.m}$$

$i_p = i_c = 0.05$ for full-scale deflection

$$3.92 \times 10^{-5} = (0.05)^2 \frac{dM}{d\theta}$$

$$\frac{dM}{d\theta} = \frac{3.92 \times 10^{-5}}{25 \times 10^{-4}} = \frac{39.2}{25} \times 10^{-2} = 0.0157$$

When the meter reads half-scale reading the value of

$$C_v = 45 + \frac{10}{2} = 50 \text{ µµF}$$

Hence, multiplying constant $= 1 + \dfrac{50}{6.12} = 1 + 8.17 = 9.17$

If the meter is to read correctly at half full scale the m.c must be 10

Hence, the error in the multiplying factor $= 10 - 9.17 = 0.83$

Hence, percentage error $= \dfrac{0.83}{9.17} \times 100 = 9.04\%$

Example 2.32 *It is required to construct a resistance of 5 Ω with a resistance temperature coefficient of 8×10^{-6}/0 ℃. Platinoid and manganin wire of cross-sectional area 0.4 mm^2 are available and are to be connected in series. Calculate the length required if the resistivity of platinoid is 34.4 µΩ-cm and 2×10^{-5} per 0 ℃. Corresponding figures for manganin are 48 µΩ-cm and 2×10^{-5} per ℃. All figures are refer to 0 ℃.*

Solution

l_1 be the length of the platinoid used (e_m)

l_2 be the length of the manganin used (c_m)

R_p = Resistance of the platinoid $= \dfrac{34.4 \times 10^{-6} l_1}{0.4 \times 10^{-2}}\ \Omega$

R_m = Resistance of the manganin $= \dfrac{48 \times 10^{-6} l_2}{0.4 \times 10^{-2}}\ \Omega$

Change in resistance per degree temperature.

$$5 = R_p + R_m = \frac{10^{-4}}{0.4}[34.4 l_1 + 48 l_2]$$

$$5 \times 8 \times 10^{-5} = \frac{34.4 \times 10^{-4} l_1}{0.4} \times 2.5 \times 10^{-4} + \frac{48 \times 10^{-4} l_2}{0.4} \times 0.2 \times 10^{-5}$$

$$4 = \frac{34.4}{0.4} \times 10^{-4} l_1 \times 2.5 + \frac{48 \times 10^{-4} l_2}{0.4} \times 0.2$$

$$4 = \frac{10^{-4}}{0.4}[34.4 \times 2.5 + 9.6 l_2]$$

$$= \frac{5 \times 2.5 = \dfrac{10^{-4}}{0.4}[34.4 \times 2.5 + 48 \times 2.5 l_2]}{8.5 = \dfrac{10^{-4}}{0.4}[48 \times 2.5 - 9.6]\, l_2}$$

$$8.5 = \frac{10^{-4}}{0.4} \times 110 \times l_2$$

$l_2 = \dfrac{8.5 \times 4 \times 10^3}{110} = 308$ cm

$l_1 = 151.3$ cm

Length of platinoid = 151.3 cm

Length of manganin = 308 cm

Example 2.33 *An electrometer force $e = 2000 \sin \omega t + 400 \sin 3\omega t + 100 \sin 5\omega t$ is connected to a circuit consisting of a resistance of 10 Ω, a variable inductance, and a capacitance of 30 μF arranged in series with a hot-wire ammeter. Find the value of inductance which will give resonance with the triple frequency component of the voltage and to estimate the readings on the ammeter and on a hot-wire voltmeter connected across the supply when resonant conditions exist (ω = 300).*

Solution

Condition for resonance for the third harmonic $3\omega L + \dfrac{1}{3\omega C}$

$$L = \frac{1}{(3\omega)^2 C} = \frac{1}{(3 \times 300)^2 \times 30 \times 10^{-6}} = 0.0411 \text{ H}$$

Impedance offered to the third harmonic $= R = 10\ \Omega$

$$i_3 = \frac{400}{10} \sin 3\omega t = 40 \sin 3\omega t$$

Inductance reactance offered to the fundamental $= 0.0411 \times \omega = 300 \times 0.0411 = 12.33\ \Omega$

Capacitance reactance offered to the fundamentals

$$\frac{1}{\omega C} = \frac{1}{300 \times 30 \times 10^{-6}} = \frac{10^3}{9} = 111.2\ \Omega$$

Inductance reactance offered to 5th harmonic $= 12.33 \times 5 = 61.65\ \Omega$

Capacitive reactance offered to the 5th harmonic $= \dfrac{111.2}{5} = 22.24\ \Omega$

Impedance offered to the fundamental $= 10 + j(12.33 - 111.2) = 10 - j98.87$

$$|Z| = \sqrt{10^2 + 98.87^2} = 99.4\ \Omega$$

$$\alpha_1 = \tan^{-1} 98.87 \quad \tan^{-1}(9.887) = 84.23^\circ$$

$$|Z_5| = \text{Impedance offered to 5th harmonic}$$

$$= 10 + j(61.65 - 22.24) = 10 + j39.41$$

$$= \sqrt{10^2 + 39.41^2} = \sqrt{100 + 1555} = \sqrt{1655} = 40.65$$

$E_3 = 0.3E_1 \quad E_5 = 0.1E_1$

$$e_{r.m.s.} = \frac{1}{\sqrt{2}}\left[E_1{}^2 + (0.3E_1)^2 + (0.1E_1)^2\right]^{1/2}$$

$$= \frac{E_1}{\sqrt{2}}[1 + 0.09 + 0.01]^{1/2} \quad = \frac{E_1}{\sqrt{2}}[1.1]^{1/2}$$

$$e_{r.m.s.} = 150 = E_1 \times \frac{1.05}{\sqrt{2}}$$

$$E_1 = \frac{\sqrt{2} \times 150}{1.05} = 202 \text{ V}$$

$E_3 = 60.6$ V

$E_5 = 20.2$ V

$$E = 202 \sin \omega t + 60.6 \sin 3(\omega t - 200) + 20.2 \sin(5\omega t - 10)$$
$$202 \sin \omega t + 60.6 \sin(3\omega t - \pi/3) + 20.2 \sin(5\omega t - 5\pi/18)$$

Example 2.36 *The movable vane of a quadrant electrometer turned through 25 scale divisions when idiostatically connected to a potential of 60 V. When used hetrostatically with the quadrants connected to a small voltage v and the hudle to a 900 V, the deflection was 10 scale divisions. Determine v.*

Solution

$T_n \propto$ (applied voltage)$(2v - v_1 - v_2) \propto v(1800 - v_1 - v_2)$

$T_i \propto$ (applied voltage)$(v_2 - v_1) \propto 60^2$

$$(2v - v_1 - v_2) \times v \propto 25$$

$$(v_1 - v_2)^2 \propto 10$$

$$\frac{1800 - (v_1 + v_2) \times v}{60^2} = \frac{25}{10}$$

$$[1800 - (v_1 + v_2)] \times v = 150 \times 60 = 9000$$

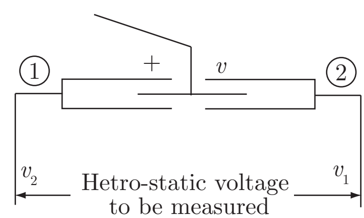


Fig. 2.75(a)

but $v = (v_2 - v_1)$

Idiostatic case: $T_i \propto (v_2 - v_1) \propto 25$

Hetrostatic: $T_n \propto (v_2 - v_1)\ (2v - v_1 - v_2) \propto 10$

But in the given problem $(v_2 - v_1)^2 = 60^2$

Assuming quadrant (1) is earthed $V_2 = 0$

$$T_n \propto v(2 \times 300 - v) \propto 10$$

$$T_i \propto 60^2 \propto 25$$

$$\frac{v(2 \times 900 - v)}{60^2} = \frac{10}{25}$$

$$\left[1800v - v^2\right] = 1440$$

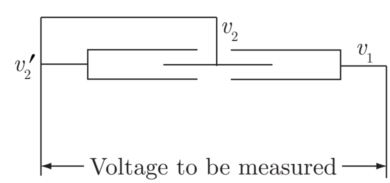


Fig. 2.75(b)

It is given v is small (v can be neglected) $1800v = 1440$

$$v \cong \frac{1440}{1800} = 0.8 \text{ V}$$

Review Questions

1. Define the various sensitivities of galvanometer.
2. How are measuring instruments classified? Also explain the basic issues concerned with the measurement of electrical quantities.
3. Explain the constructional details of PMMC instrument with neat sketch.
4. How is the current range of a PMMC instrument extended with the help of shunts? Explain a method of reducing errors due to temperature changes in the shunt connected instruments with suitable example.
5. Explain why PMMC instruments are the most widely used instruments. Explain their advantages and disadvantages.
6. Explain the working and constructional details of an attraction type moving iron instrument. Discuss its advantages and disadvantages.
7. Explain the working of a universal shunt used for multi range ammeters. Derive expressions for resistances of different sections of a universal shunt used for a 3 range ammeter.
8. Explain how a potential divider arrangement is used for multipliers used for multi range voltmeters. Derive the expressions for resistance of different sections for a 4 range voltmeter.

Exercise Problems

1. The coil of a ballistic galvanometer has 115 turns of mean area 25×10 mm^2. The flux density in the air gap is 0.12 W/m^2 and the moment of inertia is 0.5×10^{-6} kg-m^2. The stiffness constant of springs is 45×10^{-6} Nm/rad. What current must be passed to give a deflection of 1000 and what resistance must be added in series with the movement to give critical damping?

2. The coil of PMMC voltmeter is 40 mm long and 30 mm wide and has 100 turns on it. The control spring exerts torque of 240×10^{-6} N-m when the deflection is 100 divisions on full scale. If the flux density of the magnetic field in the air gap is 1d.0 wb/m^2; estimate the resistance that must put in series with the coil to give one volt per division. The resistance of the voltmeter coil may be neglected.

3. A moving coil instrument whose resistance is 25 Ω gives a full-scale deflection with a current of 1 mA. The instrument is to be used with a manganin shunt to extend its range to 100 mA. Calculate the error caused by a 10° rise in temperature when;
   (a) Copper moving coil is connected directly across the manganin shunt.
   (b) A 75 Ω manganin resistance is used in series with the instrument moving coil.
   The temperature coefficient of copper is 0.004 per ℃ and that of manganin is 0.00015 per ℃.

4. In a moving coil galvanometer with free periodic time of 4 seconds a current of 1 mA gives a deflection of 15 cm on a scale distant 250 cm. The moving system has moment of inertia

14. A sinusoidal alternating voltage of amplitude 200 V is applied to a circuit containing a rectifying device which entirely prevents the flux of current in one-directional resistance to the flow of current in other direction is 20 Ω. Determine the readings on (i) hot-wire and (ii) moving coil ammeters connected in the circuit.

15. A electrostatic has two flat parallel plates, each 12 cm^2 in area. If these plates are 5 mm apart estimate the force of attraction when there is a potential difference of 2000 V between them.

16. Determine the value of potential required to give (i) a full-scale deflection of 1000 and (ii) 500 for the variation in C, the capacitance between fixed and moving vanes of an electrostatic voltmeter with the angle of deflection as in the table, if the control of the instrument has a torsional constant of 5.5×10^{-6} N-m/radian.

θ (in degrees)	0	10	20	30	40	50	60	70	80	90	100
C in µf	37	55	71	86	100	112	124	134	144	152	160

17. The movable vane of a quadrant electrometer is turned through 25 scale divisions when idiostatically connected to a supply voltage of 60 V. When used hetrostatically with the quadrant connected to a small voltage V and the needle to a 900 V supply, the deflection was 10 scale divisions. Determine V.

18. A basic d'Arsonval meter movement with an internal resistance $R_m = 100$ and a full-scale current of $I_m = 1$ mA is to be converted into a multirange D.C. voltmeter with ranges of 0-10 V, 0-50 V, 0-250 V, 0-500 V. Find the values of various resistances using the potential divider arrangement.