

2 CHAPTER

First Law of Thermodynamics

2.1 INTRODUCTION

The first law of thermodynamics is a special case of the fundamental and general law of conservation of energy, which is applied to thermal phenomena in thermodynamic system.

The basic law of energy conservation which was discovered by M.V. Lomonsov states that energy can neither be created nor destroyed, if mass is conserved. However, it can be converted from one form to another form. Where, the sum of all kinds of energy in the isolated system (e.g., universe) will remain constant. Whereas, the first law includes systems in which there is a flow of work and heat energy only.

The conversion of mechanical work into heat has been known to man since long, but conversion of heat into mechanical work was realized only when in 1766 a reciprocating engine was created by Russian inventor I.I. Polzunov. Although, the first attempt to convert heat into mechanical work had been undertaken in first century B.C., when Heron of Alexandria invented a ball that rotated under the action of reactive forces created by water vapours (steam) that escaped from the nozzle placed on the closed cylindrical shell, when water was heated in the shell.

Regarding flow of heat there were two rival theories of heat – caloric theory and molecular theory. According to caloric theory, heat is an indestructible fluid which spreads through the matter and flows from hot body to cold body whereas molecular theory believes that the rapid vibrations of the molecules of a matter are responsible for the flow of heat energy. The caloric theory found greater support until the middle of the nineteenth century. However, most significant experiments based on the caloric theory were done much earlier. Such one observation was made by Black in 1761. According to his observations two pails each containing ice cold water and ice, were placed in a warm room, the temperature of a pail of ice cold water rose quite quickly, whereas, the temperature of the pail containing ice remained constant for longer duration, till ice melted. On the basis of his observation he argued that ice-cold water contains more heat than water.

Such concepts and theories continued to prevail until 1840s, when Joule in his work put molecular theory on sound basis by demonstrating quantitative equivalence of work and heat. Further, in 1842 J.R. Mayer established direct relationship between work and heat.

discussed that δQ and δW are path dependent terms. Therefore, a quantity ($\delta Q - \delta W$) is a differential term and in the first law of thermodynamics it is represented by a new term dE

Therefore,

$$\delta Q - \delta W = dE \tag{2.4}$$

The above expression represents the first law of thermodynamics for a close system. Where Q and W represent the transfer of amount of heat and work energy, respectively and E represents the total energy of the system during the process. The symbol d before the term E in Eq. (2.4), has exact differential and hence it is a point function which means the term E , total energy, can be calculated between two end states.

For any given finite process integrate the term in Eq. (2.4)

$$\int_1^2 \delta Q - \int_1^2 \delta W = \int_1^2 dE$$

$$\Rightarrow \quad {}_1Q_2 - {}_1W_2 = E_2 - E_1$$

The term ' E ' is a stored energy within the system. To understand this new term, let us consider a piston cylinder assembly with stoppers and filled with a compressible fluid (Fig. 2.2). Initially, fluid is at temperature T_1 and on adding heat, it attains temperature T_2 . Mathematically, expression for this is

$${}_1Q_2 - {}_1W_2 = E_2 - E_1 \tag{2.5}$$

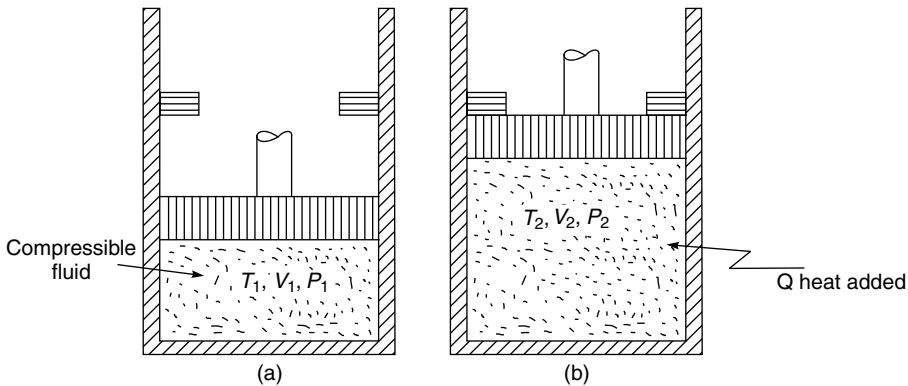


Fig. 2.2 Piston cylinder assembly

In thermodynamics work and heat are two forms of transient energy, therefore according to the first law heat and work are mutually convertible; since energy can neither be created nor destroyed, the energy of the system disappears in one form, simultaneously it appears in other form but the total energy of the system associated with energy conversion will remain same.

The total energy change ΔE of the system is the summation of several forms of energy like kinetic energy ΔKE , potential energy, ΔPE , internal energy, ΔU , magnetic energy, ΔME , surface tension, ΔSE , etc.

$$\Delta E = \Delta KE + \Delta PE + \Delta U + \Delta ME + \Delta SE + \dots \tag{2.6}$$

In closed thermodynamic system the energies (heat and work) are transferred across the boundary and the system as a whole is considered to be stationary. In such cases the ΔKE and ΔPE of the system do not change and other forms of energies like ΔME and ΔSE did not have considerable impact and neglected. Thus, above Eq. (2.6) for the first law can be written as

$$\delta Q - \delta W = \delta U \quad (2.7)$$

The above Eq. (2.7) is known as the non-flow energy equation as it does not consider the flow of mass.

Perpetual Motion Machine of First Kind (PMMFK)

It is an imaginary device that produces a work without absorbing energy from the surrounding or the system. Such device violates the first law of thermodynamics by producing work from nothing is called perpetual motion machine of first kind (PMMFK).

2.3 INTERNAL ENERGY

This different form of energy contained in a body or system of bodies is called internal energy. This energy can be represented as the sum of several forms of energy like the kinetic energy of molecules, comprising the energy of their translational and rotational motion and also of the oscillatory motion of atoms. Other forms of energy include the the energy stored in a battery, i.e., electron energy, nuclear energy, the potential energy or the energy of molecules placed in some external fields. The internal energy of a body can be expressed as,

$$U = U_{\text{kin}} + U_{\text{pot}} + U_0 \quad (2.8)$$

where U_{kin} and U_{pot} represent the kinetic and potential energies of molecules. U_0 is integration constant and represents internal energy at the absolute zero temperature. At $T = 0$ the internal motion of molecules and atoms ceases, but not the motion of particles (electron, proton) inside the atoms. The motion of electrons can takes place at any temperature, including absolute zero is therefore cannot be considered thermal energy. Therefore, in thermodynamic analysis U_0 is usually assumed zero.

For ideal gas the internal energy is summation of kinetic energy and potential energy and it is a function of state only, independent to path of process. Also the internal energy of an ideal gas, in which there are no forces of molecular interaction, is independent of volume or pressure, and depends only on temperature, expressed as,

$$\Delta U = f(T_2) - f(T_1) \quad (2.9)$$

Thus, according to the first law of thermodynamics, which is always associated with the system in definite amount with a given change of state is brought about by the performance of work alone. Therefore, the work done on the systems is simply the change in the internal energy in going from the initial to the final state,

$$\Delta U = W \quad (2.10)$$

where, U is a function of state because W is independent of path.

2.4 HEAT

It is evident from Eq. (2.10) that in a thermally isolated system it is possible to change the state of a system with doing work on it. But when a system is not thermally isolated Eq. (2.10) is no longer valid, it must be modified to,

$$\Delta U = Q + W \quad (2.11)$$

where Q represents heat, which can be measured to the extent that the change is not adiabatic. In case of adiabatic system Q is considered to be zero. Therefore, in the statement of the first law it is considered that "energy is conserved" if heat is taken into account.

As stated earlier, heat is a form of energy which is entirely equivalent in its effect on the total energy of the system as communicated by the performance of some kind of work on the same system. Therefore, sometimes it is difficult to distinguish whether a contribution of particular energy on the system should be categorised as heat or work. Suppose an electric heater and fluid which is required to be heated were placed in a close box, in this case, we should say certain amount of electric work is done. On the other hand, instead of putting fluid and electric heated in a close box, we attach heater from outside and choose not to make the heater part of the system. In such case, we should instinctively have considered the energy to be supplied as heat. In both the cases energy was communicated but in former case, it is considered to be work in the latter by heat. Thus, the most convenient way to make the distinction between the kind of energy enters into the system is a macroscopic or microscopic way. Suppose, a piston moves in a cylinder to compress a gas, the piston movement is considered macroscopic in the sense, since the velocity of piston is superimposed on gas molecules, and piston does work on the gas. On the other hand, if piston is hot, then energy is communicated to gas as heat, without the actual movement of piston, and can be ordered on microscopic scale. It is clear from the above discussion that it is always impossible to make a sharp distinction between heat and work, therefore, it is stated in the first law that they are equivalent, in certain ways.

According to first law, it is necessary to define ΔU , whenever a system undergoes a change. Since U is a function of state, and so to define the sum of Q and W , that how the system passes from initial to final state, ΔU is need to know or define. Thus, for a given infinitesimal change we write

$$dU = \delta Q + \delta W \quad (2.12)$$

Equation (2.12) is known as the differential form of the first law. Here symbol δ indicates that the infinitesimal quantities δQ and δW are not exact differentials: they cannot be evaluated from a knowledge of the initial and final states alone: dU is of course always exact since U is a function of state.

If we consider a number of systems, interacts with each other but isolated from their surroundings, then total internal energy among the group of systems must be conserved, i.e., $\Delta U = 0$. Such a group of system together form a composite system, in which neither heat nor work can enters. But in the special case when the system interact by exchange of heat alone, heat is also said to be conserved, we have

$$\Delta U_{\text{total}} = \sum \Delta U_i = \sum (Q_i + W_i) = 0 \quad (2.13)$$

While using the first law of thermodynamics in the form of Eq. (2.11) or (2.12), it is important to be clear about the signs of the terms. If ΔU is the change in internal energy of the system in going from its initial to its final state, then W must be the work done on the system and taken as -ve work, while Q is the heat transfer to the system and assigned +ve sign. Having adopted these sign convention, it is possible to define whether body is getting hotter or colder.

2.5 WORK DONE IN VARIOUS SYSTEMS

2.5.1 Work Done by Hydrostatic Pressure

Consider a compressible fluid (gas) contained in an air tight, frictionless cylinder piston assembly (Fig. 2.3a). Let the surface area of the piston be A and pressure exerted by the gas on it is p . Then the force, exerted on the piston by the gas is given

$$F = p \cdot A$$

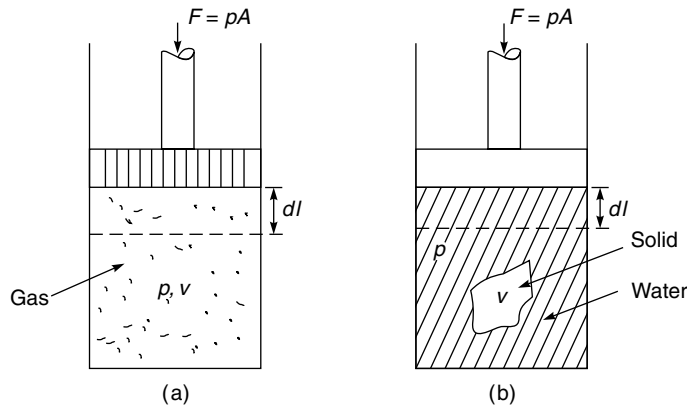


Fig. 2.3 Work done by hydrostatic pressure

Now suppose the piston is moved to a small distance dl . The work done on the gas is

$$\delta W = F \cdot dl = p \cdot A \cdot dl = -p \cdot dv \quad (2.14)$$

where dv is the change in volume of the fluid. The sign should be noted. While increasing its volume a gas does work on its surrounding and positive (+ve) work is said to be done. We shall use this result a great deal, since it is customary to follow the sign convention and generally use a fluid system as a convenient model through the development of thermodynamics.

In the case of work done on a solid by hydrostatic pressure we immerse the solid in an incompressible fluid (water) and again put the whole system in a cylinder piston assembly (Fig. 2.3b). Since the fluid is incompressible, no work can be done on it by changing the pressure and all the work done by the piston must be communicated to the solid. Thus, for work done on the system by hydrostatic pressure is always given by Eq. 2.14.

But from the definition of the work done on the surface of liquid due to surface tension is

$$\delta W = \sigma dA = 8\pi\sigma r dr \quad (2.17)$$

These two works done will be equal and opposite under equilibrium conditions, i.e.,

$$p \cdot 4\pi r^2 dr = 8\pi\sigma r dr$$

$$p = \frac{2\sigma}{r} \quad (2.18)$$

This is the pressure difference across a single spherical surface and pressure intensity inside the droplet increases with decrease of diameter of the droplet.

2.6 ENTHALPY

If we require to use T and V as independent variables. Then we must express dU in terms dT and dV as

$$dU = \left(\frac{\partial U}{\partial T}\right)_V dT + \left(\frac{\partial U}{\partial V}\right)_T dV \quad (2.19)$$

We know,

$$dQ = \partial U + p dV \quad (2.20)$$

Substituting,

$$\therefore dQ = \left(\frac{\partial U}{\partial T}\right)_V dT + \left[p + \left(\frac{\partial U}{\partial V}\right)_T\right] dV \quad (2.21)$$

If we assume that V is constant then specific heat at constant volume in its simple form can be written as,

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V \quad (2.22)$$

We may obtain similar expression if we choose p and T as our independent variables to substitute in Eq. 2.20 to obtain dU and dV in terms of dp and dT :

$$dU = \left(\frac{\partial U}{\partial p}\right)_T dp + \left(\frac{\partial U}{\partial T}\right)_p dT$$

and

$$dV = \left(\frac{\partial V}{\partial p}\right)_T dp + \left(\frac{\partial V}{\partial T}\right)_p dT$$

Then

$$dQ = \left[\left(\frac{\partial U}{\partial p}\right)_T + p\left(\frac{\partial V}{\partial p}\right)_T\right] dp + \left[\left(\frac{\partial U}{\partial T}\right)_p + p\left(\frac{\partial V}{\partial T}\right)_p\right] dT$$

In these cases two heat capacities are obtained

$$C_T = \left(\frac{\partial U}{\partial p} \right)_T + p \left(\frac{\partial V}{\partial p} \right)_T \quad (2.23)$$

$$C_p = \left[\left(\frac{\partial U}{\partial T} \right)_p + p \left(\frac{\partial V}{\partial T} \right)_p \right] \quad (2.24)$$

Equation 2.24 can be written in a further simplified way by constructing an energy function H , and we have

$$C_p = \left(\frac{\partial H}{\partial T} \right)_p \quad (2.25)$$

Equating Eqs. (2.24) and (2.25)

$$\left(\frac{\partial H}{\partial T} \right)_p = \left(\frac{\partial U}{\partial T} \right)_p + p \left(\frac{\partial V}{\partial T} \right)_p = \left[\frac{\partial}{\partial t} (U + pV) \right]_p$$

Therefore, from the above equations

$$H = U + pV \quad (2.26)$$

H is called enthalpy. As all terms U , p and V on right hand side of Eq. (2.26), defining enthalpy, are functions of state, hence, enthalpy is also a function or property of state. Its differential form gives another mathematical statement of the first law of thermodynamics when pressure p and temperature T are chosen as independent parameters:

$$dH = dU + pdV + Vdp = dQ + Vdp \quad (2.27)$$

We see from Eqs. (2.25) and (2.26) that when the system undergoes a reversible isobaric change, all the heat involved in the process is expended to change the enthalpy. Hence, Eq. (2.27) will reduce to

$$dH = dQ_p \quad (2.28)$$

From Eq. (2.27) enthalpy H , is

$$dH = H_2 - H_1 = Q + \int_{p_1}^{p_2} V dp \quad (2.29)$$

Sometimes enthalpy H has been given misleading name of 'heat content'. But it is obvious from Eq. (2.29) that the enthalpy is greater than external heat added to the system by the amount of work, Vdp , which is represented on a p - V diagram by the elementary area $abcd$ as shown in Fig. 2.5.

A change in enthalpy is fully determined by initial and final states of working medium and is independent of the intermediate states. In a cyclic process the change in enthalpy is zero.

$$\oint dH = 0$$

Therefore, like internal energy, enthalpy of an ideal gas is also a function of temperature.

2.7 NON-FLOW PROCESSES

As described in Section 2.5 whenever one or more properties (pressure, volume, temperature, etc.) of a system change, we may say that a change in state has occurred. That successive path through which the system passes from one state to another is called the process. Any change in the property or properties of the system during the process results in the flow of energy across the boundary of the system. On the basis of these facts the thermodynamic process may be classified as

- Constant volume (isochoric)
- Constant pressure (isobaric)
- Constant temperature (isothermal)
- Reversible adiabatic (isentropic)
- Polytropic
- Throttling or restricted flow

2.7.1 Constant Volume Process

A constant volume process for a perfect gas is shown in a p - V diagram in Fig. 2.7. In this case when a gas is heated in a closed vessel, it does not undergo change in volume but the process results in an increase of pressure. We know that the general expression for first law of thermodynamics is given in Eq. (2.12). Evaluating each term of the equation, we get:

Work. Since there is no change in volume, i.e., $dv = 0$ and area under curve 1-2 is zero, therefore,

$$\delta W = p \cdot dv = 0 \quad (2.31)$$

Internal Energy. For a perfect gas in constant volume process the internal energy of the system is

$$\Delta U = U_2 - U_1 = mc_v(T_2 - T_1) \quad (2.32)$$

Heat Transferred. From Eqs. (2.12), (2.31) and (2.32), we get

$$\begin{aligned} Q &= W + \Delta U \\ &= 0 + mc_v(T_2 - T_1) \end{aligned} \quad (2.33)$$

It is, therefore, concluded that in this process all the heat supplied during the process is stored in the system to raise its internal energy.

Relation between P , V and T :

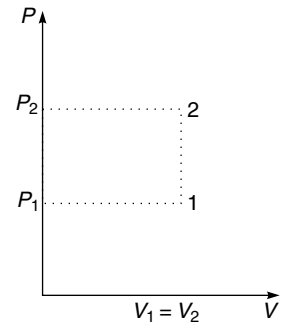


Fig. 2.7

From characteristic gas Eq. (1.23)

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad (2.34)$$

Since the process is isochoric, i.e., $V_1 = V_2$, then Eq. (2.34) can be modified to

$$\therefore \frac{P_1}{P_2} = \frac{T_1}{T_2} \quad (2.35)$$

2.7.2 Constant Pressure Process

A gas enclosed inside the cylinder beneath a sliding piston is heated. The heating of the gas will take place at constant pressure, because the force exerted by the expansion of gases due to heat, make the piston to move through a certain distance. Then work is said to be done by the gas on its surroundings. The process is represented on the p - v diagram by a horizontal line 1-2 as shown in Fig. 2.8.

Work. If the state point moves from state 1 to 2, it represents expansion process and if end state point moves in the reverse direction, from 2 to 1, it is said to be compression process.

From Eq. 2.14

$$w = \int_{v_1}^{v_2} p \cdot dv \text{ for reversible process}$$

Since p is constant, therefore

$$w = p \int_{v_1}^{v_2} dv = p(v_2 - v_1) \quad (2.36)$$

From characteristic gas equation $pV = mRT$ for constant pressure Eq. (2.36) can be written as

$$w = mR(T_2 - T_1) \quad (2.37)$$

Internal Energy. For an ideal gas in the constant pressure process the equation for internal energy can be written as

$$\Delta U = mc_p(T_2 - T_1) \quad (2.38)$$

Heat Transfer. From the non-flow energy Eq. (2.12), we can write

$$\begin{aligned} Q &= (u_2 - u_1) + w \\ Q &= mc_v(T_2 - T_1) + mR(T_2 - T_1) \\ Q &= m(T_2 - T_1)(C_v + R) \\ Q &= mc_p(T_2 - T_1) \quad \because [(C_v + R) = C_p] \end{aligned} \quad (2.39)$$

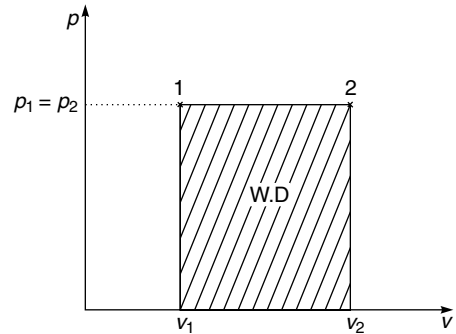


Fig. 2.8

i.e.,
$$w = p_1 v_1 \log_e \frac{v_2}{v_1} \quad (2.41)$$

Also, since $p_1 v_1 = p_2 v_2$ then

$$\frac{v_2}{v_1} = \frac{p_1}{p_2}$$

Hence, substituting in Eq. (2.41)

$$w = p_1 v_1 \log_e \frac{p_1}{p_2} \quad (2.42)$$

Using Eq. (1.23), we can write

$$p_1 v_1 = RT_1$$

Hence, substituting in Eq. (2.42)

$$w = RT_1 \log \frac{p_1}{p_2} \quad (2.43)$$

or for mass m , of the gas

$$w = mRT_1 \log \frac{p_1}{p_2}$$

Internal Energy. Since the temperature is constant during the process, therefore the internal energy remain constant, hence,

$$\Delta u = u_2 - u_1 = 0$$

Heat Transfer. Heat transfer during the process can be derived from the non-flow energy Eq. (2.12).

$$\begin{aligned} Q &= u_2 - u_1 + w \\ Q &= w \quad \text{since } u_2 = u_1 \end{aligned}$$

Therefore, it is concluded that in an isothermal process the amount of heat added to the system is converted into work and vice versa.

2.7.4 Reversible Adiabatic Process

An adiabatic process is one in which no heat is transferred to or from the system (gas) during the process. Such a process may be reversible or irreversible. In this section it is considered that the process is reversible.

From non-flow energy Eq. (2.12), for an adiabatic process, we can write

$$w = u_2 - u_1 \quad \text{Since } Q = 0$$

The above equation is true for an adiabatic process irrespective of the fact whether the process is reversible or irreversible. During adiabatic expansion, the work done by the gas is at the expense of reduction of internal energy of the gas. On the other hand,

during adiabatic compression all the work done on the system (gas) results into increase of internal energy. Since there is no transfer of heat energy, hence whole of the work contributed to raise the value of internal energy.

Governing equation for a reversible adiabatic process:

From non-flow energy Eq. (2.12),

$$\delta Q = du + \delta w$$

For a reversible process, $dw = p \cdot dv$, hence,

$$dq = du + p \cdot dv = 0 \quad (2.44)$$

Since $du = C_v dt$ Eq. (2.44) can be expressed as,

$$du + p \cdot dv = 0$$

$$C_v dt + p \cdot dv = 0$$

Since

$$pv = RT$$

$$p = \frac{RT}{v}$$

$$\therefore C_v dt + RT \frac{dv}{v} = 0$$

Dividing above equation by T , we get

$$C_v \frac{dT}{T} + R \frac{dv}{v} = 0 \quad (2.45)$$

Integrated the Eq. (2.45), we get

$$C_v \log_e T + R \log_e v = \text{const}$$

Using characteristic gas equation we have $T = pv/R$

$$C_v \log_e \frac{pv}{R} + R \log_e v = \text{constant}$$

Dividing through by C_v , we get

$$\log_e \frac{pv}{R} + \frac{R}{C_v} \log_e v = \text{constant} \quad (2.46)$$

Also from Eq. (1.33)

$$C_v = \frac{R}{\gamma - 1}$$

or

$$\frac{R}{C_v} = \gamma - 1$$

Hence, substituting the value of $\frac{R}{C_v}$ in Eq. (2.46),

$$W = \frac{p_1 v_1 - p_2 v_2}{\gamma - 1} \quad (2.49)$$

Since $pv = mRT$, the expression of W (work done) in Eq. (2.49) can be written as

$$W = \frac{mR(T_1 - T_2)}{\gamma - 1} \quad (2.50)$$

Internal Energy. In this process change in internal energy is equal to the work done. Since process is adiabatic, the amount of work done on the system contributes to raise its internal energy or work is done by the system at the expense of internal energy.

Therefore, gain in internal energy is,

$$u_2 - u_1 = mC_v(T_2 - T_1) \quad (2.51)$$

Heat Transfer. For an adiabatic process, $Q = 0$

Relation between p , v and T

Using Eq. $pv = RT$, relationship between T & v and T & p , may be derived as

$$pv = RT$$

$$\therefore p = \frac{RT}{v}$$

Substituting value of p in Eq. (2.47), we get

$$\begin{aligned} \frac{RT}{v} v^\gamma &= \text{constant} \\ Tv^{\gamma-1} &= \text{constant} \end{aligned} \quad (2.52)$$

Also, $v = (RT)/p$; hence substituting in Eq. (2.47), we get

$$\begin{aligned} p \left(\frac{RT}{p} \right)^\gamma &= \text{constant} \\ \therefore \frac{T^\gamma}{p^{\gamma-1}} &= \text{constant} \\ \frac{T}{p^{(\gamma-1)/\gamma}} &= \text{constant} \end{aligned} \quad (2.53)$$

Therefore, for a perfect gas for a reversible adiabatic process, we can write
From Eq. (2.48)

$$p_1 v_1^\gamma = p_2 v_2^\gamma \quad \text{or} \quad \frac{p_1}{p_2} = \left(\frac{v_2}{v_1} \right)^\gamma \quad (2.54)$$

From Eq. (2.52)

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1} \quad \text{or} \quad \frac{T_1}{T_2} = \left(\frac{v_2}{v_1} \right)^{\gamma-1} \quad (2.55)$$

From Eq. (2.53)

$$\frac{T_1}{p_1^{(\gamma-1)/\gamma}} = \frac{T_2}{p_2^{(\gamma-1)/\gamma}} \quad \text{or} \quad \frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\gamma-1/\gamma} \quad (2.56)$$

From Eqs. (2.55) and (2.56), we have

$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma-1} = \left(\frac{p_1}{p_2}\right)^{\gamma-1/\gamma}$$

2.7.5 Polytropic Process

The polytropic process is a process that can be described by the general equation $pv^n = \text{constant}$, where n is a constant having any value between 0 and ∞ . Many actual processes have expansion and compression curve defined by the relation $pv^n = \text{constant}$. From the definition of polytropic process it is concluded that the basic thermodynamic processes, the isochoric, isobaric, isothermal and adiabatic are special cases of the polytropic process. As example, for a chosen value of n , the general equation $pv^n = C$ will change as,

value of index n	Equation	Process
when $n = 0$	$pv^0 = C$ or $p = C$	Isobaric
when $n = \infty$	$pv^\infty = C$ or $p^{1/\infty}v = C$ i.e., $v = C$	Isochoric
when $n = 1$	$pv = C$	Isothermal
when $n = \gamma$	$pv^\gamma = C$	Adiabatic

For the given value of n the above expression is illustrated in Fig. 2.11.

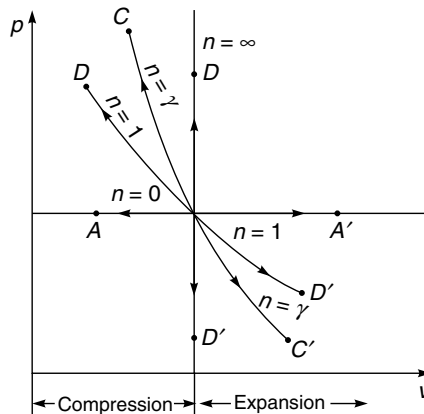


Fig. 2.11

Work. The polytropic process of an ideal gas is mathematically similar to the adiabatic process except that the index n is used in place of index γ . Therefore, Eq. (2.49) can be used to calculate the work done by substituting n for γ . Thus,

Note that

If $n = \gamma$ then $\frac{\gamma - n}{\gamma - 1} = 0$, so $Q = 0$, i.e., this is the adiabatic case.

If $n = 1$ then $\frac{\gamma - n}{\gamma - 1} = 1$, so $Q =$ work done this is the isothermal case.

We know that

$$Q = \frac{(\gamma - n)}{(\gamma - 1)} \frac{mR(T_1 - T_2)}{n - 1}$$

Since $R = C_v(\gamma - 1)$

$$\therefore Q = \frac{\gamma - n}{\gamma - 1} mC_v(\gamma - 1) \frac{(T_1 - T_2)}{n - 1}$$

$$Q = mC_v \frac{\gamma - n}{n - 1} (T_1 - T_2)$$

$$Q = mC_n(T_1 - T_2) \quad (2.62)$$

where

$$C_n = C_v \frac{\gamma - n}{n - 1}$$

C_n is called the polytropic specific heat capacity.

SOLVED EXAMPLES

EXAMPLE 2.1 A system undergoes the cyclic process 1–2–3–4–1. The value of Q , W and ΔU for individual process are given below:

Process	ΔU (kJ)	Q (kJ)	W (kJ)
1–2	–60	–	35
2–3	–	44	60
3–4	65	55	–
4–1	–	–	88

Complete the above table.

Solution: The system undergoing a cyclic process 1–2–3–4–1 is shown in Fig. Ex. 2.1

For each process applying the first law of thermodynamic:

Process: 1–2

$$\Delta u = dQ - dw$$

$$-60 = dQ - 35$$

$$dQ = -25 \text{ kJ}$$

Process: 2–3

$$\Delta u = 44 - 60$$

$$du = -16 \text{ kJ}$$

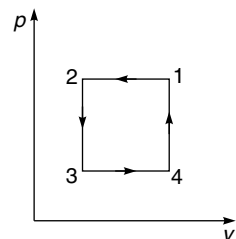


Fig. Ex. 2.1

Process: 3-4

$$65 = 55 - dw$$

$$dw = -10 \text{ kJ}$$

Process: 4-1

Since it is a cyclic process, therefore, sum of internal energy for all the processes will be zero.

$$\Delta U_{1-2} + \Delta U_{2-3} + \Delta U_{3-4} + \Delta U_{4-1} = 0$$

$$-60 - 16 + 65 + \Delta U_{4-1} = 0$$

$$\Delta U_{4-1} = 11 \text{ kJ}$$

Applying first law of thermodynamics

$$\Delta U_{4-1} = dQ - dw$$

$$11 = dQ - 88$$

$$dQ = 99 \text{ kJ}$$

EXAMPLE 2.2 One kg of gas at 50°C is heated at constant volume until the pressure is doubled. Determine (a) the final temperature, (b) the change in internal energy, and (c) the change in enthalpy. Take $C_v = 1.005 \text{ kJ/kg-K}$ and $\gamma = 1.69$ for the gas.

Solution:

Given data: $m = 1 \text{ kg}$
 $T_1 = 50^\circ\text{C}, 323 \text{ K}$
 $v_1 = v_2$
 $p_2 = 2p_1$
 $\gamma = 1.69, C_v = 1.005 \text{ kJ/kg-K}$

For constant volume process,

$$\frac{p_1}{T_1} = \frac{p_2}{T_2}$$

Final temperature, $T_2 = T_1 \times \frac{p_2}{p_1} = 323 \text{ K} \times \left(\frac{2p_1}{p_1}\right) = 646 \text{ K}$ **Ans.**

Internal energy, $\Delta u = mC_v(T_2 - T_1) = 1.005 \text{ kJ/kg K}(646 - 323)\text{K} \times (1 \text{ kg})$
 $= 324.61 \text{ kJ}$ **Ans.**

Enthalpy, $\Delta H = mC_p(T_2 - T_1)$ $\therefore \frac{C_p}{C_v} = 1.69$
 $= 1.698 \text{ kJ/kg K}(646 - 323)\text{K} \times (1 \text{ kg})$ $C_p = 1.698$
 $= 548.45 \text{ kJ}$ **Ans.**

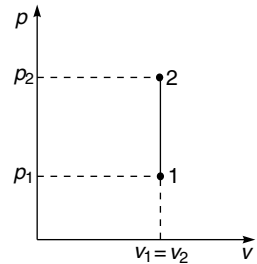


Fig. Ex. 2.2

EXAMPLE 2.3 One kg of air at 150 kPa and 30°C undergoes a constant pressure process until volume is trippled. Determine the change in internal energy and enthalpy. Take $C_p = 1.0035 \text{ kJ/kg K}$ and $C_v = 0.7165 \text{ kJ/kg K}$ for air.

Change in internal energy,

$$U_2 - U_1 = mC_v(T_2 - T_1)$$

$$\Delta U = 2.25 \times 0.84 (561 - 1104)$$

$$= 1026.27 \text{ kJ}$$

Work done by the gas,

$$W_{1-2} = \int_{v_1}^{v_2} p \cdot dv = P(v_2 - v_1)$$

$$W_{1-2} = 7 \times 10^5 (0.584 - 1.15)$$

$$W_{1-2} = -3.96 \times 10^5 \text{ J}$$

$$= -3.96 \times 10^2 \text{ kJ}$$

or

Heat transfer,

$$Q_{1-2} = mC_p(T_2 - T_1)$$

$$Q_{1-2} = 2.25 \times 1.164 (561 - 1104)$$

$$Q_{1-2} = -1422.11 \text{ kJ}$$

Since the process is isobaric, hence, $Q = \Delta H$

EXAMPLE 2.5 Four kg of nitrogen is expanded from initial state $p_1 = 1176 \text{ kPa}$ and $T_1 = 250^\circ\text{C}$ to the final pressure $p_2 = 196 \text{ kPa}$. Find, the work done, the change in internal energy and the heat transfer if the expansion is isothermal, polytropic ($n = 1.22$), and isentropic.

Solution:

Given data: $m = 4 \text{ kg}$ $p_1 = 1176 \text{ kPa}$ $T_1 = 250^\circ\text{C}$ and $p_2 = 196 \text{ kPa}$

For Isothermal Process:

$$T_1 = T_2 = 250^\circ\text{C} = 250 + 273 = 523^\circ\text{K}$$

Applying characteristic equation for initial state of gas,

$$p_1 v_1 = mRT_1$$

$$v_1 = \frac{4 \times 0.297 \times 523}{1176} = 0.528 \text{ m}^3$$

For isothermal process, $p_1 v_1 = p_2 v_2$

$$v_2 = \frac{p_1 v_1}{p_2} = \frac{1176 \times 0.528}{196}$$

$$v_2 = 3.16 \text{ m}^3$$

Work done by gas,

$$W = p_1 v_1 \log_e \frac{v_2}{v_1} = 1176 \times 0.528 \log_e \left(\frac{3.16}{.528} \right)$$

$$= 1110.9 \text{ kJ}$$

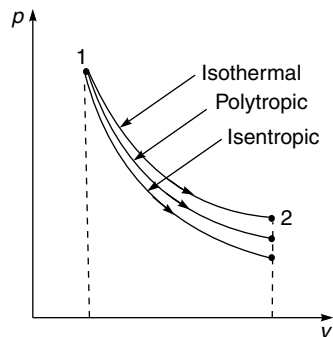


Fig. Ex. 2.5

Since there is no change in temperature, change in internal energy is zero. Here the amount of work done by the gas has been done at the cost of heat.

∴ Heat given by gas

$$Q_{1-2} = W_{1-2}$$

$$Q_{1-2} = 1110.9 \text{ kJ}$$

Polytropic Process:

Considering polytropic process

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = 523 \left[\frac{196 \times 10^5}{1176 \times 10^5} \right]^{\left(\frac{1.22-1}{1.22} \right)} = 378.6 \text{ K}$$

Work done by gas,

$$W = \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{mR(T_1 - T_2)}{n-1}$$

$$W = \frac{4 \times 0.297(523 - 378.6)}{(1.22 - 1)}$$

$$W = 779.76 \text{ kJ}$$

Change in internal energy,

$$U_2 - U_1 = mC_v(T_2 - T_1)$$

$$\Delta U = 4 \times 0.741(378.6 - 523)$$

$$\Delta U = 428.0 \text{ kJ}$$

$$\text{where } C_v = (C_p - R)$$

$$C_v = (1.038 - .297)$$

$$C_v = 0.741 \text{ kJ/kg K}$$

Heat transfer from the gas,

$$Q = W \cdot D \times \frac{\gamma - n}{\gamma - 1} = 779.76 \times \frac{(1.4 - 1.22)}{(1.4 - 1)} = 350.89 \text{ kJ}$$

Isentropic Process:

Considering isentropic process,

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = 523 \left(\frac{196 \times 10^5}{1176 \times 10^5} \right)^{\frac{1.4-1}{1.4}} = 313.4^\circ\text{K}$$

Work done by gas,

$$W = \frac{mR(T_1 - T_2)}{\gamma - 1}$$

$$= \frac{4 \times 0.297(523 - 313.4)}{(1.4 - 1)} = 622.5 \text{ kJ}$$

Change in internal energy,

$$U_2 - U_1 = mC_v(T_2 - T_1)$$

$$= 4 \times 0.741(313.4 - 523)$$

$$= -621.25 \text{ kJ}$$

Since the process is an isentropic (reversible adiabatic) there is no transfer of heat.

$$T_1 = 300 \text{ K}$$

$$v_2 = 4v_1$$

Initial volume of the air is determined by characteristic gas equation,

$$v_1 = \frac{mRT_1}{p_1} = \frac{(10 \text{ kg}) \times (0.287 \text{ kJ/kg K}) \times (300 \text{ K})}{600 \times 10^3 \text{ N/m}^2} = 1.435 \text{ m}^3$$

Final volume,

$$v_2 = 4v_1 = 4 \times 1.435 \text{ m}^3 = 5.74 \text{ m}^3$$

Final pressure,

$$p_2 = \frac{p_1 v_1}{v_2} = (600 \text{ kPa}) \times \frac{(1.435 \text{ m}^3)}{(5.74 \text{ m}^3)} = 150 \text{ kPa}$$

Work done,

$$\begin{aligned} W &= p_1 v_1 \log_e \frac{v_2}{v_1} = (600 \times 10^3 \text{ N/m}^2) \times (1.435 \text{ m}^3) \log_e (4) \\ &= 1193599.4 \text{ J} = 1193.6 \text{ kJ} \end{aligned}$$

Ans.

In isothermal process, amount of work done is equal to the heat added.

$$Q = 1193.6 \text{ kJ}$$

Ans.

EXAMPLE 2.8 The pressure of 0.15 m^3 of air increases from 500 kPa to 1.5 MPa while undergoing isothermal compression. Determine the value of the transferred heat.

Solution:

Given data:

$$v_1 = 0.15 \text{ m}^3$$

$$p_1 = 500 \text{ kPa}, 500 \times 10 \text{ N/m}^2$$

$$p_2 = 1.5 \text{ MPa}, 1500 \times 10 \text{ N/m}^2$$

Final volume,

$$v_2 = \frac{p_1 v_1}{p_2} = \frac{500 \text{ kPa}}{1500 \text{ kPa}} \times 0.15 \text{ m}^3$$

$$v_2 = 0.05 \text{ m}^3$$

In case of isothermal process work done is equal to heat transferred,

$$WD = Q = p_1 v_1 \log_e (v_2/v_1)$$

$$Q = (500 \times 10^3 \text{ N/m}^2) (0.15 \text{ m}^3) \log_e \left(\frac{0.05 \text{ m}^3}{0.15 \text{ m}^3} \right)$$

$$Q = -82395.92 \text{ J} = -82.4 \text{ kJ}$$

Ans.

EXAMPLE 2.9 One kg of an ideal gas at 30°C and 100 kPa is allowed to expand isothermally till the pressure reduces to one half of the original pressure followed by a constant volume cooling till its pressure reduces to one fourth of the original value. Then it is resorted to the initial state by a reversible adiabatic process. Calculate the net work done by the gas. Take $R = 0.294 \text{ kJ/kg K}$.

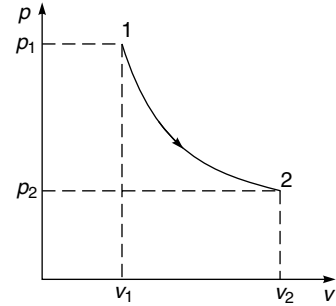


Fig. Ex. 2.7

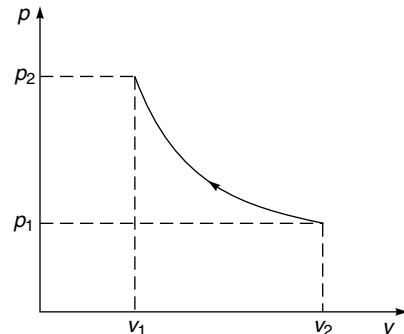


Fig. Ex. 2.8

Solution:

Given data: $p_1 = 100 \text{ kPa}$
 $T_1 = 30 + 273 = 303^\circ\text{K}$
 $p_2 = p_1/2 = 50 \text{ kPa}$
 $p_3 = p_1/4 = 25 \text{ kPa}$

Isothermal process,

$$W_{12} = RT_1 \log_e p_1/p_2$$

$$W_{12} = 0.294 \times 303 \times \log_e \frac{100}{50}$$

$$W_{12} = 61.74 \text{ kJ}$$

Constant volume process

During this process $W_{23} = 0$, because change in volume does not takes place.

Temperatures at state 3

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}$$

$$T_3 = \frac{p_3 \times T_2}{p_2} = \frac{T_2}{2} = \frac{303}{2} = 151.5^\circ\text{K}$$

Adiabatic pressure

$$W_{31} = \frac{mR}{\gamma - 1} (T_1 - T_3)$$

$$= \frac{1 \times 0.294}{(1.4 - 1)} (303 - 151.5)$$

$$W_{31} = -111.35 \text{ kJ}$$

Net $WD = W_{12} + W_{23} - W_{31} = 61.74 + 0 - 111.35$
 $= -49.6 \text{ kJ}$

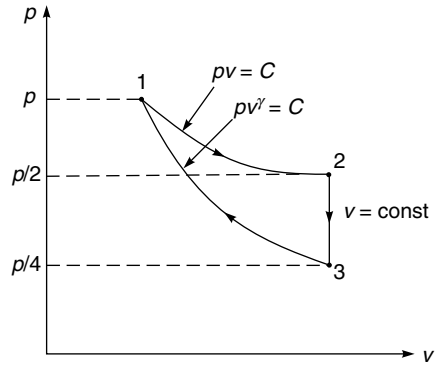


Fig. Ex. 2.9

EXAMPLE 2.10 Half kg of an ideal gas ($Cv = 0.741 \text{ kJ/kg K}$) at 300°K and 100 kPa is contained in an insulated and rigid cylinder. A paddle wheel is inserted into the gas space and rotated by an electric motor of 50 W capacity for 2 minutes. Estimate the final temperature and pressure of the gas.

Solution:

Given data: $m = 0.5 \text{ kg}$, $T_1 = 300 \text{ K}$, $p_1 = 100 \text{ kPa}$

Time = 2 minutes

Work done by a paddle wheel in 2 minutes,

$$W_{\text{paddle}} = 50 \text{ W} = 50 \text{ J/s} \times \text{Time}$$

$$W_{\text{paddle}} = 50 \times 120 = 6000 \text{ J} = 6 \text{ kJ}$$

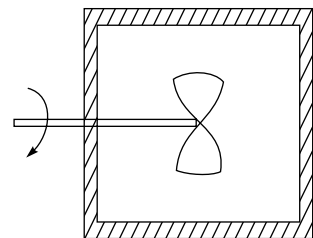


Fig. Ex. 2.10

Final temperature

Since the cylinder is rigid and insulated, whole of the work done by paddle wheel is converted into raising the internal energy of the gas.

Heat rejected

$$Q = mC_v \left[\frac{n-\gamma}{n-1} (T_2 - T_1) \right] = (3 \text{ kg}) \times (0.72 \text{ kJ/kg K}) \times \left(\frac{1.23-1.4}{1.23-1} \right) \times (500 - 300) \text{ K}$$

$$Q = -319.3 \text{ kJ}$$

Ans.

EXAMPLE 2.12 Two kg of oxygen is compressed polytropically from a pressure of 148 kPa and 17°C to 740 kPa. If the compression is according to the law, $p v^{1.3} = \text{constant}$, find (a) the final temperature and volume of the gas, (b) the amount of work done, (c) the change in internal energy, and (d) the heat transferred. Take $R = 0.26 \text{ kJ/kg K}$ and $C_p = 0.913 \text{ kJ/kg K}$ for oxygen.

Solution:

Given data: $m = 2 \text{ kg}$

$$p_1 = 148 \text{ kPa} \quad p_2 = 740 \text{ kPa}$$

$$T_1 = 17^\circ\text{C}, 290 \text{ K}$$

$$n = 1.3 \quad C_v = (C_p - R) = (0.913 - 0.26) = 0.653 \text{ kJ/kg K}$$

Initial volume

$$v_1 = \frac{mRT_1}{p_1} = \frac{(2 \text{ kg}) \times (0.26 \text{ kJ/kg K}) \times (290 \text{ K})}{(148 \times \text{kPa})} = 1.018 \text{ m}^3$$

Final volume

$$v_2 = \left(\frac{p_1}{p_2} \right)^{\frac{1}{n}} v_1 = \left(\frac{148 \text{ kPa}}{740 \text{ kPa}} \right)^{\frac{1}{1.3}} \times (1.018 \text{ m}^3) = 0.295 \text{ m}^3$$

Final temperature

$$T_2 = T_1 \left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} = (290 \text{ K}) \left(\frac{740 \text{ kPa}}{148 \text{ kPa}} \right)^{\frac{1.3-1}{1.3}} = 420.4 \text{ K}$$

Work done

$$w = \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{(148 \times 10^3 \text{ N/m}^2) \times (1.018 \text{ m}^3) - (740 \times 10^3 \text{ N/m}^2) \times (0.295 \text{ m}^3)}{(1.3-1)}$$

$$= -233320 \text{ J} = -233.32 \text{ kJ}$$

Change in internal energy

$$\Delta U = mC_v (T_2 - T_1) = 2 \text{ kg} \times (0.653 \text{ kJ/kg K}) (420.4 \text{ K} - 290 \text{ K})$$

$$= 170.3 \text{ kJ}$$

Heat transferred

$$Q = \frac{\gamma - n}{\gamma - 1} \times \text{WD}$$

$$Q = \frac{1.4 - 1.3}{1.4 - 1} \times -233.32 \text{ kJ} = -58.33 \text{ kJ}$$

EXAMPLE 2.13 A gas of mass 0.675 kg having a pressure and temperature, 14 MPa and 275°C is expanded to four times the original volume according to the law $pV^{1.3} = C$. Determine:

- the initial and final volume of the gas;
- the final pressure of the gas; and
- the final temperature of the gas.

Take $R = 0.287$ kJ/kg K.

Solution:

Given data: $m_1 = 0.675$ kg $p_1 = 14$ MPa $T_1 = 275^\circ\text{C}$ or 548 K
 $R = 0.287$ kJ/kg K, $v_2 = 4V_1$

$$p_1 v_1 = mRT_1$$

$$v_1 = \frac{mRT_1}{p_1}$$

$$v_1 = \frac{0.675 \times 0.287 \times 548}{14 \times 10^3 \text{ (kPa)}} = 7.5 \times 10^{-3} \text{ m}^3$$

$$v_2 = 4v_1 = 4 \times 7.5 \times 10^{-3} = 0.03 \text{ m}^3$$

$$p_1 v_1^n = p_2 v_2^n$$

$$p_2 = p_1 \left(\frac{v_1}{v_2} \right)^n = 14 \times 10^3 \text{ kPa} \left(\frac{1}{4} \right)^{1.3} = 2.3 \times 10^3 \text{ kPa}$$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

$$T_2 = \frac{p_2}{p_1} \times \frac{v_2}{v_1} \times T_1 = \frac{2.3 \times 10^3}{14 \times 10^3 \text{ kPa}} \times \frac{4}{1} \times 548 \text{ K} = 360.1 \text{ K}$$

$$T_2 = 360.1 - 273 = 87.1^\circ\text{C}$$

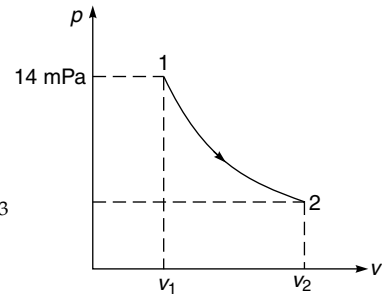


Fig. Ex. 2.13

EXAMPLE 2.14 A gas of 0.27 kg occupies volume 0.15 m³ at a pressure of 140 kPa and compressed to a pressure 1.4 MPa according to the law, $pV^{1.25} = C$. Determine:

- the change of internal energy of the air;
- the work done on or by the air; and
- the heat received or rejected by the air.

Take $C_p = 1.005$ kJ/kg K, $C_v = 0.718$ kJ/kg K

Solution:

Given data: $m_1 = 0.27$ kg $v_1 = 0.15$ m³ $p_1 = 140$ kPa, $p_2 = 1400$ kPa (1.4 MPa)

$$R = C_p - C_v$$

$$R = 1.005 - 0.718 = 0.287 \text{ kJ/kg K}$$

During isothermal process

$$p_1 v_1 = p_3 v_3$$

$$\frac{p_1}{p_3} = \frac{v_3}{v_1}, \quad \frac{v_3}{v_1} = \frac{1400 \text{ kPa}}{220 \text{ kPa}} = 6.36$$

For adiabatic process

$$p_1 v_1^\gamma = p_2 v_2^\gamma$$

$$\frac{P_1}{P_2} = \left(\frac{v_2}{v_1} \right)^\gamma = \left(\frac{v_3}{v_1} \right)^\gamma$$

Since

$$v_3 = v_2$$

$$\frac{1400 \text{ kPa}}{101 \text{ kPa}} = (6.36)^\gamma$$

$$\gamma = 1.42$$

Also,

$$\frac{C_p}{C_v} = \gamma$$

or

$$C_v = \frac{C_p}{\gamma} = \frac{1.005 \text{ kJ/kg K}}{1.42} = 0.707 \text{ kJ/kg K}$$

Temperature at end state 2.

For constant volume process,

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$T_2 = \frac{p_2}{p_3} \times T_3 = \frac{101 \text{ kPa}}{220 \text{ kPa}} \times 630 \text{ K} = 289.2 \text{ K}$$

$$T_2 = 289.2 \text{ K or } 16.2^\circ\text{C}$$

Change in internal energy,

$$\begin{aligned} \Delta U &= m C_v (T_2 - T_1) \\ &= 0.23 \text{ kg} \times 0.707 \text{ kJ/kg K} (289.2 \text{ K} - 630 \text{ K}) \\ &= -55.41 \text{ kJ} \quad (\text{loss of internal energy}) \end{aligned}$$

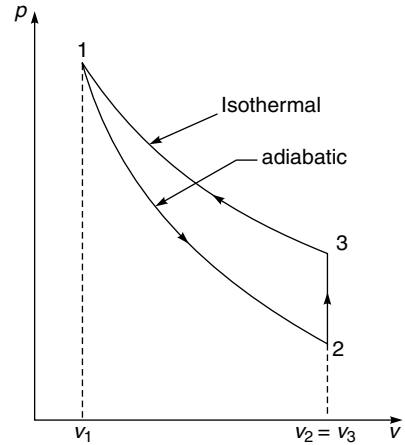


Fig. Ex. 2.15

EXAMPLE 2.16 A gas cylinder has a capacity of 290 litres and contains gas at a pressure of 3.0 MPa at temperature 290 K. The stop valve is open and some gas is used. If the temperature and pressure of the gas falls to 1.6 MPa and 287 K, respectively after the release of gas. Determine the mass of gas used.

If after the stop valve closed the oxygen remaining in the cylinder gradually attains its initial temperature of 290 K, determine the amount of heat transferred through the cylinder wall from the atmosphere. The density of gas at 0°C and 101.3 kPa may be taken as 1.43 kg/m³ and $\gamma = 1.4$.

Solution:

Given data: Cylinder capacity, $v_1 = 300$ liters or $300 \times 10^{-3} \text{ m}^3$
 $p_1 = 3.0$ MPa or 3×10^3 kPa
 $T_1 = 290$ K

After release of stop valve, $p_2 = 1.6$ MPa or 1.6×10^3 kPa
 $T_2 = 287$ K

at ambient condition, $p = 101.3$ kPa $T = 273$ K
density, $\rho = 1.43$ kg/m³

At ambient condition value of R will be,

$$PV = mRT$$

$$R = \frac{pv}{mT} = \frac{101.3 \text{ kPa} \times 1}{1.43 \text{ kg/m}^3 \times 273 \text{ K}} = 0.26 \text{ kJ/kg K}$$

Initially, mass of gas inside the cylinder,

$$m_1 = \frac{p_1 v_1}{RT_1} = \frac{3.0 \times 10^6 \text{ N/m}^2 \times 300 \times 10^{-3} \text{ m}^3}{0.26 \times 10^3 \text{ J/kg K} \times 290 \text{ K}}$$

$$m_1 = 11.93 \text{ kg}$$

Final mass of gas after release of stop valve,

$$m_2 = \frac{p_2 v_2}{RT_2} = \frac{1.6 \times 10^6 \text{ N/m}^2 \times 300 \times 10^{-3} \text{ m}^3}{0.26 \times 10^3 \text{ J/kg K} \times 287 \text{ K}}$$

$$m_2 = 6.43 \text{ kg}$$

Mass of gas used = $m_1 - m_2 = 11.93 - 6.43 = 5.5$ kg

Heat transfer during the process:

Above is the case of non-flow process, therefore,

$Q = \Delta U + W$ $W = 0$, because volume of gas remains same, hence no work is done.

$$\therefore Q = \Delta U = m_2 C_v (T_1 - T_2)$$

$$Q = m_2 \frac{R}{(\gamma - 1)} (T_1 - T_2) \quad \text{Since } C_v = \frac{R}{(\gamma - 1)}$$

$$Q = 6.43 \text{ kg} \times \frac{0.26 \text{ kJ/kg K}}{(1.4 - 1)} (290 \text{ K} - 287 \text{ K})$$

$$Q = 12.5 \text{ kJ}$$

Ans.

- (a) the temperature and pressure of the air at the end of the compression;
- (b) the temperature and pressure of the air at the end of the constant volume process; and
- (c) the heat transfer required to carry out constant volume process.

For the air, take $C_p = 1.005 \text{ kJ/kg K}$, $R = 0.24 \text{ kJ/kg K}$.

Solution:

Given data: $v_s v_1 = 15 \text{ liter or } 15 \times 10^{-3} \text{ m}^3$

(compression ratio) $r = 14 : 1 = \frac{v_1}{v_2}$

$$p_1 = 95 \text{ kN/m}^2 \quad T_1 = 30^\circ\text{C or } 303 \text{ K}$$

$$\frac{p_3}{p_2} = 1.6 : 1$$

$$PV^{1.34} = C \quad n = 1.34$$

$$p_1 v_1^{1.34} = p_2 v_2^{1.34}$$

$$p_2 = p_1 \left(\frac{v_1}{v_2} \right)^{1.34}$$

$$p_2 = 95 \text{ kN/m}^2 \left(\frac{14}{1} \right)^{1.34} = 3262.3 \text{ kN/m}^2$$

Temperature at the end of compression,

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{\gamma-1} = 303 \times (14)^{(1.34-1)} = 743.2 \text{ K or } 470.2^\circ\text{C}$$

Pressure and temperature at the end of constant volume process from state 2-3,

$$\frac{p_3}{p_2} = 1.6 : 1$$

$$p_3 = p_2 \times 1.6 = 3262.3 \text{ kN/m}^2 \times 1.6 = 5219.68 \text{ kN/m}^2$$

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

$$T_3 = \frac{p_3}{p_2} \times T_2 = 1.6 \times 743.2 \text{ K} = 1189.2 \text{ K or } 916.12^\circ\text{C}$$

Mass of air entered into the cylinder,

$$p_1 v_1 = mRT_1$$

$$m = \frac{p_1 v_1}{RT_1} = \frac{95 \text{ kN/m}^2 \times 15 \times 10^{-3} \text{ m}^3}{0.24 \text{ kJ/kg K} \times 303 \text{ K}}$$

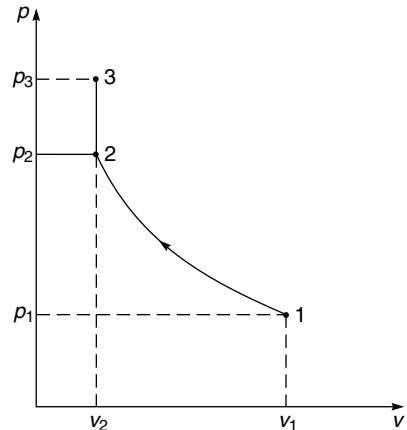


Fig. Ex. 2.18

$$m = 0.0195 \text{ kg}$$

Also $Cv = Cp - R = 1.005 - 0.24 = 0.765 \text{ kJ/kg K}$

Heat transfer carrying the constant volume process 2–3

$$Q = mC_v(T_3 - T_2)$$

$$= 0.0195 \text{ kg} \times 0.765 \text{ kJ/kg K} (1189.2 \text{ K} - 743.2 \text{ K})$$

$$Q = 6.65 \text{ kJ}$$

EXAMPLE 2.19 The cylinder of an engine has a stroke of 300 mm and a bore of 250 mm. The volume ratio of compression is 14:1. Air in the cylinder at the beginning of compression has a pressure of 96 kN/m² and temperature of 93°C. The air is compressed for the full stroke according to the law $PV^{1.3} = C$. Determine:

- the mass of air,
- the work transfer, and
- the heat transfer.

For air, take $\gamma = 1.4$, $C_p = 1.006 \text{ kJ/kg K}$.

Solution:

Given Data: Stroke, $l = 300 \text{ mm}$, bore, $d = 250 \text{ mm}$ compression ratio $r = \frac{v_1}{v_2} = 14:1$
 $p_1 = 96 \text{ kN/m}^2$, $T_1 = 93^\circ\text{C}$ or 366 K

Gas law, $PV^{1.3} = C$

$$v_1 = \frac{\pi}{4} d^2 \times l = \left(\frac{\pi}{4} (0.25)^2 \times (0.3) \right) \text{m}^3$$

$$v_1 = 0.0147 \text{ m}^3$$

Volume at the end of compression,

$$v_2 = \frac{v_1}{r} = \frac{0.0147}{14} = 1.05 \times 10^{-3} \text{ m}^3$$

Pressure at the end of compression,

$$p_2 = p_1 \left(\frac{v_1}{v_2} \right)^{\gamma-1} = 96 \text{ kN/m}^2 (14)^{(1.3-1)} = 211.88 \text{ kN/m}^2$$

Temperature at the end of compression,

$$T_1 v_1^{n-1} = T_2 v_2^{n-1}$$

$$T_2 = T_1 \left(\frac{v_1}{v_2} \right)^{n-1} = 366 \text{ K} (14)^{0.3} = 807.8 \text{ K}$$

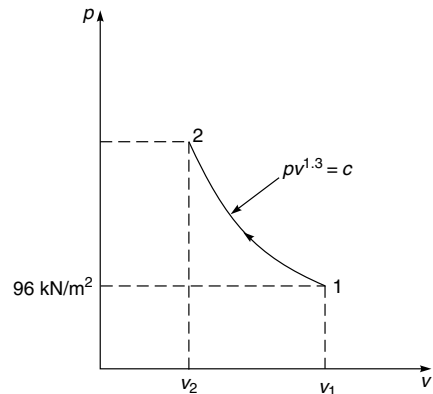


Fig. Ex. 2.19

Mass of air inside the cylinder,

$$p_1 v_1 = mRT_1$$

$$m = \frac{p_1 v_1}{RT_1} = \frac{96 \text{ kN/m}^2 \times 0.0147 \text{ m}^3}{0.287 \text{ kJ/kg K} \times 366 \text{ K}}$$

$$m = 0.0134 \text{ kg}$$

Work transfer during the process,

$$W = \frac{mR(T_1 - T_2)}{n - 1}$$

$$W = \frac{0.0134 \text{ kg} \times 0.287 \text{ kJ/kg K} (366 \text{ K} - 807.8 \text{ K})}{(1.3 - 1)}$$

$$W_2 = -5.66 \text{ kJ (work is done on the system)}$$

Heat transfer during the process,

$$Q = \frac{\gamma - n}{n - 1} \times W.D$$

$$Q = \frac{1.4 - 1.3}{1.3 - 1} \times (-5.66) \text{ kJ}$$

$$Q = -1.88 \text{ kJ (heat rejected by the system)}$$

$$\begin{aligned} \text{Since } R &= C_p - C_v \\ &= C_p - \frac{C_p}{\gamma} \\ &= C_p \frac{\gamma - 1}{\gamma} \\ &= 1.006 \left(\frac{1.4 - 1}{1.4} \right) \\ R &= 0.287 \text{ kJ/kg K} \end{aligned}$$

EXAMPLE 2.20 An air main is connected to a cylinder through a valve. The air in the main is maintained at a constant pressure and temperature of 1 MN/m^2 and 400°C respectively. The initial pressure and volume of air in the cylinder are 140 kN/m^2 and 3 litres , respectively. The valve is opened, 0.11 kg of air enters the cylinder then the valve is closed. As a result of this mass transfer, the pressure in the cylinder becomes 700 kN/m^2 and the volume becomes 15 litres . Assuming the process to be adiabatic, determine the work done on the piston. For air, take $c_p = 1.006 \text{ kJ/kg K}$, $c_v = 0.717 \text{ kJ/kg K}$.

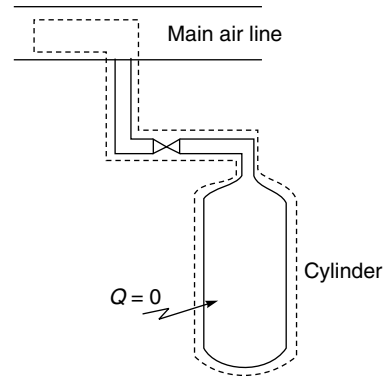


Fig. Ex. 2.20

Solution:

Assume

M_0 = initial mass of air in the cylinder

M_f = final mass of air in the cylinder

P = Pressure of air in main line

v = sp. volume of air in main line

u = sp. internal energy of air in main line

$$\frac{700 \times 942.2 \times 15}{T_f} = 10080 + \frac{942.2 \times 140 \times 3}{300} = 11399.08$$

$$T_f = \frac{700 \times 942.2 \times 15}{11399.08} = 867.8 \text{ K or } 594.8^\circ\text{C}$$

At 700 kN/m^2 , 1193°C

$$\text{sp. volume } v = 0.99 \text{ m}^3/\text{kg}$$

Work done,

$$\begin{aligned} W &= (m_f - m_0)v \\ &= 0.11 \times 0.99 \times 4889.4 \\ &= 532.4 \text{ kJ} \end{aligned}$$

EXAMPLE 2.21 A pressure vessel contains a gas at an initial pressure of 3.5 MN/m^2 and at a temperature of 60°C . It is connected through a valve to a vertical cylinder in which there is a piston. Initially, there is no gas under the piston. The valve is opened, gas enters the vertical cylinder, and work is done in moving the piston. The valve is closed and the pressure and temperature of the remaining gas in the vessel are 1.7 MN/m^2 and 25°C , respectively. Determine the temperature of the gas in the vertical cylinder if the process is assumed to be adiabatic. Take $\gamma = 1.4$.

Solution:

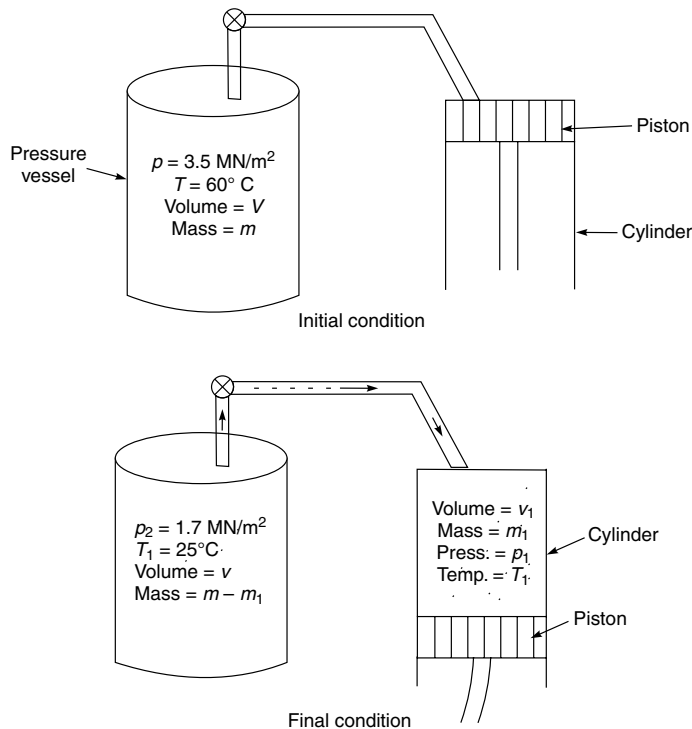


Fig. Ex. 2.21

At initial condition in pressure vessel,

$$P_1 V_1 = mRT_1$$

$$R = \frac{P_1 V_1}{mT_1} \quad (\text{i})$$

At final condition in pressure vessel,

$$P_2 V_2 = (m - m_1)RT_2$$

$$R = \frac{P_2 V_2}{(m - m_1)T_2} \quad (\text{ii})$$

Equating Eqs. (i) and (ii),

$$\frac{P_1 V_1}{mT_1} = \frac{P_2 V_2}{(m - m_1)T_2}$$

$$\frac{3.5 \times 10^6 \text{ N/m}^2 \times V_1}{m \times 333 \text{ K}} = \frac{1.7 \times 10^6 \text{ N/m}^2 \times V_2}{(m - m_1) \times 298 \text{ K}}$$

Since $V_1 = V_2 =$ cylinder volume

$$3.5 \times 298(m - m_1) = (1.7 \times 333)m$$

$$[(3.5 \times 298) - (1.7 \times 333)]m = (3.5 \times 298)m_1$$

$$476.9m = 1043m_1$$

$$\frac{m}{m_1} = 2.187$$

Assuming the process is non-flow, apply the equation of the first law,

$$Q = \Delta U + W$$

$$Q = 0, \text{ since the process is adiabatic}$$

$$\therefore W + \Delta U = 0 \quad (\text{iii})$$

$$W = -\Delta U = -(U_1 - U_2)$$

$$= (\text{Internal energy before the valve is opened} - \text{Internal energy after valve is opened})$$

Internal energy before valve is opened,

$$U_1 = mC_v T \quad (\text{iv})$$

Internal energy after valve is opened,

$$U_2 = \text{Internal energy of gas in pressure vessel} + \text{Internal energy of gas in cylinder}$$

$$U_2 = (m - m_1)C_v T_2 + m_1 C_v T_1 \quad (\text{v})$$

Work required to move the piston,

$$W = p_1 v_1 = m_1 R T_1 = m_1 (C_p - C_v) T_1 = m C_v (\gamma - 1) T_1 \quad (\text{vi})$$

From Eqs. (iii), (iv), (v), and (vi), we have,

$$m_1 C_v (\gamma - 1) T_1 = m C_v T - [(m - m_1)C_v T_2 + m_1 C_v T_1]$$

during compression the pressure of the gas at the given volume, V_1 will not be equal to that of what it was during expansion process.

Only reversible processes can be shown on P - V diagrams, since on these diagrams each point represents an equilibrium state of the gas.

2.9 STEADY STATE FLOW ENERGY EQUATION

Steady state flow processes are of primary importance in engineering because mass production of materials and energy flow demand continuous operation of processes. A flow process in thermodynamics constitutes an open system through which working medium (gas) is allowed to flow in and out, also the interaction of energy (heat, work, internal energy, KE and PE) between system and surroundings. Where steady state implies that inflow and outflow of mass and energies at any instant of time must be equal, in other words there should be no accumulation of mass and energy in the system.

Consider the schematic diagram of Fig. 2.13, which shows a region/space bounded by a dotted line is called control surface and the region thus bounded is also called control volume. During a small time interval, dt the fluid of mass, m enters the control volume at section 1-1 with average velocity, V_1 , pressure P_1 , sp. volume v_{s1} , and internal energy U_1 , whereas at section 2-2 mass leaves the control volume and the properties are mentioned by using subscript 2 instead of 1.

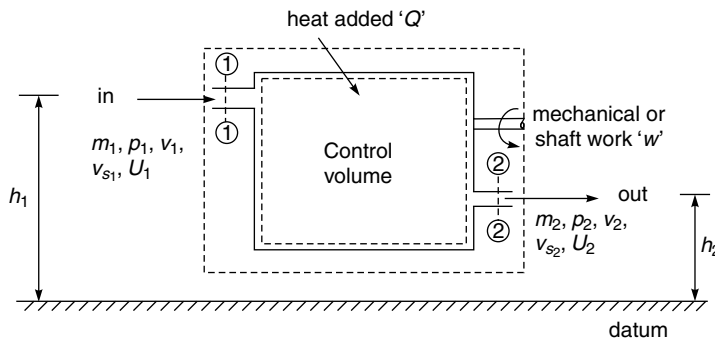


Fig. 2.13 Schematic diagram of steady state flow process

The kinetic energy of mass, m flowing in the control volume is given by,

$$\text{KE} = \frac{mV_1^2}{2}$$

and potential energy can be expressed as,

$$\text{PE} = mgh_1$$

where, h_1 is the height above some arbitrary chosen datum. Also, when the fluid flow across the boundary of any chosen system, heat ' Q ' and mechanical or shaft work ' W ', may also cross the boundary.

Now to write the steady state flow energy equation we may apply the first law in a form already developed (Eq. 2.12) provided that we add the terms to account for change in kinetic energy, potential energy and flow work which require to push the fluid in and out of the control volume considering the universal definition of the law of conservation of energy that “energy can neither be created nor destroyed, it can be transformed from one form to another”, the steady flow energy equation, appear as:

$$m \left(u_1 + p_1 v_1 + \frac{V_1^2}{2} + gh_1 \right) + Q = m \left(u_2 + p_2 v_2 + \frac{V_2^2}{2} + gh_2 \right) + W_s \quad \dots(2.63)$$

$$m \left(H_1 + \frac{V_1^2}{2} + gh_1 \right) + Q = m \left(H_2 + \frac{V_2^2}{2} + gh_2 \right) + W_s \quad \dots(2.64)$$

2.10 WORK DONE IN STEADY FLOW PROCESS

Equation 2.64, for steady flow process, for unit mass of gas flowing in and out of system can be written as

$$H_1 + \frac{v_1^2}{2} + gh_1 + Q = H_2 + \frac{v_2^2}{2} + gh_2 + W_s \quad (2.65)$$

or

$$Q - W = (H_2 - H_1) + \left(\frac{v_2^2 - v_1^2}{2} \right) + (h_2 - h_1)g \quad (2.66)$$

In differential form

$$\delta Q - \delta W = dH + d(v^2/2) + d(gh) \quad (2.67)$$

Now, throughout the discussion mechanical work done during steady flow process and non-flow process can be differentiated by putting suffix *sf* for steady flow and *nf* for non-flow processes.

Therefore, Eq. (2.67) can be written as,

$$\delta Q - \delta W_{sf} = dH + d(v^2/2) + d(gh) \quad (2.68)$$

Equation (2.12) represents non-flow process, considering the effect of motion and gravity it can be written as,

$$\delta Q - \delta W_{nf} = dU + dv^2/2 + d(gh) \quad (2.69)$$

or,

$$\delta Q = du + \delta W_{nf} + dv^2/2 + d(gh) \quad (2.70)$$

Substituting the value of δQ from Eq. (2.70) in Eq. (2.67), we have,

$$du + \delta W_{nf} + dv^2/2 + d(gh) - \delta W_{sf} = dH + dv^2/2 + d(gh) \quad (2.71)$$

i.e.,

$$du + \delta W_{nf} - \delta W_{sf} = dH \quad (2.72)$$

$$-\delta W_{sf} = dH - du - \delta W_{nf}$$

$$-\delta W_{sf} = \delta u + pv - \delta u - \delta W_{nf} \quad \therefore dH = du + pv$$

$$\delta W_{sf} = \delta W_{nf} - pv$$

$$\begin{aligned}\delta W_{sf} &= \delta W_{nf} - (p_2 v_2 - p_1 v_1) \\ \delta W_{sf} &= p_1 v_1 + \delta W_{nf} - p_2 v_2\end{aligned}\quad (2.73)$$

For non-flow process, $\delta W_{nf} = \int P dv$

Substituting the value of δW_{nf} in Eq. (2.73), we have

$$\delta W_{sf} = p_1 v_1 + \int P dv - p_2 v_2 = \int v dp \quad (2.74)$$

Note that the quantity of work required during the steady flow process is given by the shaded area in Fig. 2.14(a). This area can be obtained by using Eq. (2.74).

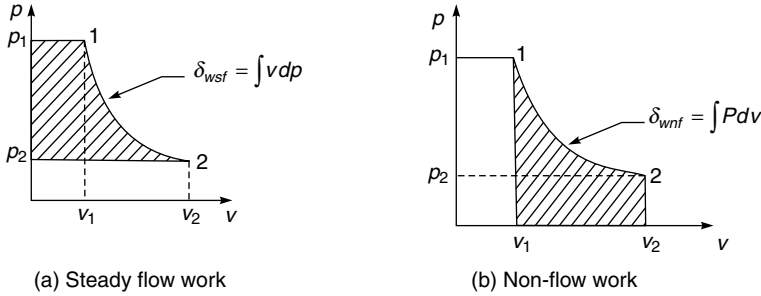


Fig. 2.14

In Eq. (2.74), the term $p_1 v_1$ represents the flow work brought into the control volume and the term $-p_2 v_2$ shows the work leaving the control volume by virtue of flowing fluid. Where, the term $\int P dv$ represents the work done within the control volume. The sum of three terms therefore, represents the work done during steady flow process and can be expressed as

$$\delta W_{sf} = \int v dp \quad (2.75)$$

The steady flow work δW_{sf} given in the above equation if applied to a process follows the law, $Pv^n = C$, can be calculated as

$$\begin{aligned}v &= \frac{C^{1/n}}{P^{1/n}} \\ \int_1^2 v dP &= C^{1/n} \int_1^2 P^{-1/n} dP = C^{1/n} \left[\frac{P^{1/n+1} - P_1^{1/n+1}}{(-1/n+1)} \right]\end{aligned}$$

Using $C^{1/n} = P_1^{1/n} v_1 = P_2^{1/n} v_2$

$$\delta W_{sf} = \int_1^2 v dP = n \left[\frac{P_2 v_2 - P_1 v_1}{n-1} \right] \quad (2.76)$$

Place the value δW_{sf} in Eq. (2.68) we get,

$$\delta Q - n \left(\frac{P_2 v_2 - P_1 v_1}{n-1} \right) = dH + \frac{dv^2}{2} + dgh$$

$$\begin{aligned}
 \text{Power output} &= \text{W.D./sec} = 550.68 \text{ kJ/kg} \times 4 \text{ kg/s} \\
 &= 2202.75 \text{ kJ/s} \\
 &= 2202.75 \text{ kW}
 \end{aligned}$$

Example 2.23 Lead is extruded slowly through a horizontal die. The pressure difference across the die is 154.45 MN/m^2 . Assuming there is no cooling through the die, determine the temperature rise of the lead. Assume the lead is incompressible and to have a density of 11360 kg/m^3 and specific heat capacity of 130 J/kg K .

Solution:

Consider the steady flow energy equation (2.63) applied to the given extrusion process:

$$u_1 + p_1v_1 + \frac{v_1^2}{2} + gh_1 + Q = u_2 + p_2v_2 + \frac{v_2^2}{2} + gh_2 + W \quad (\text{i})$$

Since the die is horizontal, there will be no change in the potential energy, therefore,

$$gh_1 = gh_2$$

The velocity of lead is also low, and hence kinetic energy can be neglected.

There is no cooling, and no external work is done, hence term Q and W can be neglected.

Now the energy equation becomes,

$$\begin{aligned}
 p_1v_1 + u_1 &= p_2v_2 + u_2 \\
 u_2 - u_1 &= p_1v_1 - p_2v_2 \quad (\text{ii})
 \end{aligned}$$

$$\begin{aligned}
 \text{Rise in temperature of any substance} &= \text{mass} \times \text{sp. heat} \times \text{temp. rise,} \\
 &= mC\Delta t
 \end{aligned}$$

Here in this particular case lead is incompressible, so energy due to rise in temperature of lead is associated with change in internal energy instead of work of expansion or compression

$$mC\Delta t = u_2 - u_1$$

Also, lead is incompressible, there is no change in volume,

$$v_1 = v_2 = v$$

Therefore, Eq. (ii) can be written as,

$$mC\Delta t = v(p_1 - p_2)$$

$$\Delta t = \frac{1 \text{ m}^3 \times 154.45 \times 10^6 \text{ N/m}^2}{11360 \text{ kg/m}^3 \times 130 \text{ J/kg K}}$$

$$\Delta t = 104.58 \text{ K}$$

Example 2.24 Air enters a gas turbine system with a velocity of 105 m/s and has a specific volume of $0.8 \text{ m}^3/\text{kg}$. The inlet area of the gas turbine system is 0.05 m^2 . At the exit the air has a velocity of 135 m/s and specific volume of $1.5 \text{ m}^3/\text{kg}$. In its passage through the turbine system, the specific enthalpy of the air is reduced by 145 kJ/kg and the air also has a heat transfer loss of 27 kJ/kg . Determine:

- (a) the mass flow rate of the air through the turbine system in kg/s;
- (b) the exit area of the turbine system in m²; and
- (c) the power developed by the turbine system in kW.

Solution:

Given data: $v_1 = 105 \text{ m/s}$ $v_2 = 135 \text{ m/s}$
 $v_{s_1} = 0.8 \text{ m}^3/\text{kg}$ $v_{s_2} = 1.5 \text{ m}^3/\text{kg}$
 $A_1 = 0.05 \text{ m}^2$ $H_1 - H_2 = 145 \text{ kJ/kg}$
 $Q = -27 \text{ kJ/kg}$

Mass of air at inlet,

$$m = \frac{A_1 v_1}{v_{s_1}} = \frac{0.05 \text{ m}^2 \times 105 \text{ m/s}}{0.8 \text{ m}^3/\text{kg}} = 6.56 \text{ kg/s}$$

Area at outlet of the turbine,

$$m = \frac{A_2 v_2}{v_{s_2}}$$

$$A_2 = \frac{m v_{s_2}}{v_2} = \frac{6.56 \text{ kg/s} \times 1.5 \text{ m}^3/\text{kg}}{135 \text{ m/s}}$$

$$A_2 = 0.072 \text{ m}^2$$

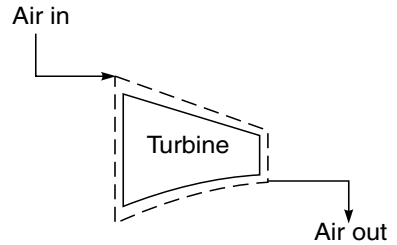


Fig. Ex. 2.24

Steady flow energy equation,

$$H_1 + \frac{v_1^2}{2} + Q = H_2 + \frac{v_2^2}{2} + W$$

$$W = (H_1 - H_2) + \frac{v_1^2 - v_2^2}{2} - Q$$

$$W = 145 \text{ kJ/kg} + \left(\frac{105^2 - 135^2}{2 \times 10^3} \right) \text{ m}^2/\text{s}^2 - 27 \text{ kJ/kg}$$

$$W = 114.4 \text{ kJ/kg}$$

$$\text{Power developed} = 114.4 \text{ kJ/kg} \times 6.56 \text{ kg/s}$$

$$= 750.46 \text{ kW}$$

EXAMPLE 2.25 Steam at 50 kPa and 473 K enters a nozzle with a velocity of 1 m/s at the rate of 1 kg/s and leaves as dry saturated steam at 10 kPa. The energy loss as heat from the nozzle is 10 kJ/s. Calculate the exit velocity of steam.

Solution:

Given data: $p_1 = 50 \text{ kPa}$, $T_1 = 473 \text{ K}$, $v_1 = 1 \text{ m/s}$, $m = 1 \text{ kg/s}$
 $p_2 = 10 \text{ kPa}$, $Q = 10 \text{ kJ/s}$

A compressor is a device, which takes in ambient air and compresses it to higher pressure. Compressor may or may not be insulated. Here, work is taken negative, since it is done on the gas to compress it.

In this case if compressor is insulated and change in kinetic and potential energies are zero then SFEE can be written as,

$$m(h_2 - h_1) = -W_s$$

Here gases are compressed adiabatically, therefore, it can be calculated as,

$$-W_s = \dot{m} C_p (T_2 - T_1)$$

$$W_s = \frac{mR\gamma}{\gamma - 1} (T_2 - T_1)$$

$$W_s = \frac{2 \times 0.294 \times 1.4}{(1.4 - 1)} (750 - 300)$$

$$W_s = 926.1 \text{ kW}$$

EXAMPLE 2.27 A compressor is required to provide 500 kPa and 483 K air for a stationary power plant. The air enters into the compressor at 101 kPa and 293 K. The outlet velocity is 25 m/s. Evaluate the work per unit mass need for an adiabatic compressor.

Solution

<i>Given data:</i>	$p_1 = 500 \text{ kPa}$	$T_1 = 483 \text{ K}$	$m = \text{unity}$
	$p_2 = 10 \text{ kPa}$	$T_2 = 295 \text{ K}$	$v_2 = 25 \text{ m/s}$

In this problem initial kinetic energy (v_1) is taken zero because in practice inlet area to the compressor is often larger than the outlet area. Also the compressor is adiabatic, so $Q = 0$. Thus, Eq. (2.63) reduces to

$$m(H_1 + gh_1) = m(H_2 + \frac{v_2^2}{2} + gh_2) + (-W_s)$$

Compressor work is done on the system, therefore, W_s is taken negative.

Assuming, $h_1 = h_2$, we get

$$mH_1 = mH_2 + \frac{v_2^2}{2} - W_s$$

$$W_s = m(H_2 - H_1) + \frac{v_2^2}{2}$$

$$W_s = mC_p(T_2 - T_1) + \frac{v_2^2}{2} \quad \text{since } (H = C_p dT)$$

Considering per unit mass flow of air, i.e., $m = 1 \text{ kg/s}$

$$W_s = [(1.005 \text{ kJ/kg K}) \times (483 - 295) \text{ K}] + \left[\frac{1}{2} \times \frac{25^2}{1000} \text{ kJ/kg} \right]$$

$$W_s = 189.25 \text{ kJ/kg}$$

EXAMPLE 2.28 A compressor compresses air of an initial specific volume of $0.89 \text{ m}^3/\text{kg}$ at the rate of $5 \text{ m}^3/\text{min}$. During compression the enthalpy of air increases by 85.4 kJ/kg . The compressor is considered to be water jacketed. While in operation heat is transferred to the cooling water at the rate of 250 kJ/min . If change in kinetic and potential energies are considered negligible, determine the power required to drive the compressor.

Solution:

Given data: sp. volume of air, $v_s = 0.89 \text{ m}^3/\text{kg}$

Rate of compression of air = $5 \text{ m}^3/\text{min}$

$$\therefore \text{Mass of air, } m = \frac{5}{0.89} = 5.61 \text{ kg/min}$$

Increase in enthalpy of air = 85.4 kJ/kg

Rate of heat transfer to cooling water, $Q = 250 \text{ kJ/min}$

In this case $\Delta KE = 0$, $\Delta PE = 0$

For this case SFEE (Eq. 2.63) can be written as

$$mH_1 + Q = mH_2 + (-W_s)$$

$$(-W_s) = m(H_2 - H_1) + Q = m\Delta H + Q$$

$$W_s = (5.61 \text{ kg/min}) \times (85.4 \text{ kJ/kg}) + 250 \text{ kJ/min}$$

$$W_s = 729.09 \text{ kJ/min or } 12.15 \text{ kJ/s}$$

$$W_s = 12.15 \text{ kW}$$

Ans.

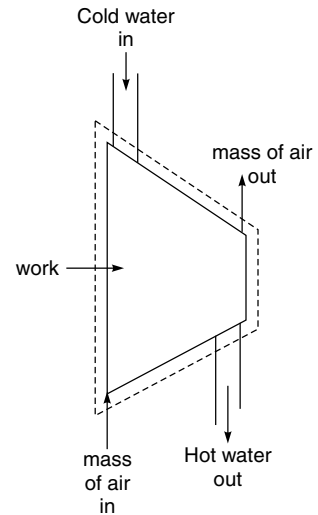


Fig. Ex. 2.28

EXAMPLE 2.29 A centrifugal air compressor compresses 30 kg/min of air from 110 kPa , 22°C to a final pressure of 345 kPa . The compression is polytropic with a polytropic index of 1.35 . The volume of air changes from $3 \text{ m}^3/\text{kg}$ to $0.85 \text{ m}^3/\text{kg}$. If the suction line diameter is 25 cm and discharge line diameter is 10 cm , and delivery connection is 10 m above the inlet, calculate power output.

Solution:

Polytropic index, $n = 1.35$

EXAMPLE 2.30 A steam turbine, designed to operate under steady flow condition, receives steam at the rate of 2.5 kg/s. The steam enters the turbine at a velocity of 50 m/s at an elevation of 6 m and has specific enthalpy of 2780 kJ/kg. The steam leaves the turbine at an elevation of 1 m with 100 m/s and enthalpy of outgoing steam is 2260 kJ/kg. The rate of heat transfer to the surrounding from the turbine is 5 kJ/s. Determine the power output of the turbine.

Solution:

Given data: $m = 2.5 \text{ kg/s}$

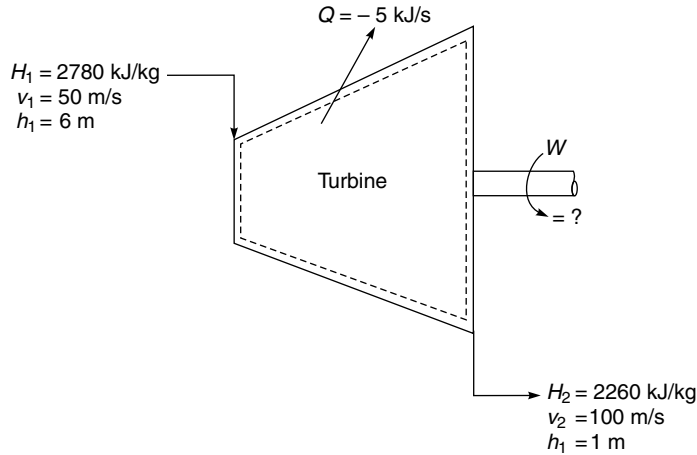


Fig. Ex. 2.30

Steady flow energy equation

$$Q - W = m(H_2 - H_1) + \frac{m}{2}(v_2^2 - v_1^2) + g(h_2 - h_1)$$

$$\begin{aligned} -5 \text{ kJ/s} - W &= (2.5 \text{ kg/s}) [(2260 \text{ kJ/kg}) - (2780 \text{ kJ/kg})] + \left(\frac{2.5 \text{ kg/s}}{2} \right) \times \left(\frac{(100 \text{ m/s})^2 - (50 \text{ m/s})^2}{1000} \right) \\ &\quad + \left[\frac{(1 \text{ m} - 6 \text{ m}) \times 9.81 \text{ m/s}^2}{1000} \right] 2.5 \text{ kg/s} \end{aligned}$$

$$-5 \text{ kJ/s} - W = 2.5 \text{ kg/s} [-(520 \text{ kJ/kg}) + (3.37 \text{ kJ/kg}) - (0.049 \text{ kJ/kg})]$$

$$-5 \text{ kJ/s} - W = (-516.6 \text{ kJ/kg}) 2.5 \text{ kg/s}$$

$$W = 5 \text{ kJ/s} + 1291.7 \text{ kJ/s}$$

$$W = 1296.7 \text{ kJ/s or } 1296.7 \text{ kW}$$

Ans.

EXAMPLE 2.31 A steam turbine is designed to operate under steady flow condition and receives steam at the rate of 1.5 kg/s. The steam entering at the rate of 60 m/s has pressure and temperature equal to 2000 kPa and 400°C, respectively. The steam leaves the turbine with a velocity of 150 m/s and is 98% dry, the pressure of the steam at the exit of the turbine is 100 kPa. The difference in elevation between the inlet to the outlet

is 1 m and rate of heat loss is 3000 kJ/min. Calculate: (i) power output of the turbine, and (ii) Diameter of inlet and exit pipe.

Solution:

Given data: $m = 1.5 \text{ kg/s}$
 $p_1 = 2000 \text{ kPa}$ $p_2 = 100 \text{ kPa}$, $v_2 = 150 \text{ m/s}$
 $T_1 = 400^\circ\text{C}$ $h_1 - h_2 = 1 \text{ m}$
 $v_1 = 60 \text{ m/s}$ Dryness fraction, $x = 0.98$
 $Q = 3000 \text{ kJ/min}$, 50 kJ/s

The steam turbine is a device in which steam expands, work is produced on the shaft of the turbine. Unlike the compressor the work done by the turbine is considered to be positive.

In steam turbine, most often elevation (height of inlet and outlet steam pipe) is neglected, hence $\Delta PE = 0$. Also the expansion of steam is considered to be adiabatic, thus $Q = 0$. But in this problem both elevation and heat transfer are considered. Therefore, in the steady flow energy equation. Equation 2.63 will be written as

$$Q - W = m(H_2 - H_1) + \frac{m}{2} (v_2^2 - v_1^2) + mg(h_2 - h_1)$$

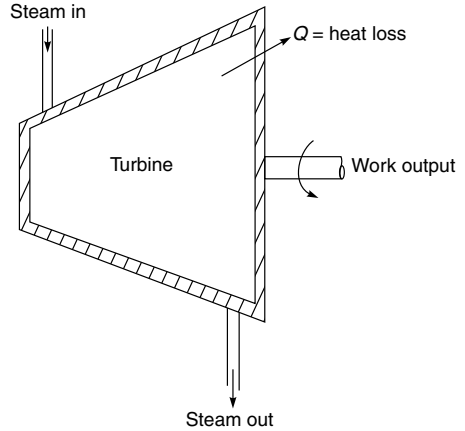


Fig. Ex. 2.31

Properties from Steam Table

State	$P_1 \text{ kPa}$	$T_1^\circ\text{C}$	x	$H \text{ kJ/kg}$	$v_1, \text{m}^3/\text{kg}$
Steam in	2000	400	-	3247.6	0.12010
Steam out	100	-	0.98	2630.3	1.660

Enthalpy H for steam leaving turbine will be

$$H = h_f + xh_{fg} = 417.44 + .98 (225.02) = 26303$$

$$-50 \text{ kJ/s} - W = (1.5 \text{ kg/s}) \left[(2630.3 \text{ kJ/kg}) - 3247.6 \text{ kJ/kg} \right] + \left\{ \frac{(150 \text{ m/s})^2 - (60 \text{ m/s})^2}{2 \times 1000} \right\} + \left\{ \frac{9.81 \text{ m/s}^2 \times (-1 \text{ m})}{1000} \right\}$$

$$-50 \text{ kJ/s} - W = (1.5 \text{ kg/s}) [(-617.3 \text{ kJ/kg}) + (9.45 \text{ kJ/kg}) - (0.0098 \text{ kJ/kg})]$$

$$W = 911.8 \text{ kJ/s or } 911.8 \text{ kW}$$

Ans.

$$Q - W = m(H_2 - H_1) + \frac{m}{2} (V_2^2 - V_1^2) + mg(h_2 - h_1)$$

$$-W = (18.9 \text{ kg/s}) \left[\{(2186 \text{ kJ/kg}) - (3501 \text{ kJ/kg})\} + \left\{ \frac{(129 \text{ m/s})^2 - (37.5 \text{ m/s})^2}{2 \times 1000} \right\} + \left\{ \frac{9.81(0 - 10 \text{ m})}{1000} \right\} \right]$$

$$-W = (18.9 \text{ kg/s}) [(-1315 \text{ kJ/kg}) + (7.61 \text{ kJ/kg}) - (0.0981 \text{ kJ/kg})]$$

$$-W = -24711.5 \text{ kJ/s}$$

$$-W = 24711.5 \text{ kW}$$

Ans.

Case II

If heat loss, $Q = 879 \text{ kJ/min}$

From above equation,

$$-Q - W = -24711.5 \text{ kJ/s}$$

$$-\frac{879}{60} \text{ kJ/s} - W = -24711.5 \text{ kJ/s}$$

$$W = 24696.8 \text{ kW}$$

Ans.

EXAMPLE 2.33 A gas turbine receives air at 1035 kPa and 555 K and discharges to pressure of 103.42 kPa. The actual temperature at discharge is 333 K. If sp. heat of the gas, C_p , can taken to be constant over this temperature range and is equal to 1.826 kJ/kg K, determine the work output of the turbine per kg of working fluid. At inlet condition the specific volume is $0.15 \text{ m}^3/\text{kg}$, and at outlet it is $0.92 \text{ m}^3/\text{kg}$.

Solution:

In this case $Q = 0$ and inlet and outlet pipes are placed at same datum level. Therefore, steady flow energy equation is reduced to

$$W = m(H_1 - H_2)$$

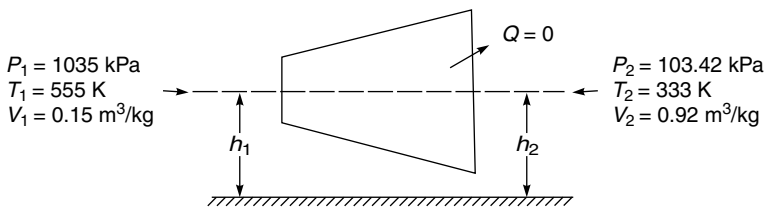


Fig. Ex. 2.33

Change in enthalpies

$$H_1 - H_2 = C_p(T_1 - T_2)$$

$$= (1.826 \text{ kJ/kg K}) \times [(555 \text{ K}) - (333 \text{ K})]$$

$$H_1 - H_2 = 405.3 \text{ kJ/kg}$$

Work out per kg,

$$W = (1 \text{ kg/s}) \times (405.3) \text{ kJ/kg} \quad (\text{Taking mass flow rate as unit mass per unit time})$$

$$W = 405.3 \text{ kJ/s or kW}$$

Ans.

EXAMPLE 2.34 Steam is generated in a boiler. The water at 7000 kPa as saturated liquid supplied to the boiler and leaves as superheated steam at 6 MPa and 500°C temperature. How much heat is added to the steam to generate 1 kg of steam under these conditions. Neglect kinetic and potential energy, also assume that there is no heat loss.

Solution:

Boiler, condenser and heat exchanger are the devices. Their objective is to transfer heat energy therefore heat transfer rate cannot be taken zero.

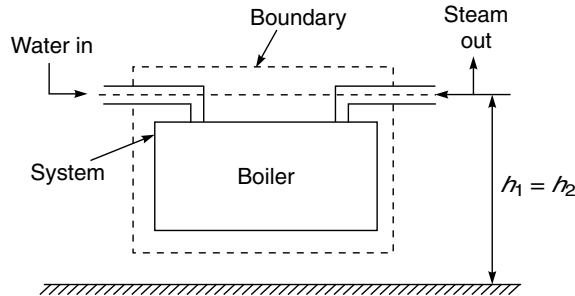


Fig. Ex. 2.34

If change in kinetic energy and potential energies are negligible, as in present case, then

$$V_1 = V_2 = 0$$

and

$$h_1 = h_2$$

There is no shaft work, therefore,

∴

$$W_S = 0$$

Therefore, steady flow energy equation can be expressed as,

$$Q = m(H_2 - H_1)$$

Properties from Steam Table

State	Pressure kPa	Temperature °C	Enthalpy kJ/kg
Entry	7000	–	1266.9
Exit	6000	500	3422.12

$$Q = (1 \text{ kg}) \times [(3422.12 \text{ kJ/kg}) - (1266.9 \text{ kJ/kg})]$$

$$Q = 2155 \text{ kJ}$$

Ans.

EXAMPLE 2.35 A heat exchanger is used to cool 182 kg/min of lubricating oil. Hot oil enters at 102°C and leaves at 52°C. The sp. heat of oil is 1.59 kJ/kg K. Cooling water enters the unit at 15.5°C and leaves a 32.2°C. If sp. heat of the water is 1.8723 kJ/kg K, determine the required water flow rate if heat losses are negligible.

For this case steady flow energy equation is

$$u_1 + p_1v_1 + Q = u_2 + p_2v_2$$

$$Q = (u_2 - u_1) + (p_2v_2 - p_1v_1)$$

$$Q = C_v(T_2 - T_1) + (p_2v_2 - p_1v_1)$$

$$Q = (0.599 \text{ kJ/kg K})[577 \text{ K} - 783 \text{ K}] + [(524 \text{ kPa} \times 0.24 \text{ m}^3/\text{kg}) - (689 \text{ kPa} \times 0.25 \text{ m}^3/\text{kg})]$$

$$Q = -123.39 \text{ kJ/kg} - 46.49 \text{ kJ/kg}$$

$$Q = -169.88 \text{ kJ/kg}$$

Ans.

EXAMPLE 2.37 If the pipe referred to in Example 2.36 is a vertical run of pipe such that section 2-2 is 30 m above section 1-1 determine the direction and magnitude of the heat transfer.

Solution:

Given data: $h_1 = 0$

$$h_2 = 30 \text{ m}$$

Other parameters are same as in the previous example.

Steady flow energy equation

$$u_1 + p_1v_1 + Q = u_2 + p_2v_2 + h_2$$

$$Q = (u_2 - u_1) + (p_2v_2 - p_1v_1) + h_2$$

From previous example

$$u_2 - u_1 = C_v(T_2 - T_1) = -123.39 \text{ kJ/kg}$$

$$p_2v_2 - p_1v_1 = -46.49 \text{ kJ/kg}$$

For given value of h_2 , PE is

$$PE = h_2 = \frac{gh_2}{100} \text{ kJ/kg} = \frac{(9.81 \text{ m/s}^2) \times (10 \text{ m})}{1000}$$

$$PE = h_2 = 0.0981 \text{ kJ/kg}$$

$$Q = -123.39 \text{ kJ/kg} - 46.49 \text{ kJ/kg} + 0.0981 \text{ kJ/kg}$$

$$Q = -169.78 \text{ kJ/kg}$$

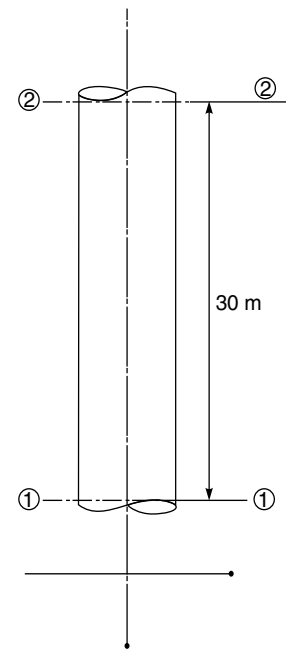


Fig. Ex. 2.37

Ans.

EXAMPLE 2.38 Determine the power required to drive a pump which extract the water from the deep well at the rate of $0.016 \text{ m}^3/\text{s}$. Water is extracted from 65 m deep well and raises 4 m above the ground. The diameter of the pipe is 18 cm at the inlet and 12 cm at exit. Neglect heat and frictional losses.

About System

The centrifugal pump is a device that transfers the mechanical energy of a motor or an engine into potential energy of a liquid. These devices pump the liquid from lower level to higher level and therefore power consuming devices.

Here work done is taken as negative because it is done on the system,

$$W = -ve$$

Also $Q = 0$ and $\Delta U = 0$ as there is no change in temperature and sp. volume of water.

$$v_1 = v_2 = v$$

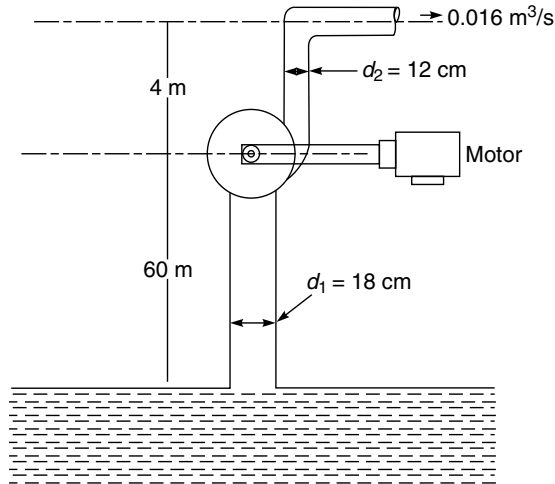


Fig. Ex. 2.38

Therefore, steady flow energy equation reduces to,

$$W = m \left[p_2 v - p_1 v + \frac{V_2^2 - V_1^2}{2} + (h_2 - h_1)g \right]$$

Water flow rate, $q = 0.016 \text{ m}^3/\text{s}$

Mass flow rate of water $m = (0.016 \text{ m}^3/\text{s}) \times (1000 \text{ kg}/\text{m}^3) = 16 \text{ kg}/\text{s}$

Steady flow energy equation,

$$Q - W = m \left[(H_2 - H_1) + \left(\frac{V_2^2 - V_1^2}{2} \right) + g(h_2 - h_1) \right]$$

$$\text{Velocity at inlet, } V_1 = \frac{q}{A_1} = \frac{0.016 \text{ m}^3/\text{s}}{\frac{\pi}{4} (0.18 \text{ m})^2} = 0.628 \text{ m/s}$$

$$\text{Velocity at exit, } V_2 = \frac{q}{A_2} = \frac{0.016 \text{ m}^3/\text{s}}{\frac{\pi}{4} (0.12 \text{ m})^2} = 1.41 \text{ m/s}$$

Velocity at exit,

$$V_2 = \frac{q}{A_2} = \frac{(0.075 \text{ m}^3/\text{kg})}{\frac{\pi}{4} (0.1 \text{ m})^2} = 9.54 \text{ m/s}$$

1. Change in internal energy is zero, since temperature remains constant,

$$\Delta u = 0$$

2. Change in kinetic energy,

$$\Delta \text{KE} = \frac{V_2^2 - V_1^2}{2 \times 1000} = \frac{(9.54 \text{ m/s})^2 - (4.24 \text{ m/s})^2}{2 \times 1000} = 0.036 \text{ kJ/kg}$$

3. Change in potential energy,

$$\Delta \text{PE} = \frac{g(h)}{1000} = \frac{(9.81 \text{ m/s})^2 - (9 \text{ m})}{1000} = 0.088 \text{ kJ/kg}$$

4. Change in flow work,

$$\begin{aligned} \Delta p v &= p_2 v_2 - p_1 v_1 = v(p_2 - p_1) \\ &= (0.0015 \text{ m}^3/\text{kg}) \times (420 \times 10^3 \text{ N/m}^2 - 80 \times 10^3 \text{ N/m}^2) \\ &= 0.51 \text{ kJ/kg} \end{aligned}$$

Substituting the value of each term in steady flow energy equation we get,

$$Q - W = 50 [0 + 0.51 + 0.036 + 0.088] = 31.7 \text{ kJ/s}$$

or

$$W = 31.7 \text{ kW}$$

Ans.



2.11 NON-STEADY FLOW PROCESSES

In many engineering problems the rate of mass flow in and out of the system is not same. Similarly, amount of heat transfer to or from the system and or the rate at which work is done on or by the fluid is not necessarily constant with time. Therefore, in such cases the total energy of the system within the boundary is no longer constant, such processes are called non-steady flow processes.

For such a process energy equation can be written as:

$$\left[\begin{array}{c} \text{Energy entering} \\ \text{system} \end{array} \right] - \left[\begin{array}{c} \text{Energy leaving} \\ \text{system} \end{array} \right] = \left[\begin{array}{c} \text{Increase of energy} \\ \text{stored in the system} \end{array} \right]$$

While writing steady flow energy Eq. (2.65) if we consider that all the energy and mass of flowing fluid are time dependent, then it becomes non-steady flow equation and can be expressed as:

$$\left[\delta m_1 \left(u_1 + \frac{V_1^2}{2} + gh_1 \right) + \delta Q + \delta m_1 p_1 v_1 \right] - \left[\delta m_2 \left(u_2 + \frac{V_2^2}{2} + gh_2 \right) + \delta W + \delta m_2 p_2 v_2 \right] = \delta E \quad (2.78)$$

During finite time the total work done is given by $\sum \delta W = W$, and the total heat out transferred is given by $\sum \delta Q = Q$. Also the mass flow in and out will be m_1 and m_2 , respectively.

Let the initial and final masses and energies within the system boundaries be $m_1 u_1$ and $m_2 u_2$ respectively

$$\therefore \quad \sum \delta E = m_2 u_2 - m_1 u_1$$

Substituting the values in Eq. (2.78), we have

$$\left[Q + m_1 \left(u_1 + \frac{V_1^2}{2} + gh_1 \right) + m_1 p_1 v_1 \right] - \left[W + m_2 \left(u_2 + \frac{V_2^2}{2} + gh_2 \right) + m_2 p_2 v_2 \right] = (m_2 u_2 - m_1 u_1) \quad (2.79)$$

Assuming that change in potential energy is zero and that of kinetic energy, $V^2/2$ is very small and can be neglected, we have:

$$\begin{aligned} [Q + m_1 u_1 + m_1 p_1 v_1] - [W + m_2 u_2 + m_2 p_2 v_2] &= m_2 u_2 - m_1 u_1 \\ [Q + m_1(u_1 + p_1 v_1)] - [W + m_2(u_2 + p_2 v_2)] &= m_2 u_2 - m_1 u_1 \end{aligned} \quad (2.80)$$

$$Q - W + m_1 H_1 - m_2 H_2 = m_2 u_2 - m_1 u_1 \quad (2.81)$$

Equation (2.80), is said to be non-steady flow energy equation.

These types of equation find their applications while charging or discharging the cylinder/tanks with gas. For example, LPG cylinder used in the kitchen, when its gas is utilized for cooking food, during the process mass and energy of gas change with time. Similarly, inverse/reverse is true while filling the cylinder with LPG. Other examples are, water flowing through water jacket in an I.C. engine cylinder and water flowing through cooling tower exhibits the property of non-steady flow energy.

2.11.1 Charging of Tank

Consider a tank, to be filled with a gas through a pipeline. The schematic diagram, Fig. 2.15 shows the condition of mass and energy in the tank and pipeline.

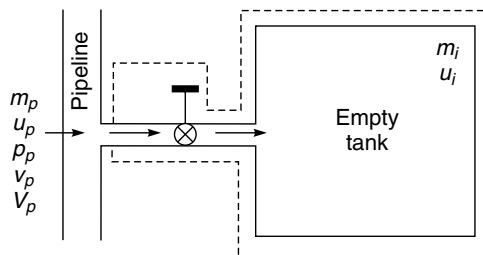


Fig. 2.15 Filling process of a tank

EXAMPLE 2.40 A single cylinder four-stroke internal combustion engine is running at 3000 rpm. During the suction stroke the volume flow rate of air is $0.055 \text{ m}^3/\text{s}$ and ambient temperature is 27°C . After completion of cycle, at the end of exhaust stroke 0.001 kg of residual gases left in the cylinder, the temperature of residual gases is 530°C . The suction process occurs at constant pressure and heat transfer is negligible. Determine the temperature of air inside the cylinder after completion of suction stroke.

Solution:

$$\begin{aligned} \text{Given data:} \quad N_1 &= 3000 \text{ rpm, } 50 \text{ rps} & T_2 &= 530^\circ\text{C, } 803 \text{ K} \\ V_a &= 0.055 \text{ m}^3/\text{s} & m_2 &= 0.001 \text{ kg} \\ T_1 &= 27^\circ\text{C, } 300 \text{ K} \end{aligned}$$

Assumption:

Density air, $\rho_{\text{air}} = 1000 \text{ kg/m}^3$

Sp. heat of mixture inside the cylinder $C_p = 1 \text{ kJ/kg}$

(i) The mass of air sucked during the suction stroke

Time required for suction strike:

$$t = \frac{1}{50 \times 4} \text{ s}$$

$$m_{\text{air}} = V_a \times \rho_{\text{air}} \times \text{time}$$

$$= (0.055 \text{ m}^3/\text{s}) \times (1000 \text{ kg/m}^3) \times \left(\frac{1}{50 \times 4} \text{ s} \right) = 0.275 \text{ kg}$$

Non steady flow energy Eq. (2.82) can be reduced to

$$(m_f - m_i) \left(H_p + \frac{V_p^2}{2} \right) + Q = (m_f u_f - m_i u_i) + W$$

Since, heat transfer is negligible, i.e., $Q = 0$ and kinetic energy is also negligibly low, then

$$(m_f - m_i)(H_p) = m_f u_f - m_i u_i + W \quad (\text{i})$$

Work done at constant pressure for delivering m kg of air

$$W = m_f \int_i^f p \cdot dv = m_f [p(v_f - v_i)]$$

$$W = m_f [p_f v_f - p_i v_i]$$

Substitute the value of W in above equation (i), we get

$$(m_f - m_i)(H_p) = m_f u_f - m_i u_i + m_f (p_f v_f - p_i v_i)$$

$$(m_f - m_i)(H_p) = m_f (u_f + p_f v_f) - m_i (u_i + p_i v_i)$$

$$(m_f - m_i)(H_p) = m_f H_f - m_i H_i$$

$$\text{or} \quad (m_f - m_i) C_p T_p = m_f C_p T_f - m_i C_p T_i$$

$$\text{or} \quad (m_f - m_i) T_p = m_f T_f - m_i T_i$$

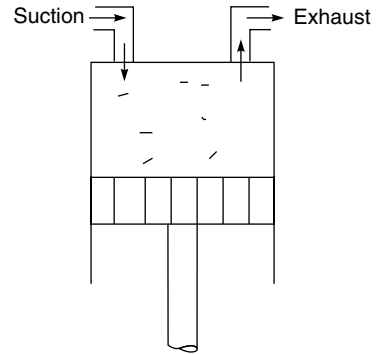


Fig. Ex. 2.40

Substituting the given values, we get,

$$(0.275 - 0.001) \times 300 = 0.275 \times T_f - 0.001 \times 803$$

$$T_f = 301.8 \text{ K}$$

EXAMPLE 2.41 Water is flowing through a pipe of 5.5 cm diameter with the velocity 0.49 m/s into an initially empty tank of 0.22 m³ capacity. Determine the flow rate of water through the pipe and time taken to fill the drum?

Solution:

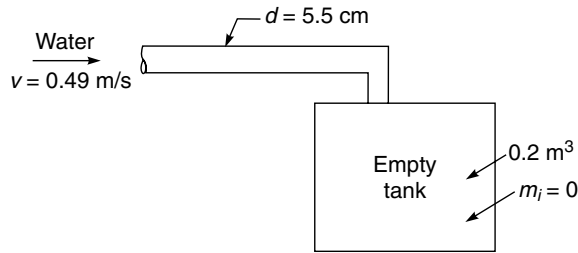


Fig. Ex. 2.41

Mass flow rate

$$m = \frac{VA}{v} = \frac{(0.49 \text{ m/s}) \times \pi(0.0275 \text{ m})^2}{(0.001 \text{ m}^3/\text{kg})} = 1.164 \text{ kg/s}$$

In this case the tank is initially empty ($m_i = 0$) and suppose water of mass m_f enters into the tank flowing with certain rate then energy equation can be expressed as,

$$m_f - m_i = m_{\text{in}} (t_2 - t_1) \quad (\text{i})$$

t_2 and t_1 are times after drum is full and initially when drum is empty,

$$m_f = \frac{\text{Capacity of drum}}{\text{Sp. volume of water}} = \frac{0.22 \text{ m}^3}{0.001 \text{ m}^3/\text{kg}} = 220 \text{ kg}$$

Substituting the values in above equation (i), we get,

$$220 \text{ kg} - 0 \text{ kg} = 1.164 \text{ kg/s} (t_2 - 0)$$

$$t_2 = \frac{220 \text{ kg}}{1.164 \text{ kg/s}} = 189 \text{ s} = 3 \text{ min } 9 \text{ seconds}$$

REVIEW QUESTIONS

1. Explain the law of conservation of energy.
2. What is the statement of the first law of thermodynamics and discuss its importance?
3. What is meant by internal energy of a gas?
4. Show that internal energy is the consequence of the first law of thermodynamics.

23. A mass of gas has an initial pressure of 1 bar and a temperature of 20°C. The temperature of the gas is now increased to 550°C while the volume remains constant. Determine the final pressure of the gas. [Ans. 2.81 bar]
24. A mass of air has an initial pressure of 1.3 MN/m², volume 0.014 m³ and temperature 135°C. It is expanded until its final pressure is 275 kN/m² and its volume becomes 0.056 m³. Determine:
 (a) the mass of air, and
 (b) the final temperature
 Take $R = 0.287 \text{ kJ/kg K}$ [Ans. (a) 0.155 kg; (b) 72°C]
25. A quantity of gas has an initial pressure and volume of 0.1 MN/m² and 0.1 m³, respectively. It is compressed to a final pressure of 1.4 MN/m² according to the law $pV^{1.26} = \text{constant}$. Determine the final volume of the gas. [Ans. 0.0123 m³ = 12.35 litres]
26. A quantity of gas has an initial volume and temperature of 1.2 litres and 150°C, respectively. It is expanded to a volume of 3.6 litres according to the law $pV^{1.4} = \text{constant}$. Determine the final volume of the gas. [Ans. 0°C]
27. A mass of gas has an initial pressure and temperature of 0.11 MN/m² and 15°C, respectively. It is compressed according to the law $pV^{1.3} = \text{constant}$ until the temperature is 90°C. Determine the final pressure of the gas. [Ans. 0.299 MN/m²]
28. A certain gas, having volume of 0.075 m³ at 60°C and 690 kPa, expands to 0.35 m³ according to the law $pV^{1.2} = C$. Determine: (a) the final temperature, (b) the work done, and (c) the heat received or rejected during the process.
 Take $\gamma = 1.4$ and $R = 0.286 \text{ kJ/kg K}$ for the gas.
 [Ans. (a) -60.7°C; (b) 68.65 kJ; (c) 34.33 kJ (received)]
29. 0.23 kg of air has an initial pressure of 1.7 MN/m² and a temperature of 200°C. It is expanded to a pressure of 0.34 MN/m² according to the law $pV^{1.35} = \text{constant}$. Determine the work transferred during the expansion.
 Take $R = 0.29 \text{ kJ/kg K}$. [Ans. 30.72 kJ]
30. One kilogram of nitrogen is compressed from the initial state $p_1 = 196 \text{ kPa}$ and $v_1 = 40^\circ\text{C}$ to the final pressure $p_2 = 1176 \text{ kPa}$. Find the final parameters, the work done, the change in internal energy, and the heat transferred if the expansion is isothermal, polytropic ($n = 1.25$), and isentropic.
 Take $R = 0.297 \text{ kJ/kg K}$ and $k_p = 1.038$ for nitrogen.

Process	Final temp °C	Final volume $v_2 \text{ m}^3/\text{kg}$	Work done kJ/kg	Change in internal energy kJ /kg	Heat transferred kJ/kg
Isothermal	40	0.079	-166	0	-166
Polytropic	175	0.1127	-160	100	-60
Isentropic	249	0.132	-155.7	155.7	0

31. Two kg of air at a pressure of 690 kPa occupies a volume of 0.3 m^3 . This air is then expanded to volume of 1.5 m^3 . Find the final temperature, the work done and the heat received or rejected by this air for each of the following methods of expansion:

- (a) at constant pressure,
- (b) isothermally, and
- (c) according to the law $pV^{1.2} = \text{constant}$.

Take $R = 0.287 \text{ kJ/kg K}$, $k_p = 1.00$ and $k_v = 0.7077$ for air.

[Ans. (a) $1,521^\circ\text{C}$, 824 kJ, 2854 kJ received;
 (b) 85.7°C , 331.5 kJ 331.5 kJ received;
 (c) -13°C , 283.5 kJ, 143 KJ received]

32. 1 kg of gas initially at 1180 kPa and 320°C expands adiabatically until its pressure is 102 kPa. The gas is then heated at constant volume until its temperature is 320°C and its pressure is found to be 206 kPa. Finally, the gas is compressed isothermally to original pressure of 1180 kPa. Determine: (a) the ratio of specific heats, (b) the temperature of the gas at the end of adiabatic expansion, and (c) the change in internal energy of the gas during adiabatic expansion.

Take k_p for gas = 1.005 kJ/kg K

[Ans. (a) 1.404; (b) 20.7°C ; (c) 214.3 kJ (loss of internal energy)]

33. A quantity of oxygen occupies 0.9 m^3 at 5°C and 147 kPa. It is compressed according to law $pV^{1.25} = \text{constant}$, until the pressure is 206 kPa. If the gas constant is $0.26 \text{ kJ per kg per degree kelvin}$, determine: (a) the final volume and temperature of the oxygen, (b) the heat interchange between the oxygen and its surroundings, stating whether this is a heat reception.

Take k_v for gas = 0.712 kJ/kg K

[Ans. (a) 0.109 m^3 , 198.2°C ; (b) -116 kJ (rejected)]

34. One kg of gas, at an initial pressure of 0.11 MN/m^2 and a temperature of 15°C , is compressed isothermally until the volume becomes 0.1 m^3 . Determine:

- (a) the final pressure,
- (b) the final temperature, and
- (c) the heat transfer.

If the compression had been adiabatic, determine:

- (d) the final pressure,
- (e) the final temperature, and
- (f) the work transfer.

For the gas, take $C_p = 0.92 \text{ kJ/kg K}$, $C_v = 0.66 \text{ kJ/kg K}$

[Ans. (a) 0.748 MN/m^2 ; (b) 15°C ; (c) -143.8 kJ ;
 (d) 1.591 MN/m^2 ; (e) 339.9°C ; (f) -214.4 kJ]

35. A gas has an initial pressure, volume and temperature of 140 kN/m^2 , 0.012 m^3 and 100°C , respectively. The gas is compressed to a final pressure of 2.8 MN/m^2 and volume of 0.0012 m^3 . Determine:

- (a) the index of compression if the compression is assumed to follow the law $pV^n = C$;
- (b) the final temperature of the gas;
- (c) the work transfer; and
- (d) the change of internal energy of the gas.

For the gas, take $R = 0.287$ kJ/kg K, $C_v = 0.717$ kJ/kg K.

[Ans. (a) $n = 1.3$; (b) 471°C ; (c) -5.6 kJ; (d) 4.2 kJ]

36. One kg of gas occupying 0.15 m³ at a pressure of 1.5 MPa is expanded at constant pressure to 0.25 m³. The gas is then expanded adiabatically to 0.9 m³. Calculate:
- (a) the temperature at the end of constant pressure expansion;
 - (b) the temperature and pressure at end of adiabatic expansion;
 - (c) the heat supplied during each portion of the process; and
 - (d) the total work done during the whole process.

Take $k_p = 1.013$ kJ/kg K and $k_v = 0.72$ kJ/kg K for gas.

[Ans. (a) $1,007^\circ\text{C}$; (b) 487°C , 247.4 kPa; (c) 526.75 kJ, No heat is supplied during adiabatic expansion; (d) 524.3 kJ]

37. 0.1 kg of gas is heated by means of an electric heater for a period of 10 min, during which time the pressure of the gas remains constant. The temperature of the gas is increased from 16°C to 78°C . The power used by the heater is 20 watts. Assuming no losses, determine:
- (a) the specific heat capacity of the gas at constant pressure;
 - (b) the specific heat capacity of the gas at constant volume;
 - (c) the characteristic gas constant; and
 - (d) the density of the gas at a temperature of 16°C and with a pressure of 0.12 MN/m²

For the gas, take $\gamma = 1.38$.

[Ans. (a) 1.935 kJ/kg K; (b) 1.402 kJ/kg K; (c) 0.533 kJ/kg K; (d) 0.78 kg/m³]

38. A classroom that normally contains 60 students is to be air conditioned with window air conditioner units of 5 kW rating. A person at rest may be assumed to dissipate heat at a rate of 360 kJ/h. There are 10 tube lights in the room of rating 40 W. The rate of heat transferred to the classroom through the walls and window is estimated to be 12000 kJ/h. If the room air is to be maintained at a constant temperature of 21°C , determine the number of window air conditioner units required for the classroom.

[Ans. 2 units]

39. A quantity of gas has mass of 0.2 kg and an initial temperature of 15°C . It is compressed adiabatically through a volume ratio of $4:1$. The final temperature after compression is 237°C . The work transfer during compression is 33 kJ. For the gas, determine:
- (a) the specific heat capacity at constant volume;
 - (b) the adiabatic index;

is 160 kJ/kg, and heat loss to the surroundings is 720 kJ/min. Assuming that inlet and discharge lines are at the same level, find:

(i) motor power required to drive the compressor, and

(ii) ratio of inlet to outlet pipe diameter. [Ans. (i) 52.78 kW (ii) $d_1/d_2 = 5.16$]

45. In a steady flow process, the working fluid flow at a rate of 240 kg/min. The fluid rejects 120 kJ/s passing through the system. The conditions of fluid at inlet and outlet are given as $V_1 = 300$ m/s, $p_1 = 6.2$ bar, $u_1 = 21000$ kJ/kg, $v_1 = 0.37$ m³/kg and $V_2 = 150$ m/s, $p_2 = 1.3$ bar, $u_2 = 1500$ kJ/kg, $v_2 = 1.2$ m/kg. The suffix 1 indicates the conditions at inlet and 2 indicates at outlet of the system. Neglecting the change in potential energy, determine the power capacity of the system in MW.

[Ans. 2.7086 MW]