

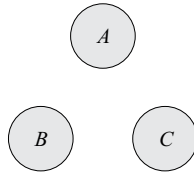
# Laws of Thermodynamics

## 2.1 SIGNIFICANCE AND SCOPE OF LAWS

Thermodynamics is based on the following four laws. These laws cannot be proved mathematically. Their validity stems from the fact that these laws have not been violated. These laws are based on experimental results and observations of common experience.

### 2.1.1 Zeroth Law

When a body  $A$  is in thermal equilibrium with body  $B$ , and also separately with body  $C$ , then bodies  $B$  and  $C$  will be in thermal equilibrium with each other. This is known as the zeroth law of thermodynamics.



**Fig. 2.1** Zeroth law

If,  $T_A = T_B$  and also  
 $T_A = T_C$ , then  
 $T_B = T_C$

where,  $T$  = Temperature.

This law deals with thermal equilibrium, concept of equality of temperature and forms the basis for all temperature measurements.

**Example:** If body  $C$  is the thermometer, body  $B$  is the reference temperature (i.e., triple point of water), body  $A$  is the unknown temperature, the thermometer compares the unknown temperature with the known reference temperature.

### 2.1.2 First Law

Energy can neither be created nor destroyed; it is always conserved. However, it can change from one form to another.

The first law deals with the conservation of energy and introduces the concept of internal energy. In general, all thermal machines are designed from first law of thermodynamics,

$$Q_{1-2} - W_{1-2} = dU = U_2 - U_1$$

For a closed system where changes in kinetic energy and potential are negligible, the difference in heat supplied and work done is stored as internal energy.

### 2.1.3 Second Law

It is the directional law of energy and also the law of degradation of energy. It is based on the following statements.

#### (a) Kelvin-Planck statement

“It is impossible to construct an engine working on a cyclic process, whose sole purpose is to convert heat energy from a single reservoir into an equivalent amount of work.” No cyclic engine can convert whole of heat into equivalent work. Second law dictates limits on the conversion of heat into work.

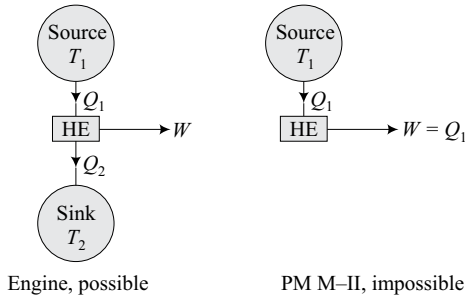


Fig. 2.2 Kelvin-Planck statement

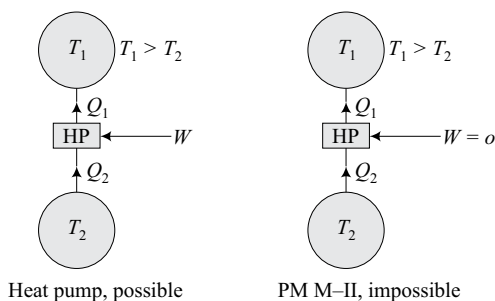
#### (b) Clausius statement

“It is impossible for a self-acting machine, working in a cyclic process, to transfer heat from a body at a lower temperature to a body at a higher temperature without the aid of an external energy”.

Heat cannot flow itself from a cold body to a hot body without expenditure of mechanical work. Second law deals with the direction of flow of heat energy.

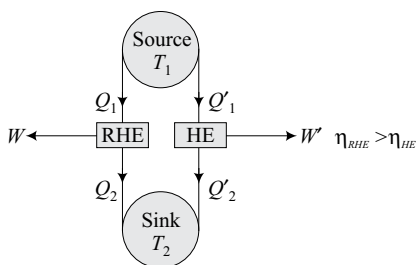
#### (c) Carnot statement

“No heat engine operating in a cycle between two given thermal reservoirs, with fixed temperatures, can be more efficient than a reversible engine operating between the same thermal reservoirs”.



**Fig. 2.3** Clausius statement

Second law provides a yardstick to evaluate the performance of an engine or a device. The portion of heat energy which is not available for conversion into work is measured by entropy.



**Fig. 2.4** Carnot statement

### 2.1.4 Third Law

Entropy has zero value at absolute zero temperature. The third law defines the absolute zero of entropy.

“At absolute zero temperature, the entropy of all homogeneous crystalline (condensed) substances in a state of equilibrium becomes zero”.

This law is used in chemical engineering for measurement of chemical affinity, analysis of chemical equilibrium and study of behaviour of solids at very low temperatures.

$$\text{As } T \rightarrow 0, \quad S \rightarrow 0.$$

## 2.2 CLASSIFICATION OF THERMODYNAMIC PROCESSES

The thermodynamic processes are grouped into:

### 2.2.1 Non-flow Processes

The processes occurring in a closed system where there is no transfer of mass across the boundary are called non-flow processes. In such processes, the energy in the form of heat and work crosses the boundary of the system.

The heating (or cooling) and expansion (or compression) of a gas may be performed in the following ways:

**(a) Reversible non-flow processes**

- (i) Constant volume process (or isochoric process)
- (ii) Constant pressure process (or isobaric process)
- (iii) Hyperbolic process ( $p\nu = c$ )
- (iv) Constant temperature process (or isothermal process)
- (v) Adiabatic process (or isotropic process), and
- (vi) Polytropic process.

**(b) Irreversible non-flow process**

Free expansion process (or unrestricted process)

### 2.2.2 Flow Processes

The processes in open systems permit the transfer of mass to and from the system. Such processes are called flow processes. The mass enters the system and leaves after exchanging energy.

The flow processes may:

**(a) Steady flow process**

The mass flow rate, heat flow rate, workflow rate through the system remain constant and there is no change in any properties or chemical composition of the working fluid at any given point within the system. Nozzles, turbines, compressors and other thermal machines operate as steady flow process.

**(b) Unsteady flow process**

The filling and evacuation of vessels are unsteady flow processes.

The complete classification of thermodynamic processes is illustrated in Fig. 2.4. The governing equations are also given in the diagram.

## 2.3 FIRST LAW OF THERMODYNAMICS

The first law of thermodynamics was formulated on the basis of Paddle Wheel Experiment conducted by Joule. A number of experiments were conducted by him wherein a paddle wheel was rotated by different forms of inputs. His findings were that the work expended was proportional to increase in thermal energy.

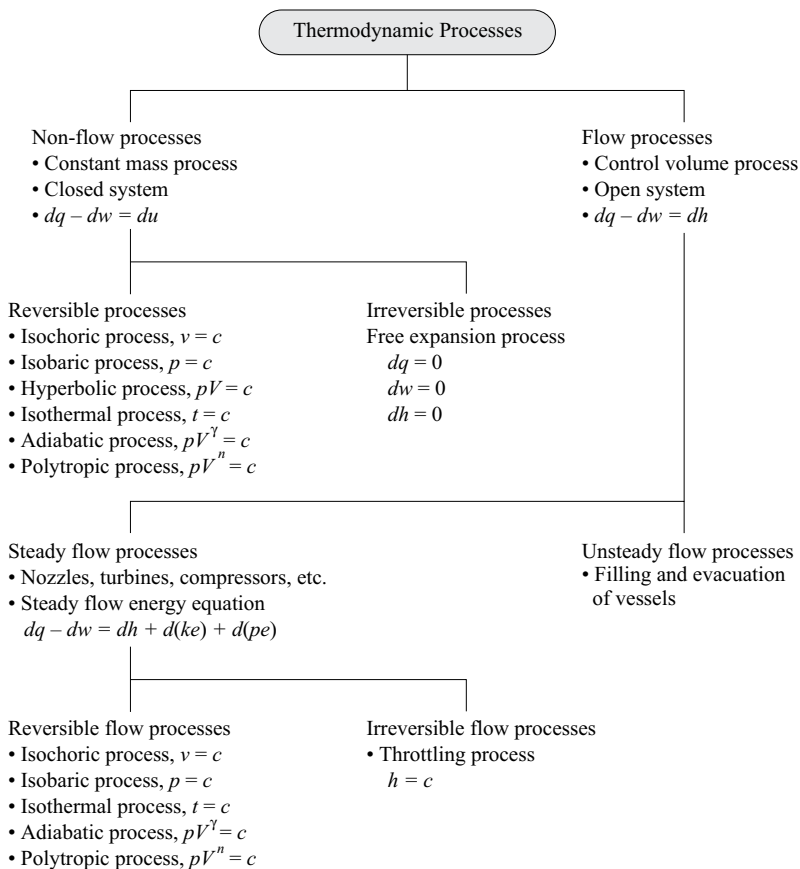
$$Q \propto W$$

or 
$$Q = \frac{W}{J}$$

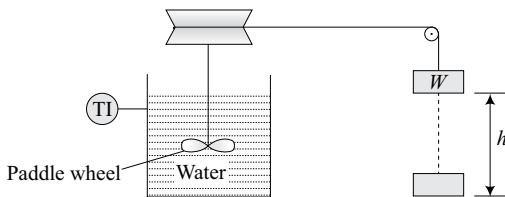
where,  $J$  = proportionality constant called mechanical equivalent.

In SI system,

$$J = 1$$



**Fig. 2.5** Classification of thermodynamic processes



**Fig. 2.6** Joule's paddle wheel experiment

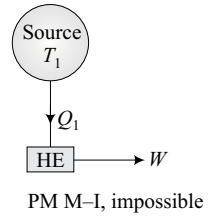
The first law of thermodynamics states that work and heat are mutually convertible. The present tendency is to include all forms of energy.

The first law can be stated in many ways:

1. Energy can neither be created nor destroyed; it is always conserved. However, it can change from one form to another.

2. All energy that goes into a system comes out in some form or the other. Energy does not vanish and has the ability to be converted into any other form of energy.
3. If the system is carried through a cycle, the summation of work delivered to the surroundings is equal to summation of heat taken from the surrounding.
4. Total energy of an isolated system, in all its forms, remains constant.
5. No machine can produce energy without corresponding expenditure of energy. It is impossible to construct a perpetual motion machine of the first kind.

$$Q_1 \neq 0$$



**Fig. 2.7** PMM-1

### 2.3.1 Limitations of First Law of Thermodynamics

There are some important limitations of First Law of Thermodynamics.

1. When a closed system undergoes a thermodynamic cycle, the net heat transfer is equal to the net work transfer.

$$\sum_i Q_i = \sum_i W_i$$

The cyclic integral of heat transfer is equal to cyclic integral of work transfer.

$$\oint \delta Q = \oint \delta W$$

where  $\oint$  stands for cyclic integral (integral around complete cycle),  $\delta Q$  and  $\delta W$  are small elements of heat and work transfer and have same units.

The law neither specifies the direction of flow of heat and work nor gives any conditions under which energy transfers can take place.

2. The heat energy and mechanical work are mutually convertible. The mechanical energy can be fully converted into heat energy but only a part of heat energy can be converted into mechanical work. Therefore, there is a limitation on the amount of conversion of one form of energy into another form.

### 2.3.2 Applications of First Law to Thermodynamic Processes

According to First law, when a system undergoes a thermodynamic process (change of state) both heat and work transfer takes place. The net (difference) energy transfer is stored within the system and is called stored energy or total energy of he system.

$$\delta Q - \delta W = dE.$$

where,  $E$  is the extensive property and represents the total energy of the system at a given state.

$$Q_{1-2} - W_{1-2} = E_2 - E_1$$

$Q$ ,  $W$  and  $E$  have same units.

For a unit mass,

$$q_{1-2} - w_{1-2} = e_2 - e_1$$

where,  $Q_{1-2}$  = Heat transferred to the system during the process 1-2. It is positive when supplied to the system and negative when rejected by the system.

$W_{1-2}$  = Work done by the system on the surrounding during the process 1-2.

$E_1$  = Total energy of the system at state 1.

$$\begin{aligned} &= PE_1 + KE_1 + U_1 + FE_1 \\ &= mgz_1 + \frac{m_1 V_1^2}{2} + U_1 + p_1 V_1 \end{aligned}$$

$E_2$  = total energy of the system at state 2.

$$\begin{aligned} &= PE_2 + KE_2 + U_2 + FE_2 \\ &= mgz_2 + KE_2 + U_2 + p_2 V_2 \end{aligned}$$

$$\begin{aligned} \therefore Q_{1-2} - W_{1-2} &= E_2 - E_1 \\ &= (PE_2 + KE_2 + U_2 + FE_2) - (PE_1 + KE_1 + U_1 + FE_1) \\ &= (PE_2 - PE_1) + (KE_2 - KE_1) - (H_2 - H_1) \quad [\because H = U + FE] \\ &= mg(Z_2 - Z_1) + \frac{m}{2}(V_2^2 - V_1^2) + (H_2 - H_1) \end{aligned}$$

For unit mass,

$$q_{1-2} - w_{1-2} = g(Z_2 - Z_1) + \frac{V_2^2 - V_1^2}{2} + (h_2 - h_1)$$

**Case I** For a closed system, there is no flow energy (FE)

$$\therefore q_{1-2} - w_{1-2} = g(Z_2 - Z_1) + \frac{V_2^2 - V_1^2}{2} + (u_2 - u_1)$$

This is called non-flow energy equation.

**Case II** When there is no change in the potential energy of a closed system, i.e.,  $PE_2 = PE_1$

$$\therefore q_{1-2} - w_{1-2} = (KE_2 - KE_1) + (u_2 - u_1) = \frac{V_2^2 - V_1^2}{2} + (u_2 - u_1)$$

**Case III** Closed or non-flow thermodynamic system, when there is no change of  $PE$  and also there is no flow of mass into or out of a system, i.e.,

$$PE_2 = PE_1 \text{ and } KE_2 = KE_1$$

$$\therefore q_{1-2} - w_{1-2} = u_2 - u_1$$

**Case IV** Isolated system,

$$q_{1-2} = 0 \text{ ; } w_{1-2} = 0$$

$$e_2 = e_1 \text{ and } u_2 = u_1$$

This shows that the first law of thermodynamics is law of conservation of energy.

**Case V** Cyclic process

There is no change in the internal energy and stored energy is zero.

$$\oint \delta q = \oint \delta w.$$

## 2.4 FIRST LAW ANALYSIS OF PROCESSES FOR IDEAL GAS

For an ideal gas,

$$du = C_v dT$$

$$dh = C_p dT$$

$$pv = RT.$$

These equations are valid for all processes.

### 2.4.1 Constant Volume Process

The first law equation for a non-flow process when kinetic energy and potential energy are negligible:

$$\delta q - \delta w = du.$$

For a reversible process,

$$\delta w = p dv \text{ and } du = C_v dT$$

$$\therefore \delta q - p dv = C_v dT$$

For constant volume process

$$\delta w = p dv = 0$$

$$\therefore \delta q = C_v dT = du$$

$$\text{or } q_{1-2} = u_2 - u_1 = \int_1^2 C_v dT$$

If the value of  $C_v$  as a function of  $T$  is known, integral can be evaluated.

### 2.4.2 Constant Pressure Process

The first law equation is:



$$\begin{aligned} \delta q - \delta w &= du \\ \text{or} \quad \delta q &= du + p \, dv \\ &= d(u + pv) && [\because p = \text{constant}] \\ &= dh = C_p \, dT \\ \therefore q_{1-2} &= \int_1^2 C_p \, dT = h_2 - h_1 \end{aligned}$$

If the value of  $C_p$  as a function of temperature is known, integral can be evaluated.

### 2.4.3 Constant Temperature Process

The first law equation is:

$$\begin{aligned} \delta q - \delta w &= du \\ du &= C_v \, dT = 0 && (\because T \text{ is constant}) \\ \therefore \delta q &= p \, dv \\ q_{1-2} &= \int_1^2 p \, dv \end{aligned}$$

For an ideal gas,

$$\begin{aligned} pv &= RT \\ \therefore p &= \frac{RT}{v} \\ \therefore q_{1-2} &= w_{1-2} = \int_1^2 \frac{RT}{v} \, dv \\ &= RT \ln \frac{v_2}{v_1} \\ &= p_1 v_1 \ln \frac{v_2}{v_1} \end{aligned}$$

$$\text{Also,} \quad w_{1-2} = p_1 v_1 \ln \frac{p_1}{p_2} \quad (\because p_1 v_1 = p_2 v_2)$$

### 2.4.4 Adiabatic Process

The first law equation is,

$$\delta q - \delta q = du.$$

No heat leaves or enters the system.

$$\therefore 0 - \delta w = du$$

$$0 - p \, dv = C_v \, dT$$

$$\therefore dT = \frac{-p \, dv}{C_v} \quad \dots(a)$$

But,  $p v = RT$

Differentiating,

$$p \, dv + v \, dp = R \, dT$$

$$\therefore dT = \frac{p \, dv + v \, dp}{R} = \frac{p \, dv + v \, dp}{(C_p - C_v)} \quad [\because R = C_p - C_v] \quad \dots(b)$$

Equating (a) and (b) for  $dT$

$$\frac{-p \, dv}{C_v} = \frac{p \, dv + v \, dp}{(C_p - C_v)}$$

$$\therefore \frac{(C_p - C_v)}{C_v} = \frac{p \, dv + v \, dp}{-p \, dv} = -1 - \frac{v \, dp}{p \, dv}$$

$$\therefore \frac{C_p}{C_v} - 1 = -1 - \left[ \frac{v}{dv} \times \frac{dp}{p} \right]$$

or  $\gamma = - \left[ \frac{v}{dv} \times \frac{dp}{p} \right] \quad \left[ \because \frac{C_p}{C_v} = \gamma \right]$

$$\therefore \gamma \left[ \frac{dv}{v} \right] = - \frac{dp}{p}$$

or  $\gamma \frac{dv}{v} + \frac{dp}{p} = 0$

Integrating,

$$\gamma \ln v + \ln p = \text{constant}$$

$$\ln p v^\gamma = \ln (\text{constant})$$

$$p_1 v_1^\gamma = p_2 v_2^\gamma = p v^\gamma = \text{constant}$$

$$\frac{p_1}{p_2} = \left( \frac{v_2}{v_1} \right)^\gamma$$

$$\frac{v_1}{v_2} = \left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}}$$

But 
$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}$$

or 
$$\frac{v_1}{v_2} = \frac{T_1}{T_2} \times \frac{p_2}{p_1}$$

$\therefore$  
$$\left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}} = \frac{T_1}{T_2} \times \frac{p_2}{p_1}$$

or 
$$\frac{T_1}{T_2} = \left( \frac{p_2}{p_1} \right)^{\frac{1}{\gamma}} \times \frac{p_2}{p_1}$$

$$= \left( \frac{p_1}{p_2} \right)^{-\frac{1}{\gamma}+1}$$

$\therefore$  
$$\frac{T_1}{T_2} = \left( \frac{p_1}{p_2} \right)^{\frac{\gamma-1}{\gamma}}$$

## 2.4.5 Polytropic Process

The first law equation is

$$\delta q - \delta w = du$$

$$q_{1-2} - w_{1-2} = (u_2 - u_1)$$

$$\delta q - pdv = du$$

For polytropic process,

$$w_{1-2} = \frac{p_1 v_1 - p_2 v_2}{n-1} = \frac{R(T_1 - T_2)}{n-1}$$

$\therefore$  
$$q_{1-2} = \frac{R(T_1 - T_2)}{n-1} + Cv(T_2 - T_1)$$

$$= \frac{R(T_1 - T_2)}{n-1} + \frac{R}{\gamma-1}(T_2 - T_1) \quad \left[ \because Cv = \frac{R}{\gamma-1} \right]$$

1. The work done is zero as there is no expansion of boundary

$$W_{1-2} = 0$$

2. The system is insulated and there is no exchange of heat with the surrounding.

$$Q_{1-2} = 0$$

3. Applying first law of thermodynamics to the closed system,

$$Q_{1-2} - W_{1-2} = U_2 - U_1$$

$$0 - 0 = U_2 - U_1$$

$$\therefore U_2 = U_1$$

The internal energy of the system during free expansion remains constant.

4.  $U_2 - U_1 = m C_v (T_2 - T_1) = 0$

$$\therefore T_2 = T_1.$$

The process of free expansion is isothermal process.

5.  $H_2 - H_1 = m C_p (T_2 - T_1) = 0$

The enthalpy of the system remains constant.

## 2.5 SUMMARY OF THERMODYNAMIC RELATIONS FOR NON-FLOW PROCESSES

The thermodynamic relations for ideal gas in different non-flow processes are summarised in Table 2.1. The following relationships have been tabulated:

1. Governing equation or  $p$ - $v$ - $T$  relationship
2. Work done
3. Change of internal energy
4. Change of enthalpy
5. Heat exchanged.

The values given are for unit mass.

**Example 2.1:** 5 m<sup>3</sup> of air at 2 bar and 27°C is compressed upto 6 bar pressure following  $pv^{1.3} = \text{constant}$ . It is subsequently expanded adiabatically to 2 bar. Considering the two processes to be reversible, determine the net work. Also plot the process on  $T$ - $s$  diagram. [U.P.T.U. I sem, 2002-03].

**Solution:**

1. Draw  $T$ - $s$  diagram to show the following process:
  - (i) Process 1–2: Reversible polytropic.
  - (ii) Process 2–3: Reversible adiabatic

$$W_{2-3} = \frac{p_2 V_2 - p_3 V_3}{\gamma - 1} = \frac{6 \times 10^5 \times 2.1476 - 2 \times 10^5 \times 4.7}{0.4} = 8.714 \times 10^5 \text{ N/m}$$

5. Total work done

$$W_{1-3} = W_{1-2} + W_{2-3} = -9.62 \times 10^5 + 8.714 \times 10^5 = -0.906 \times 10^5 \text{ N/m} \\ = -90.6 \text{ kJ}$$

The work is done on the system.

**Example 2.2:** Two kg of an ideal gas is compressed adiabatically from pressure 100 kPa and temperature 220 K to a final pressure of 400 kPa. Calculate:

- (i) Initial volume
- (ii) Final volume and temperature
- (iii) Heat exchanged
- (iv) Change in internal energy.

Take  $C_p = 1 \text{ kJ/kg-K}$  and  $C_v = 0.707 \text{ kJ/kg-K}$ .

**Solution:**

(i) *Initial volume*

$$p_1 = 100 \text{ kPa} = 100 \times 10^3 \text{ N/m}^2$$

$$T_1 = 220 \text{ K}$$

$$m = 2 \text{ kg}$$

$$R = C_p - C_v = 1 - 0.707 = 0.293 \text{ kJ/kg-K} \\ = 293 \text{ J/kg-K.}$$

$$\text{Now, } p_1 V_1 = m R T_1$$

$$\therefore V_1 = \frac{m R T_1}{p_1} = \frac{2 \times 293 \times 220}{100 \times 10^3} = 1.29 \text{ m}^3$$

(ii) *Final volume and temperature*

$$\gamma = \frac{C_p}{C_v} = \frac{1}{0.707} = 1.414$$

$$p_2 = 400 \times 10^3 \text{ N/m}^2$$

$$\text{Now, } p_1 V_1^\gamma = p_2 V_2^\gamma$$

$$\therefore V_2 = V_1 \left( \frac{p_1}{p_2} \right)^{\frac{1}{\gamma}} = 1.29 \left( \frac{100}{400} \right)^{\frac{1}{1.414}} \\ = 0.484 \text{ m}^3$$

$$p_2 V_2 = m R T_2$$

$$\therefore T_2 = \frac{p_2 V_2}{m R} = \frac{400 \times 10^3 \times 0.484}{2 \times 293} = 330.4 \text{ K}$$

(iii) *Work performed*

$$\begin{aligned} W_{1-2} &= \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} = \frac{100 \times 10^3 \times 1.29 - 400 \times 10^3 \times 0.484}{1.414 - 1} \\ &= -156 \times 10^3 \text{ J} \\ &= 156 \text{ kJ.} \end{aligned}$$

Work is done on the system during compression process.

(iv) *Heat exchanged*

During adiabatic process, no heat is exchanged

$$\therefore Q_{1-2} = 0$$

(v) *Change in internal energy*

From first law of thermodynamics

$$\begin{aligned} dU &= dQ - dW = 0 - (-156) \\ &= \mathbf{156 \text{ kJ}} \text{ (increase)} \end{aligned}$$

**Example 2.3:** The internal energy of a certain substance is expressed by the equation:

$$u = 3.62 pv + 86$$

where,  $u$  is given in kJ/kg

$p$  is given in kPa and

$v$  is in  $\text{m}^3/\text{kg}$ .

A system composed of 5 kg of this substance expands from an initial pressure of 550 kPa and a volume of  $0.25 \text{ m}^3/\text{kg}$  to a final pressure of 125 kPa, in a process in which pressure and volume are related by  $pv^{1.2} = \text{constant}$ . If the expansion process is quasi-static, determine  $Q$ ,  $\Delta u$  and  $W$  for this process.

[U.P.T.U. II Sem., 2005-06]

**Solution:**

$$m = 5 \text{ kg}$$

$$p_1 = 550 \text{ kPa}$$

$$v_1 = 0.25 \text{ m}^3/\text{kg}$$

$$p_2 = 125 \text{ kPa}$$

$$n = 1.2$$

$$V_1 = mv_1 = 5 \times 0.25 = 1.25 \text{ m}^3$$

$$p_1 v_1^n = p_2 v_2^n$$

$$\therefore v_2 = v_1 \left( \frac{p_1}{p_2} \right)^{\frac{1}{n}} = 0.25 \left( \frac{550}{125} \right)^{\frac{1}{1.2}} = 0.859 \text{ m}^3/\text{kg}.$$

$$V_2 = mv_2 = 5 \times 0.859 = 4.295 \text{ m}^3$$

(i) *Work done*

$$\begin{aligned} W_{1-2} &= \frac{p_1 V_1 - p_2 V_2}{n - 1} = \frac{550 \times 1.25 - 125 \times 4.295}{1.2 - 1} \\ &= \frac{687.5 - 536.875}{0.2} = \mathbf{753.125 \text{ kJ}} \quad \text{Ans.} \end{aligned}$$

(ii) *Change of internal energy*

$$u_1 = 3.62 p_1 v_1 + 86$$

$$u_2 = 362 p_2 v_2 + 86$$

$$\begin{aligned} \therefore u_2 - u_1 &= 3.62 (p_2 v_2 - p_1 v_1) \\ &= 3.62 (125 \times 0.859 - 550 \times 0.25) \\ &= 3.62 (107.375 - 137.5) = -109 \text{ kJ/kg} \end{aligned}$$

$$\Delta U = m(u_2 - u_1) = 5 \times 109 = \mathbf{545 \text{ kJ (decrease) Ans.}}$$

(iii) *Heat exchange*

$$\begin{aligned} Q_{1-2} &= W_{1-2} + (U_2 - U_1) \\ &= 753.125 - 545 = \mathbf{208.125 \text{ kJ}} \quad \text{Ans.} \end{aligned}$$

**Example 2.4:** A cylinder contains  $0.5 \text{ m}^3$  of air at  $1.5 \text{ bar}$  and  $100^\circ\text{C}$ . It is compressed polytropically to a volume of  $0.125 \text{ m}^3$  and final pressure is  $9.0 \text{ bar}$ . Determine:

(i) The mass of air

(ii) The value of index 'n' for  $pv^n = \text{constant}$

(iii) Work done, and

(iv) Heat supplied

[U.P.T.U. II Sem., 2003-04]

**Solution:**

Data given,

$$V_1 = 0.5 \text{ m}^3$$

$$p_1 = 1.5 \text{ bar} = 150 \text{ kPa}$$

$$T_1 = 100^\circ + 273 = 373 \text{ K}$$

$$V_2 = 0.125 \text{ m}^3$$

$$p_2 = 9 \text{ bar} = 900 \text{ kPa.}$$

(i) *Mass of air*

Assume  $R = 0.287 \text{ kJ/kg-K}$  for air

$$p_1 V_1 = m R T_1$$

$$\therefore m = \frac{p_1 V_1}{R T_1} = \frac{150 \times 0.5}{0.287 \times 373} = \mathbf{0.7 \text{ kg.}} \quad \text{Ans.}$$

(ii) *Value of index*

$$p_1 V_1^n = p_2 V_2^n$$

$$\left( \frac{V_1}{V_2} \right)^n = \frac{p_2}{p_1}$$

$$n \ln \left( \frac{V_1}{V_2} \right) = \ln \frac{p_2}{p_1}$$

$$\therefore n = \frac{\ln \left( \frac{p_2}{p_1} \right)}{\ln \left( \frac{V_1}{V_2} \right)} = \frac{\ln \left( \frac{900}{150} \right)}{\ln \left( \frac{0.5}{0.125} \right)} = \frac{1.7917595}{1.3862944}$$

$$= \mathbf{1.29} \quad \text{Ans.}$$

(iii) *Work done*

$$W_{1-2} = \frac{p_1 V_1 - p_2 V_2}{n - 1} = \frac{150 \times 0.5 - 900 \times 0.125}{1.29 - 1} = \frac{75 - 112.5}{0.29}$$

$$= -129.3 \text{ kJ}$$

Work is done on the system during compression.

(iv) *Heat supplied*

$$Q_{1-2} = \frac{\gamma - n}{\gamma - 1} W_{1-2} = - \frac{1.4 - 1.29}{1.4 - 1} \times (129.3)$$

$$= -35.56 \text{ kJ.}$$

Heat is rejected by the system. **Ans.**



$$V_1 \approx V_2$$

$$KE_1 = KE_2$$

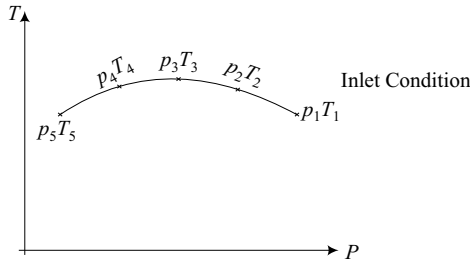
Applying steady flow energy equation to unit mass flow

$$h_1 + \frac{v_1^2}{2} + gz_1 + q_{1-2} = h_2 + \frac{v_2^2}{2} + gz_2 + w_{1-2}$$

$$\therefore h_1 = h_2$$

Therefore, throttling process is a constant enthalpy process

$$h_1 = h_2 = h_3 = h_4 = h_5 = \dots$$



**Fig. 2.10** Constant enthalpy process

If the readings of pressure and temperature of the experiment are plotted on  $T$ - $p$  diagram, a constant enthalpy line is obtained.

The slope of the constant enthalpy curve is called Joule-Thomson coefficient

$$\mu = \left( \frac{dT}{dp} \right)_h$$

For perfect gas,  $\mu = 0$ .

### Applications of Throttling

Although throttling is an energy loss process, it is used for the following.

1. To find out the dryness fraction of steam in a throttling calorimeter.
2. The speed of steam turbine is controlled in throttle governing.
3. Refrigeration effect (cooling) is obtained by throttling the refrigerant in a valve or capillary tube at inlet to evaporator.

$$\therefore h_1 = h_2$$

$$\therefore h_1 - h_2 = 0$$

$$Cp(T_1 - T_2) = 0$$

$$\therefore T_1 = T_2$$

$$Cv(T_1 - T_2) = 0$$

$$\therefore u_1 = u_2$$

For an ideal gas, throttling takes place at

1. Constant enthalpy
2. Constant internal energy
3. Constant temperature.

However, in a real gas or fluid (refrigerant),

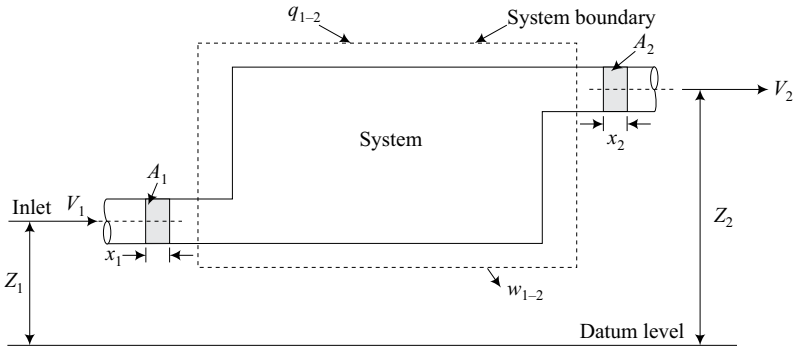
$$T_1 \neq T_2.$$

## 2.8 STEADY FLOW ENERGY EQUATION

According to the first law of thermodynamics, the total energy entering a system must be equal to total energy leaving the system. For unit mass,

$$e_1 = e_2$$

$$u_1 + p_1 v_1 + \frac{V_1^2}{2} + gz_1 + q_{1-2} = u_2 + p_2 v_2 + \frac{V_2^2}{2} + gz_2 + w_{1-2}$$



**Fig. 2.11** Steady flow process

where, suffix 1 is for inlet and 2 for outlet.

$u$  = specific internal energy

$pv$  = Flow work

$V$  = Fluid velocity

$Z$  = Height

$q_{1-2}$  = Heat exchange

$w_{1-2}$  = Work exchange

Now, 
$$h = u + pv$$

$$\therefore h_1 + \frac{V_1^2}{2} + qz_1 + q_{1-2} = h_2 + \frac{V_2^2}{2} + qz_2 + w_{1-2}$$

This is called steady flow energy equation. This equation may also be written as follows:

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + q(z_2 - z_1)$$

In differential form,

$$dq - dw = dh + d(ke) + d(pe)$$

**Case I.** If the effect of gravity can be neglected, i.e.,  $z_2 \approx z_1$

$$\begin{aligned} \therefore d(pe) &= 0 \\ dq - dw &= dh + (dke) \end{aligned}$$

or

$$q_{1-2} - w_{1-2} = (h_2 - h_1) + \left( \frac{V_2^2 - V_1^2}{2} \right)$$

**Case II.** If gravity can be neglected and the change in velocity is negligible, i.e.,  $V_2 \approx V_1$

$$\begin{aligned} d(pe) &= 0 \\ d(ke) &= 0 \\ \therefore q_{1-2} - w_{1-2} &= (h_2 - h_1). \end{aligned}$$

**Case III.** Applying the steady flow energy equation to a closed system (non-flow process)

$$\begin{aligned} d(pe) &= 0 \\ d(ke) &= 0 \\ p_1 v_1 &= 0 && \text{(Flow energy or displacement energy at} \\ &&& \text{inlet and outlet is zero)} \\ p_2 v_2 &= 0 \\ \therefore h_1 &= u_1 \\ h_2 &= u_2 \\ \therefore q_{1-2} - w_{1-2} &= u_2 - u_1 \end{aligned}$$

This is called energy equation for a non-flow process.

## 2.8.1 Equation of Continuity

The mass flow rate ( $\dot{m}$ ) of the working substance entering the system is same as leaving the system. The steady flow energy equation will be:

$$\dot{m} \left( h_1 + \frac{V_1^2}{2} + qZ_1 + q_{1-2} \right) = \dot{m} \left( h_2 + \frac{V_2^2}{2} + qZ_2 + W_{1-2} \right)$$

or

$$\dot{Q}_{1-2} - \dot{W}_{1-2} = \dot{m}(h_2 - h_1) + \frac{\dot{m}}{2}(V_2^2 - V_1^2) + \dot{m}g(z_2 - z_1)$$

where,  $\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$  (kg/s)

This is called equation of continuity.

**Example 2.6:** 0.5 kg/s of a fluid flows in a steady state process. The properties of fluid at entrance are measured as  $p_1 = 1.4$  bar, density =  $2.5 \text{ kg/m}^3$ ,  $u_1 = 920 \text{ kJ/kg}$  while at exit the properties are  $p_2 = 5.6$  bar, density =  $5 \text{ kg/m}^3$  and  $u_2 = 720 \text{ kJ/kg}$ . The velocity at entrance is  $200 \text{ m/s}$  while at exit is  $180 \text{ m/s}$ . It rejects  $60 \text{ kW}$  of heat and rises through  $60 \text{ m}$  during the flow. Find the change of enthalpy and the rate of work done. [U.P.T.U. II Sem., 2002-03]

**Solution:** The data given,

$$\dot{m} = 0.5 \text{ kg/s}$$

$$p_1 = 1.4 \text{ bar} = 140 \text{ kPa}$$

$$\rho_1 = 2.5 \text{ kg/m}^3 \quad \therefore v_1 = \frac{1}{\rho_1} = 0.4 \text{ m}^3/\text{kg}$$

$$u_1 = 920 \text{ kJ/kg}$$

$$p_2 = 5.2 \text{ bar} = 560 \text{ kPa}$$

$$\rho_2 = 5 \text{ kg/m}^3 \quad \therefore v_2 = \frac{1}{\rho_2} = \frac{1}{5} = 0.25 \text{ m}^3/\text{kg}$$

$$u_2 = 720 \text{ kJ/kg}$$

$$V_1 = 200 \text{ m/s}$$

$$V_2 = 180 \text{ m/s}$$

$$Q_{1-2} = 60 \text{ kW} = 60 \text{ kJ/s}$$

$$Z_2 - Z_1 = 60 \text{ m}$$

(i) *Change of enthalpy*

$$\begin{aligned} \Delta H &= \dot{m} (h_2 - h_1) = \dot{m} [(u_2 + p_2 v_2) - (u_1 + p_1 v_1)] \\ &= 0.5 [(720 + 560 \times 0.25) - (920 + 140 \times 0.4)] \\ &= 0.5 [860 - 976] = -58 \text{ kJ} \end{aligned}$$

There is a decrease in the enthalpy.

(ii) *Rate of work done*

Apply steady flow energy equation to the system

$$\begin{aligned} \dot{Q}_{1-2} - \dot{W}_{1-2} &= \dot{m} \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g(Z_2 - Z_1)}{10^3} \right] \\ -60 - \dot{W}_{1-2} &= 0.5 \left[ (860 - 976) + \frac{180^2 - 200^2}{2 \times 10^3} + \frac{9.81 \times 60}{10^3} \right] \\ &= 0.5 [-116 - 3.8 + 0.5886] = -59.6 \end{aligned}$$

$$\therefore \dot{W}_{1-2} = -60 + 59.6 = \mathbf{0.4 \text{ kW}} \quad \text{Ans.}$$

Work is done on the system.

## 2.9 APPLICATIONS OF STEADY FLOW ENERGY EQUATION

The steady flow energy equation can be applied to various energy systems and devices such as boilers, condensers, evaporators, nozzles, turbines, compressors etc.

### 2.9.1 Heat Exchanger

A heat exchanger is a device in which heat is transferred from one fluid to another. There are two steady flow streams, one of heating fluid and other fluid to be heated. The flow through a heat exchanger is characterized by,

1. No work exchange,

$$W_{1-2} = 0$$

2. No change in potential energy

$$Z_2 = Z_1$$

3. No change in kinetic energy

$$V_2 \approx V_1$$

4. Normally no external heat interaction, if heat exchanger is insulated.

Heat gained by cold fluid = Heat lost by hot fluid

$$\dot{m}_c(h_1 - h_2) = \dot{m}_h(h_4 - h_3)$$

where,  $\dot{m}_c$  and  $\dot{m}_h$  are flow rate of cold fluid and hot fluid respectively.

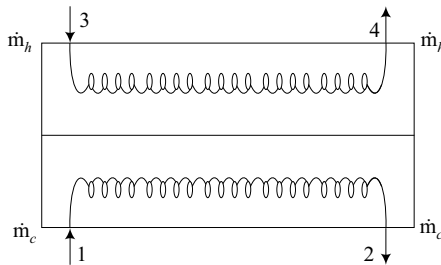


Fig. 2.12 Heat exchanger

The boiler, condenser, evaporator, etc. are designed in a similar manner by using steady flow energy equation. The heat supplied or removed is equal to change of enthalpy,

$$q_{1-2} = (h_2 - h_1)$$

**Example 2.7:** In a heat exchanger, 50 kg of water is heated per minute from 50°C to 110°C by hot gases which enter the heat exchanger at 250°C. If the flow rate of gases is 100 kg/min, estimate the net change of enthalpy. For water  $C_p = 4.186 \text{ kJ/kg-K}$  and for air,  $C_p = 1 \text{ kJ/kg-K}$ .

**Solution:**

$$\dot{m}_w = 50 \text{ kg/min}$$

$$t_{w1} = 50^\circ\text{C}$$

$$t_{w2} = 110^\circ\text{C}$$

$$\dot{m}_g = 100 \text{ kg/min}$$

$$t_{g1} = 250^\circ\text{C}$$

$$C_{p_w} = 4.186 \text{ kJ/kg-K}$$

$$C_{p_g} = 1 \text{ kJ/kg-K}$$

Using energy balance,

Heat gained by water = Heat lost by hot gas

$$\dot{m}_w C_{p_w} (t_{w2} - t_{w1}) = \dot{m}_g C_{p_g} (t_{g1} - t_{g2})$$

$$50 \times 4.186 (110 - 50) = 100 \times 1 (250 - t_{g2})$$

$$\therefore t_{g2} = 125^\circ\text{C}$$

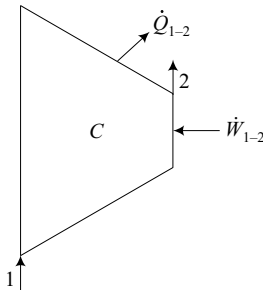
$$\begin{aligned} \text{Net change of enthalpy of gas} &= 100 \times 1 \times (250 - 125) \\ &= 12500 \text{ kJ/min} \end{aligned}$$

## 2.9.2 Compressor

Rotary and reciprocating compressors are used to increase the pressure of air and other gases. These are rotated by electric motor or engines. The main characteristics of the system are:

1. Work is done on the system and, hence, it is negative.

$\dot{W}_{1-2}$  is -ve.



**Fig. 2.13** Compressor

2. Potential energy and kinetic energy can normally be neglected.

$$Z_2 \approx Z_1$$

$$V_2 \approx V_1$$

3. Heat is lost from the compressor either by radiation or through a coolant (air or water).

The heat exchange is -ve.

$\dot{Q}_{1-2}$  is -ve or zero.

$$-\dot{Q}_{1-2} - (-\dot{W}_{1-2}) = \dot{m}(h_2 - h_1)$$

$$\therefore \dot{W}_{1-2} = \dot{Q}_{1-2} + \dot{m}(h_2 - h_1)$$

Therefore, work is done on the system to increase the enthalpy of the fluid.

**Example 2.8:** 0.8 kg of air flows through a compressor under steady state conditions. The properties of air at entry are: pressure 1 bar, velocity 10 m/s, specific volume 0.95 m<sup>3</sup>/kg and internal energy 30 kJ/kg. The corresponding values at exit are: 8 bar, 6 m/s, 0.2 m<sup>3</sup>/kg and 124 kJ/kg. Neglecting the change in potential energy, determine the power input and pipe diameter at entry and exit.

[U.P.T.U. CO. 2005-06]

**Solution:** The data given,

$$\dot{m} = 0.8 \text{ kg/s}$$

$$p_1 = 1 \text{ bar} = 100 \text{ kPa}$$

$$V_1 = 10 \text{ m/s}$$

$$v_1 = 0.95 \text{ m}^3/\text{kg}$$

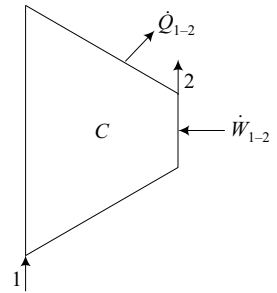
$$u_1 = 0.9 \text{ kJ/kg}$$

$$p_2 = 8 \text{ bar} = 800 \text{ kPa}$$

$$V_2 = 6 \text{ m/s}$$

$$v_2 = 0.2 \text{ m}^3/\text{kg}$$

$$u_2 = 124 \text{ kJ/kg}$$



$$\therefore h_1 = u_1 + p_1 v_1 = 30 + 100 \times 0.95 = 125 \text{ kJ/kg}$$

$$h_2 = u_2 + p_2 v_2 = 124 + 800 \times 0.2 = 284 \text{ kJ/kg}$$

(i) Apply steady flow energy equation to the compressor,

$$Q_{1-2} - W_{1-2} = \dot{m} \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3} (Z_2 - Z_1) \right]$$

$$Q_{1-2} = 0$$

$W_{1-2}$  is -ve.

$$Z_2 = Z_1$$

$$\therefore W_{1-2} = 0.8 \left[ (284 - 125) + \frac{6^2 - 10^2}{2 \times 10^3} \right] = 127 \text{ kW} \quad \text{Ans.}$$

(ii) Apply equation of continuity,

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

$$\therefore A_1 = \frac{\pi}{4} D_1^2 = \frac{\dot{m} v_1}{V_1}$$

$$\therefore D_1 = \sqrt{\frac{4 \dot{m} v_1}{\pi V_1}} = \sqrt{\frac{4 \times 0.8 \times 0.95}{\pi \times 10}} = 0.311 \text{ m}$$

$$= 311 \text{ mm} \quad \text{Ans.}$$

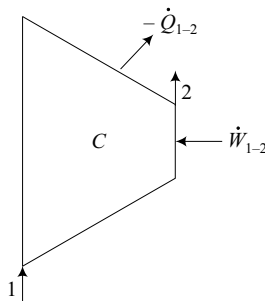
$$D_2 = \sqrt{\frac{4 \dot{m} v_2}{\pi V_2}} = \sqrt{\frac{4 \times 0.8 \times 0.2}{\pi \times 6}} = 0.184 \text{ m}$$

$$= 184 \text{ mm} \quad \text{Ans.}$$

**Example 2.9:** An air compressor compresses atmospheric air at 0.1 MPa and 27°C by ten times of inlet pressure. During compression, the heat loss to the surrounding is estimated to be 5% of compression work. Air enters the compressor with a velocity of 40 m/s and leaves with 100 m/s. Inlet and exit cross-sectional areas are 100 cm<sup>2</sup> and 20 cm<sup>2</sup> respectively. Estimate the temperature of air at exit from compressor and power input to the compressor.

[U.P.T.U. II Sem., 2001-02]

**Solution:**



1. Inlet conditions

$$p_1 = 0.1 \text{ MPa} = 0.1 \times 10^3 \text{ kN/m}^2$$

$$T_1 = 27^\circ\text{C} + 273 = 300 \text{ K}$$



$$V_1 = 40 \text{ m/s}$$

$$A_1 = 100 \text{ cm}^2 = 100 \times 10^{-4} \text{ m}^2$$

## 2. Outlet conditions

$$p_2 = 10 \times 0.1 \times 10^3 \text{ kN/m}^2 = 10^3 \text{ kN/m}^2$$

$$V_2 = 100 \text{ m/s}$$

$$A_2 = 20 \text{ cm}^2 = 20 \times 10^{-4} \text{ m}^2$$

## 3. Equation of continuity

The equation of state,

$$p_1 V_1 = mRT_1$$

or 
$$p_1 v_1 = RT_1$$

$$\begin{aligned} \therefore v_1 &= \frac{RT_1}{p_1} = \frac{0.287 \times 300}{100} \\ &= 0.861 \text{ m}^3/\text{kg} \quad [R = 0.287 \text{ kJ/kg-K for air}] \end{aligned}$$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{(100 \times 10^{-4}) \times 40}{0.861} = 0.4646 \text{ kg/s}$$

$$\dot{m} = \frac{A_2 V_2}{v_2}$$

$$\therefore v_2 = \frac{A_2 V_2}{\dot{m}} = \frac{(20 \times 10^{-4}) \times 100}{0.4646} = 0.43 \text{ m}^3/\text{kg}$$

$$p_2 v_2 = RT_2$$

$$T_2 = \frac{p_2 v_2}{R} = \frac{10^3 \times 0.43}{0.287} = 1498 \text{ K}$$

## 4. Steady flow energy equation

$$Q_{1-2} - W_{1-2} = \dot{m} \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3} (Z_2 - Z_1) \right]$$

$$-0.05 W_{1-2} - W_{1-2} = 0.43 \left[ 1.005(1498 - 300) + \frac{(100)^2 - (40)^2}{2 \times 10^3} + 0 \right]$$

$$W_{1-2} = \mathbf{546.86 \text{ kW} \quad \text{Ans.}}$$

### 2.9.3 Gas Turbine

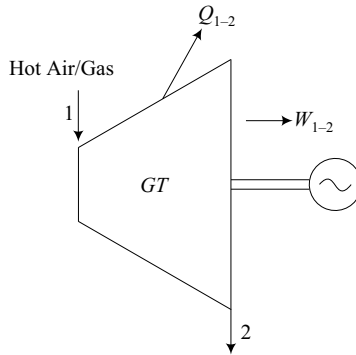
A gas turbine converts the heat energy of hot gases into mechanical work. A compressor driven by the gas turbine compresses air or gas to a higher pressure. The high pressure air or gas is heated by combustion of fuel. The high pressure and high temperature air or gas is admitted to a gas turbine. Power is produced in the turbine at the expense of enthalpy drop of the gas. The main characteristics of the system are:

1. The heat loss by radiation to the surrounding is  $-ve$ .
2. Normally,  $ke$  and  $pe$  are neglected.

$$V_2 \approx V_1$$

$$Z_2 = Z_1$$

$$\dot{W}_{1-2} = Q_{1-2} + \dot{m}(h_1 - h_2)$$



**Fig. 2.14** Gas turbine

**Example 2.10:** Air passes through a gas turbine system at the rate of 4.5 kg/s. It enters the turbine system with a velocity of 90 m/s and a specific volume of 0.85 m<sup>3</sup>/kg. It leaves the turbine system with a specific volume of 1.45 m<sup>3</sup>/kg. The exit area of the turbine system is 0.38 m<sup>2</sup>. In its passage through the turbine, the specific enthalpy of air is reduced by 200 kJ/kg and there is a heat loss of 40 kJ/kg. Determine:

- (i) The inlet area of turbine
- (ii) The exit velocity of air in m/s.
- (iii) Power developed by the turbine system in kW.

**Solution:**

1. *Inlet conditions*

$$\dot{m} = 4.5 \text{ kg/s}$$

$$V_1 = 90 \text{ m/s}$$

$$\begin{aligned} \therefore -W_{1-2} &= \dot{m} \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} \right] \\ &= 1 \left[ (1600 - 2900) + \frac{(150)^2 - (100)^2}{2 \times 10^3} \right] \\ &= -1300 + 6.25 \\ \therefore W_{1-2} &= 1293.75 \text{ kW} \end{aligned}$$

**Example 2.12:** A steam turbine operating under steady flow conditions receives 3600 kg of steam per hour. The steam enters the turbine at a velocity of 80 m/s an elevation of 10 m and specific enthalpy of 3276 kJ/kg. It leaves the turbine at a velocity of 150 m/sec, an elevation of 3 m and a specific enthalpy of 2465 kJ/kg. Heat losses from the turbine to the surrounding amount to 36 MJ/hr. Estimate the power output of turbine. [U.P.T.U. I/II Sem., 2001-02]

**Solution:**

1. *Inlet Data,*

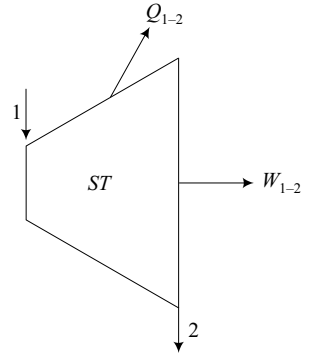
$$\dot{m} = 3600 \text{ kg/hour} = 1 \text{ kg/sec}$$

$$V_1 = 80 \text{ m/s}$$

$$Z_1 = 10 \text{ m}$$

$$h_1 = 3276 \text{ kJ/kg}$$

$$\begin{aligned} Q_{1-2} &= -36 \text{ MJ/hour} = -\frac{36 \times 10^3}{3600} \text{ kJ/sec} \\ &= -10 \text{ kJ/sec} \end{aligned}$$



2. *Outlet conditions,*

$$V_2 = 150 \text{ m/sec}$$

$$Z_2 = 3 \text{ m}$$

$$h_2 = 2465 \text{ kJ/kg}$$

3. *Steady flow energy equation,*

$$\begin{aligned} Q_{1-2} - W_{1-2} &= \dot{m} \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3} (Z_2 - Z_1) \right] \\ -10 - W_{1-2} &= 1 \left[ (2465 - 3276) + \frac{(150)^2 - (80)^2}{2 \times 10^3} + \frac{9.81}{10^3} (3 - 10) \right] \end{aligned}$$

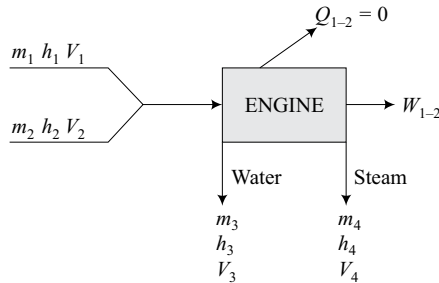
$$-10 - W_{1-2} = -811 + 8.05 - 0.06867$$

$$W_{1-2} = 793 \text{ kW} \quad \text{Ans.}$$

**Example 2.13:** The steam supply to an engine comprises two streams which mix before entering the engine. One stream is supplied at the rate of 0.01 kg/s with an enthalpy of 2950 kJ/kg and a velocity of 20 m/s. The other stream is supplied at the rate of 0.1 kg/s with an enthalpy of 2665 kJ/kg and a velocity of 120 m/s. At the exit from the engine the fluid leaves as two streams, one of water at the rate of 0.001 kg/s with an enthalpy of 421 kJ/kg and the other of steam. The fluid velocities at the exit are negligible. The engine develops a shaft power of 25 kW. The heat transfer is negligible. Evaluate the enthalpy of the second exit stream.

[U.P.T.U. II Sem., 2004-05]

**Solution:**



1. *Inlet data,*

$$m_1 = 0.01 \text{ kg/s}$$

$$h_1 = 2950 \text{ kJ/kg}$$

$$V_1 = 20 \text{ m/s}$$

$$m_2 = 0.1 \text{ kg/s}$$

$$h_2 = 2665 \text{ kJ/kg}$$

$$V_2 = 120 \text{ m/s}$$

Total inlet energy

$$\begin{aligned} E_i &= m_1 h_1 + \frac{m_1 V_1^2}{2 \times 10^3} + \frac{g}{10^3} Z_1 + Q_{1-2} + m_2 h_2 + \frac{m_2 V_2^2}{2 \times 10^3} + \frac{g}{10^3} Z_2 \\ &= 0.01 \times 2950 + \frac{0.01 \times (20)^2}{2 \times 10^3} + 0 + 0 + 0.1 \times 2665 + \frac{0.1 \times 120^2}{2 \times 10^3} + 0 \end{aligned}$$

$$E_i = 29.5 + 0.002 + 266.5 + 0.72 = 296.722 \text{ kJ}$$

$$m_i = 1 \cdot 0.01 + 0.1 = 0.11 \text{ kg/s}$$

3. The change of potential energy is negligible,

$$Z_2 = Z_1$$

4. The steady flow energy equation,

$$\frac{V_2^2 - V_1^2}{2} = (h_1 - h_2)$$

$$V_2 = \sqrt{2(h_1 - h_2) + V_1^2}$$

If  $V_2 \gg V_1$  and  $V_1$  can be neglected,

$$V_2 = \sqrt{2(h_1 - h_2)}$$

The mass flow rate,

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

The area of nozzle at the inlet and outlet can be estimated.

**Example 2.14:** In an isentropic flow through a nozzle, air flows at the rate of 600 kg/hr. At inlet to nozzle, pressure is 2 MPa and temperature is 127°C. The exit pressure is 0.5 MPa. If the initial air velocity is 300 m/s, determine:

- (i) Exit velocity of air, and
- (ii) Inlet and exit area of nozzle. (U.P.T.U. I Sem., 2000-01)

**Solution:**

1. *Inlet conditions,*

$$p_1 = 2 \text{ MPa} = 2 \times 10^3 \text{ kPa}$$

$$T_1 = 127^\circ\text{C} + 273 = 400 \text{ K}$$

$$\dot{m} = 600 \text{ kg/hr} = \frac{600}{3600} = \frac{1}{6} \text{ kg/s}$$

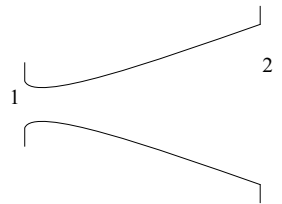
$$V_1 = 300 \text{ m/s}$$

$$Q_{1-2} = 0$$

2. *Outlet conditions,*

$$p_2 = 0.5 \text{ MPa} = 0.5 \times 10^3 \text{ kPa}$$

The flow through nozzle is isentropic,



$$\therefore \frac{T_2}{T_1} = \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}}$$

For air  $\gamma = 1.4$

$$\therefore T_2 = 400 \left( \frac{0.5}{2} \right)^{\frac{1.4-1}{1.4}} = 269.18 \text{ K}$$

$$W_{1-2} = 0$$

3. Steady flow energy equation,

$$Q_{1-2} - W_{1-2} = \dot{m} \left[ (h_2 - h_1) + \frac{V_2^2 - V_1^2}{2 \times 10^3} + \frac{g}{10^3} (Z_2 - Z_1) \right]$$

$$h_2 - h_1 = C_p(T_2 - T_1)$$

$$\therefore 0 - 0 = \frac{1}{6} \left[ 1.005(400 - 269.18) + \frac{V_2^2 - (300)^2}{2 \times 10^3} + 0 \right]$$

$$\therefore 0 = 131.474 + \frac{V_2^2 - (300)^2}{2 \times 10^3}$$

$$\therefore V_2 = \sqrt{131.474 \times 2 \times 10^3 + 300^2}$$

$$= \mathbf{594 \text{ m/s} \quad \text{Ans.}}$$

4. Equation of continuity

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{A_2 V_2}{v_2}$$

From equation of state,

$$p_1 v_1 = RT_1$$

$$v_1 = \frac{RT_1}{p_1} = \frac{0.287 \times 400}{2 \times 10^3} = 0.0574 \text{ m}^3/\text{kg}$$

For air,

$$R = 0.287 \text{ kJ/kg-K}$$

$$\therefore A_1 = \frac{\dot{m} V_1}{v_1} = \frac{\frac{1}{6} \times 0.0574}{300} = 3.1888 \times 10^{-5} \text{ m}^2$$

$$= 31.88 \text{ mm}^2$$

$$v_2 = \frac{RT_2}{p_2} = \frac{0.287 \times 269.18}{0.5 \times 10^3} = 0.1545 \text{ m}^3/\text{kg}$$

$$\therefore A_2 = \frac{\dot{m} v_2}{V_2} = \frac{\frac{1}{6} \times 0.1545}{594} = 4.335 \times 10^{-5} \text{ m}^2$$

$$= 43.35 \text{ mm}^2 \quad \text{Ans.}$$

The exit area of the nozzle is more than inlet area. Therefore, the nozzle is a convergent-divergent nozzle.

### 2.9.6 Diffuser

A diffuser has varying cross-section and reduces the velocity of the flowing fluid. There are two types of diffusers:

1. *Subsonic Diffuser*

The velocity of the fluid is less than sonic speed and the area of cross-section of diffuser increases from inlet to exit.

2. *Supersonic Diffuser*

The velocity of fluid is more than sonic velocity and the area of diffuser decreases.

The operating characteristics of a diffuser are similar to that for a nozzle.

**Example 2.15:** Water vapour at 90 kPa and 150°C enters a subsonic diffuser with a velocity of 150 m/s and leaves the diffuser at 190 kPa with a velocity of 55 m/s and during the process 1.5 kJ/kg of heat is lost to the surrounding. Determine:

- (i) The final temperature
- (ii) The mass flow rate
- (iii) The exit diameter, assuming the inlet diameter as 10 cm and steady flow.

[U.P.T.U. II Sem., 2001-02]

**Solution:**

1. *Inlet condition,*

$$p_1 = 90 \text{ kN/m}^2$$

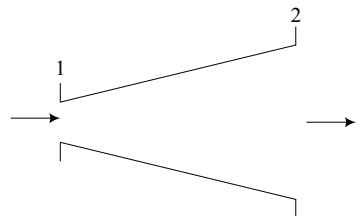
$$T_1 = 150^\circ\text{C} + 273 = 423 \text{ K}$$

$$V_1 = 150 \text{ m/s}$$

2. *Outlet condition,*

$$p_2 = 190 \text{ kN/m}^2$$

$$V_2 = 55 \text{ m/s}$$



For water vapours take  $C_p = 2.1 \text{ kJ/kg-K}$

## 3. Steady flow energy equation,

$$Q_{1-2} - W_{1-2} = (h_2 - h_1) + \frac{V_2^1 - V_1^2}{2 \times 10^3} + \frac{g}{10^3} (Z_2 - Z_1)$$

For the given diffuser,

$$W_{1-2} = 0$$

$$Z_2 = Z_1$$

$$\therefore q_{1-2} = (h_2 - h_1) + \frac{V_2^1 - V_1^2}{2 \times 10^3}$$

$$-1.5 = Cp(T_2 - T_1) + \frac{(55)^2 - (150)^2}{2 \times 10^3}$$

$$Cp(T_2 - T_1) = -1.5 + 9.7375 = 8.2375$$

$$T_2 = \frac{8.2375}{2.1} + 423 = 427 \text{ K}$$

or  $t_2 = 427 - 273 = 154^\circ\text{C}$  Ans.

## 4. Mass flow rate,

From equation of state,

$$p_1 v_1 = RT_1$$

Take,  $R = \frac{\bar{R}}{M} = \frac{8.314}{18} = 0.4619 \text{ kJ/kg-K}$

$$\therefore v_1 = \frac{0.4619 \times 423}{90} = 2.17 \text{ m}^3/\text{kg}$$

$$v_2 = \frac{0.4619 \times 427}{190} = 1.038 \text{ m}^3/\text{kg}$$

$$\dot{m} = \frac{A_1 V_1}{v_1} = \frac{\frac{\pi}{4} \times (0.10)^2 \times 150}{2.17} = 0.543 \text{ kg/s} \quad \text{Ans.}$$

## 5. Exit diameter,

$$A_2 = \frac{\dot{m} v_2}{V_2} = \frac{0.543 \times 1.038}{55} = 0.010248 \text{ m}^2$$



If  $T_\infty$  is the atmospheric temperature and  $T_1$  is the temperature of space to be heated, then

$$T_\infty = T_2$$

$$T_\infty < T_1$$

From 1st law of thermodynamics,

$$\dot{W} = \dot{Q}_1 - \dot{Q}_2$$

The schematic diagram of a heat pump is shown in Fig. 2.18. It operates as follows:

1. Heat pump receives heat  $\dot{Q}_2$  for atmosphere which is at a temperature  $T_2$  equal to atmospheric temperature,  $T_\infty$ .
2. It receives external work,  $\dot{W}$  and transfers heat from low temperature to a higher temperature.
3. It supplies heat  $\dot{Q}_1$  to the space to be heated which is maintained at a temperature  $T_1$  which is higher than atmospheric temperature.

*Coefficient of performance (COP)*

The performance of a heat pump is measured by its coefficient of performance. *COP* is the ratio of amount of heat supplied to the space and mechanical work received.

$$(COP)_{HP} = \frac{\dot{Q}_1}{\dot{W}} = \frac{\dot{Q}_1}{\dot{Q}_1 - \dot{Q}_2} = 1 + \frac{\dot{Q}_2}{\dot{Q}_1 - \dot{Q}_2}$$

For a heat pump working as reversible reversed heat engine.

$$COP = \frac{T_1}{T_1 - T_2}$$

#### 2.10.4 Refrigerator

A refrigerator is a reversed heat engine which removes heat from a body at low temperature and transfers heat to a body at higher temperature. The objective of the system is to produce refrigeration effect and is used to preserve food and drugs by storing at low temperatures.

The schematic diagram of a refrigerator is shown in Fig. 2.20.

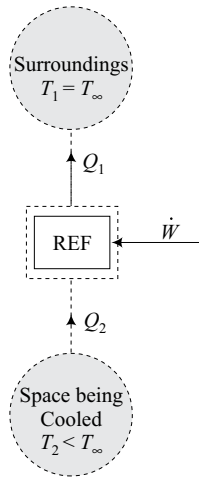
$$T_1 = T_\infty$$

$$T_\infty > T_2$$

$$W = Q_1 - Q_2 \text{ as per 1st law of thermodynamics.}$$

The refrigerator works as follows:

1. It removes heat  $\dot{Q}_2$  from space being cooled at low temperature  $T_2$  which is lower than atmospheric temperature,  $T_\infty$ .



**Fig. 2.20** Refrigerator

2. It rejects heat  $\dot{Q}_1$  to the surrounding at temperature  $T_1$  which is equal to atmospheric temperature,  $T_\infty$ .
3. It receives power in the form of work,  $\dot{W}$  to transfer heat from a lower temperature to higher temperature.

#### *Coefficient of performance (COP)*

The performance of a refrigerator is measured by its coefficient of performance. COP is the ratio of amount of heat extracted called *refrigerated effect* and work input

$$(CPO)_{REF} = \frac{\dot{Q}_2}{\dot{W}} = \frac{\dot{Q}_2}{\dot{Q}_1 - \dot{Q}_2}$$

*Observation,*

$$(COP)_{HP} = \frac{\dot{Q}_1}{\dot{Q}_1 - \dot{Q}_2} = 1 + \frac{\dot{Q}_2}{\dot{Q}_1 - \dot{Q}_2}$$

$$(COP)_{REF} = \frac{\dot{Q}_2}{\dot{Q}_1 - \dot{Q}_2}$$

$$\therefore (COP)_{HP} = 1 + (COP)_{REF}$$

If refrigerator works as reversible reversed heat engine,  $COP = \frac{T_2}{T_1 - T_2}$ .

**Example 2.16:** A reversible heat engine operates between reservoirs at 420 K and 280 K. If the output from the engine is 2.5 kJ, determine the efficiency of the engine and its heat interactions with the two heat reservoirs.

Subsequently, the engine is reversed and made to operate as heat pump between the same reservoirs. Calculate the coefficient of the heat pump and the power input required when the heat transfer rate from the 280 K reservoir is 5 kW.

[U.P.T.U. II Sem., (CO) 2003-04]

**Solution:**

1. Draw the schematic diagram of heat engine,

$$T_1 = 420 \text{ K}$$

$$T_2 = 280 \text{ K}$$

$$W = 2.5 \text{ kJ}$$

$$\begin{aligned} \eta &= \frac{T_1 - T_2}{T_1} = \frac{420 - 280}{420} \\ &= 0.3333 = 33.33\% \end{aligned}$$

$$\eta = \frac{W}{Q_1}$$

$$\therefore Q_1 = \frac{W}{\eta} = \frac{2.5}{0.3333} = 7.5 \text{ kJ}$$

$$W = Q_1 - Q_2$$

$$\therefore Q_2 = Q_1 - W = 7.5 - 2.5 = 5 \text{ kJ}$$

2. Draw the schematic diagram of heat pump

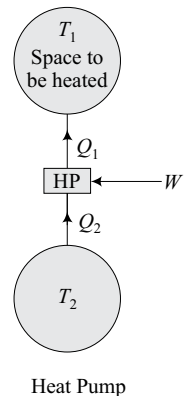
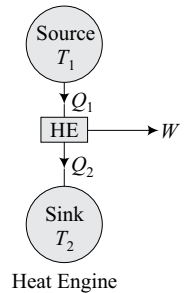
$$\begin{aligned} \text{COP} &= \frac{T_1 - T_2}{T_1} = \frac{420 - 280}{420} \\ &= 3 \end{aligned}$$

$$Q_2 = 5 \text{ kW} = 5 \text{ kJ/s}$$

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\therefore Q_1 = \frac{T_1}{T_2} Q_2 = \frac{420}{280} \times 5 = 7.5 \text{ kJ/s}$$

$$\begin{aligned} W &= Q_1 - Q_2 = 7.5 - 5 \\ &= 2.5 \text{ kJ/s} = \mathbf{2.5 \text{ kW}} \end{aligned}$$



**Example 2.17:** A reversed Carnot engine is used for heating a building. It supplies  $210 \times 10^3$  kJ/hr of heat to the building at  $20^\circ\text{C}$ . The outside air is at  $-5^\circ\text{C}$ . Find the heat taken from the outside air per hour and power of the driving motor.

**Solution:**

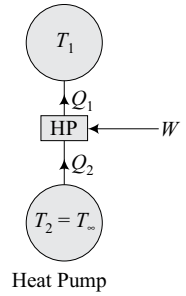
1. Draw the schematic diagram.
2. Given data,

$$T_1 = 20^\circ\text{C} + 273 = 293 \text{ K}$$

$$T_2 = -5^\circ\text{C} + 273 = 268 \text{ K}$$

$$Q_1 = 210 \times 10^3 \text{ kJ/hr} = \frac{210 \times 10^3}{3600} = 58.33 \text{ kJ/sec}$$

$$3. \quad (COP)_{HP} = \frac{T_1}{T_1 - T_2} = \frac{Q_1}{Q_1 - Q_2} = \frac{Q_1}{W}$$



$$\therefore \quad W = Q_1 \left( \frac{T_1 - T_2}{T_1} \right)$$

$$= 58.33 \left( \frac{293 - 268}{293} \right)$$

$$= 4.98 \text{ kW}$$

4. Heat taken from outside air,

$$Q_2 = Q_1 - W = 58.33 - 4.98 = 53.33 \text{ kJ/sec}$$

$$= 53.35 \times 3600 = \mathbf{192060 \text{ kJ/hr} \quad \text{Ans.}}$$

**Example 2.18:** A domestic food refrigerator maintains a temperature of  $-15^\circ\text{C}$ . The air temperature is  $30^\circ\text{C}$ . If heat leaks into the freezer is  $1.75$  kJ/sec. continuously, then what is the least power necessary to pump the heat out continuously.

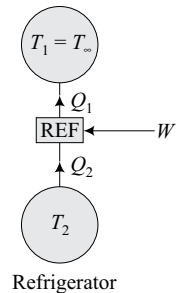
**Solution:**

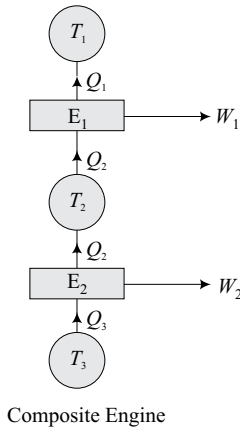
1. Draw the schematic diagram of refrigerator.
2. The refrigerator must work as reversible device so that power required is minimum.
3. Given data,

$$T_1 = 30^\circ\text{C} + 273 = 303 \text{ K}$$

$$T_2 = -15 + 273 = 258 \text{ K}$$

$$Q_2 = 1.75 \text{ kJ/sec}$$





$$1 - \eta_1 = \frac{T_2}{T_1} \quad \dots(1)$$

Similarly,  $1 - \eta_2 = \frac{T_3}{T_2} \quad \dots(2)$

Multiply equations (1) and (2),

$$\therefore (1 - \eta_1)(1 - \eta_2) = \frac{T_2}{T_1} \cdot \frac{T_3}{T_2} = \frac{T_3}{T_1} \quad \dots(3)$$

But,  $\eta_3 = \frac{T_1 - T_3}{T_1} = 1 - \frac{T_3}{T_1}$

$$\therefore 1 - \eta_3 = \frac{T_3}{T_1} \quad \dots(4)$$

From equations (3) and (4),

$$1 - \eta_3 = (1 - \eta_1)(1 - \eta_2) = \frac{T_3}{T_1}$$

$$= 1 - \eta_2 - \eta_1 + \eta_1\eta_2$$

$$\therefore \eta_3 = \eta_1 + \eta_2 - \eta_1\eta_2$$

**Example 2.21:** A cold storage of 100 tonnes of refrigeration capacity runs at 1/4th of its Carnot COP. Inside temperature is  $-15^\circ\text{C}$  and atmospheric temperature is  $35^\circ\text{C}$ . Determine the power required to run the plant. Take one tonne of refrigeration as 3.52 kW.

[U.P.T.U. I Sem., (CO), 2003]

**Solution:**

$$T_1 = 35^\circ\text{C} + 273 = 308 \text{ K}$$

$$T_2 = -15^\circ\text{C} + 273 = 258 \text{ K}$$

$$\begin{aligned} (\text{COP})_{\text{Carnot}} &= \frac{T_2}{T_1 - T_2} = \frac{258}{308 - 258} \\ &= 5.16 \end{aligned}$$

COP of actual refrigeration plant,

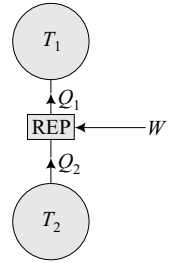
$$(\text{COP})_{\text{REF}} = \frac{1}{4} \times 5.16 = 1.25$$

Refrigeration load,

$$Q_2 = 3.52 \times 100 = 352 \text{ kW} = 352 \text{ kJ/s}$$

$$(\text{COP})_{\text{REF}} = \frac{Q_2}{Q_1 - Q_2} = \frac{Q_2}{W}$$

$$\therefore W = \frac{Q_2}{(\text{COP})_{\text{REF}}} = \frac{352}{1.25} = \mathbf{281.6 \text{ kW} \quad \text{Ans.}}$$



**Example 2.22:** Show that the minimum work needed by the heat pump for the heat transfer between two bodies at temperature  $T_1$  initially, such that one body cools down to temperature  $T_2$  shall be:

$$W = Cp \left[ \frac{T_1^2}{T_2} + T_2 - 2T_1 \right]$$

Consider the specific heat of the two bodies as  $Cp$ .

[U.P.T.U. I Sem., 2002-03]

**Solution:** Draw the schematic diagram of heat pump.

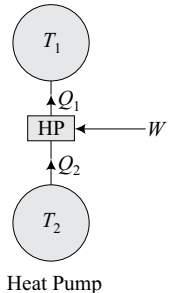
To satisfy the condition of minimum work requirement, the heat pump must be reversible.

$$\therefore \text{COP} = \frac{T_2}{T_1 - T_2}$$

For a unit mass, the heat to be extracted will be

$$Q_2 = Cp (T_1 - T_2)$$

Now, 
$$\text{COP} = \frac{Q_2}{W}$$



**Example 2.24:** Using an engine of 30% thermal efficiency to drive a refrigerator having a COP of 5, what is the heat input into the engine for each MJ removed from the cold body by the refrigerant. If the system is used as heat pump, how many MJ of heat would be available for heating for each MJ of heat input to the engine.

**Solution:**

(i) *Engine-Refrigerator system,*

The schematic diagram is shown.

The thermal efficiency of engine,

$$\eta = \frac{W}{Q_1} = 0.3$$

$$(\text{COP})_{\text{REF}} = \frac{Q_3}{W} = 5$$

$$\therefore W = 0.3Q_1$$

Also, 
$$W = \frac{Q_3}{5}$$

$$\therefore 0.3Q_1 = \frac{Q_3}{5}$$

$$Q_1 = \frac{Q_3}{5 \times 0.3}$$

For  $Q_3 = 1 \text{ MJ}$

$$Q_1 = \frac{1}{5 \times 0.3} = \mathbf{0.67 \text{ MJ} \quad \text{Ans.}}$$

(ii) *Engine-Heat Pump system,*

$$(\text{COP})_{\text{HP}} = (\text{COP})_{\text{REF}} + 1 = 5 + 1 = 6$$

$$= \frac{Q_4}{W}$$

But from engine efficiency,

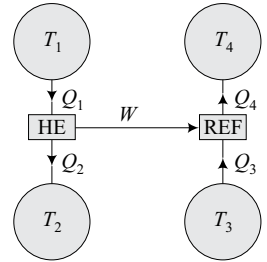
$$W = 0.3 Q_1$$

If  $Q_1 = 1 \text{ MJ}$

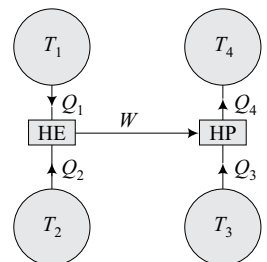
$$W = 0.3 \text{ MJ}$$

$$\therefore Q_4 = W(\text{COP})_{\text{HP}} = 0.3 \times 6$$

$$= \mathbf{1.8 \text{ MJ} \quad \text{Ans.}}$$



Engine-Refrigerator System



Engine-Heat Pump system

**Example 2.25:** Heat pump is used for heating the premises in winter and cooling the same in summer such that temperature inside remains  $25^{\circ}\text{C}$ . Heat transfer across the walls and roof is found  $2 \text{ MJ}$  per hour per degree temperature difference between interior and exterior. Determine the minimum power required for operating the pump in winter when outside temperature is  $1^{\circ}\text{C}$  and also give the maximum temperature in summer for which the device shall be capable of maintaining the premises at desired temperature for same power input.

[U.P.T.U. II Sem., 2001-02]

**Solution:**

(i) In winter, the device works as heat pump.

$$T_1 = 25^{\circ}\text{C} + 273 = 298 \text{ K}$$

$$T_2 = 1^{\circ}\text{C} + 273 = 274 \text{ K}$$

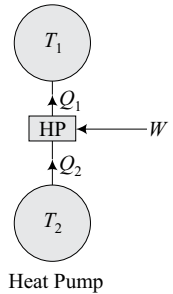
For a reversible heat pump,

$$(\text{COP})_{\text{HP}} = \frac{T_1}{T_1 - T_2} = \frac{298}{298 - 274} = 12.4$$

Also, 
$$\text{COP} = \frac{Q_1}{W}$$

$$Q_1 = \frac{2 \times 10^6}{3600} (298 - 274) = 13333.33 \text{ J} = 13.33 \text{ kJ}$$

$$W = \frac{13.33}{12.42} = \mathbf{1.074 \text{ kW} \quad \text{Ans.}}$$



(ii) In summer, the device works as air-conditioner,

$$(\text{COP})_{\text{REF}} = \frac{T_2}{T_1 - T_2} = \frac{Q_2}{W}$$

$$\therefore \frac{298}{(298 - T_2)} = \frac{2 \times 10^6}{3600} (298 - T_2) \quad 1.074$$

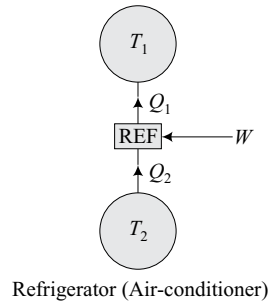
$$(298 - T_2)^2 = \frac{298 \times 1.074 \times 3600}{2 \times 10^6}$$

$$= 576$$

$$298 - T_2 = \sqrt{576} = 24$$

$$\therefore T_2 = 298 + 24 = 322 \text{ K}$$

$$= \mathbf{49^{\circ}\text{C} \quad \text{Ans.}}$$



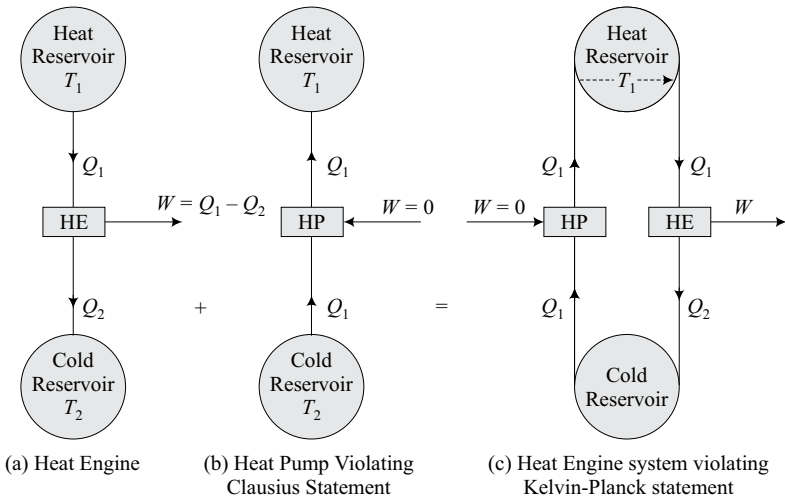


## 2.11 EQUIVALENCE OF SECOND LAW STATEMENTS

The Kelvin-Planck statement and Clausius statements are two statements of second law of thermodynamics (see para 2.1). These statements look different but these are two parallel statements and are equivalent in all respects. The equivalence of these statements will be proved by the logic that violation of one statement leads to violation of second statement and vice-versa.

### 2.11.1 Violation of Clausius Statement

A cyclic heat engine (HE) operates between two reservoirs drawing heat  $Q_1$  and producing work,  $W$  as shown in Fig. 2.20(a).



**Fig. 2.21** Violation of Clausius statement

A cyclic heat pump (HP) is shown in Fig. 2.21(b) which is transferring heat from cold reservoir ( $T_2$ ) to a hot reservoir ( $T_1$ ) with no work input ( $W = 0$ ). This violates Clausius statement as heat cannot flow from a cold temperature to a higher temperature without any external work input.

Figure 2.21(c) shows a heat engine (HE) and heat pump (HP) combined to form a heat engine system. The heat pump is supplying heat  $Q_1$  to hot reservoir which is the requirement of heat engine. Therefore, hot reservoir can be by-passed. The heat pump and heat engine  $T$  form a system operating in cycles and producing work  $W$  continuously while exchanging heat with one reservoir (cold) only. Thus, the Kelvin-Planck statement is violated. Hence, when Clausius statement is violated, simultaneously, Kelvin-Planck statement is also violated.

### 2.11.2 Violation of Kelvin-Planck Statement

A heat pump (HP) is extracting heat  $Q_1$  from low temperature reservoir ( $T_2$ ) and discharging heat to high temperature reservoir ( $T_1$ ) getting work from outside.

A heat engine (HE) is producing work  $W$  by exchanging heat with one reservoir at temperature  $T_1$  only. The Kelvin-Planck statement is violated which requires two heat reservoirs for an engine to produce work continuously.

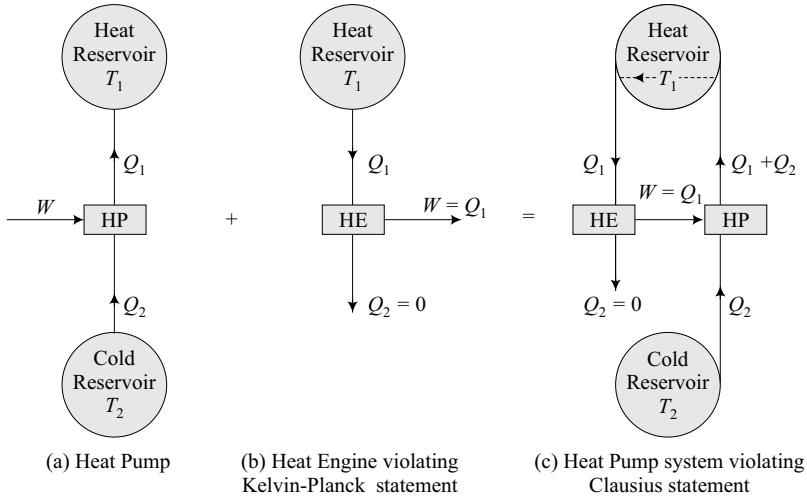


Fig. 2.22 Violation of Kelvin-Planck statement

The heat pump and heat engine are combined in Fig. 2.22(c) to create a heat pump system. The heat pump is getting work  $W$  from heat engine. Therefore, heat pump system is transferring heat from a lower temperature ( $T_2$ ) to a higher temperature ( $T_1$ ) without any external work. Therefore, the Clausius statement is also violated.

From the above two cases, it is clear that when one of the statements of second law is violated, the other statement is also violated. Therefore, the two statements are equivalent.

## 2.12 CARNOT CYCLE

Sadi Carnot proposed the concept of Carnot engine in 1824 which works on reversible cycle called Carnot cycle.

The *Carnot engine* has air enclosed in a cylinder. The cylinder receives heat  $Q_1$  from hot reservoir and rejects heat  $Q_2$  to cold reservoir. The cylinder head can be covered with insulating cap at will. The following assumptions are made:

1. The working substance is air.
2. Piston-cylinder arrangement is weightless and frictionless.

$$\begin{aligned} \therefore Q_{3-4} &= W_{3-4} = p_3 V_3 \ln \frac{V_3}{V_4} \\ &= mRT_3 \ln \frac{V_3}{V_4} \quad [\because pV = mRT] \\ &= mRT_3 \ln r \end{aligned}$$

where,  $r = \text{compression ratio} = \frac{V_3}{V_4} = \frac{V_2}{V_1}$

Also,  $Q_{3-4} = T_3(S_3 - S_4)$ .

**4. Process 4-1: Reversible Adiabatic or Isentropic Compression**

The cylinder head is disconnected from cold reservoir and covered with insulating cap. The air is allowed to be compressed isentropically from  $V_4$  to  $V_1$ .

$$Q_{4-1} = 0$$

Applying 1st law of thermodynamics,

$$Q_{4-1} = (U_4 - U_1) + W_{4-1}$$

$$\begin{aligned} \therefore (U_4 - U_1) &= \frac{p_1 V_1 - p_2 V_2}{\gamma - 1} \\ &= \frac{mR(T_1 - T_2)}{\gamma - 1} \end{aligned}$$

But,  $T_3 = T_4$

$$\therefore U_4 - U_1 = \frac{mR(T_1 - T_3)}{\gamma - 1}$$

$$\therefore U_3 - U_2 = U_4 - U_1$$

Also,  $S_4 = S_1$ .

**2.12.2 Efficiency of Carnot Cycle**

Work done,

$$\begin{aligned} W &= \text{Heat supplied} - \text{Heat rejected} \\ &= mRT_1 \ln r - mRT_3 \ln r \\ &= mR \ln r (T_1 - T_3) \end{aligned}$$

Efficiency,

$$\eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{mR \ln r (T_1 - T_3)}{mR \ln r T_1} = \frac{T_1 - T_3}{T_1}$$

From  $T$ - $s$  diagram,

$$W = T_1(S_2 - S_1) - T_3(S_3 - S_4)$$

$$\eta = \frac{T_1(S_2 - S_1) - T_3(S_3 - S_4)}{T_1(S_2 - S_1)}$$

But,  $S_3 = S_2$  and  $S_1 = S_4$

$$\therefore \eta = \frac{T_1 - T_3}{T_1}$$

For process 2-3,  $\frac{T_2}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1}$

But,  $T_2 = T_1$

$$\therefore \frac{T_1}{T_3} = \left(\frac{V_3}{V_2}\right)^{\gamma-1}$$

For process 4-1,  $\frac{T_1}{T_4} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$

But,  $T_4 = T_3$

$$\therefore \frac{T_1}{T_3} = \left(\frac{V_4}{V_1}\right)^{\gamma-1}$$

$$\therefore \frac{V_3}{V_2} = \frac{V_4}{V_1}$$

or  $\frac{V_2}{V_1} = \frac{V_3}{V_4} = r$

$$\therefore \frac{T_1}{T_3} = (r)^{\gamma-1}$$

$$\begin{aligned} \therefore \eta &= 1 - \frac{T_3}{T_1} = 1 - \left(\frac{1}{r}\right)^{\gamma-1} \\ &= 1 - \frac{1}{r^{\gamma-1}} \end{aligned}$$

### 2.12.3 Observations

$$1. \text{ Carnot efficiency, } \eta = 1 - \frac{T_2}{T_1}.$$

$$\text{Thermal efficiency, } \eta = \frac{\text{Work done}}{\text{Heat supplied}} = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}.$$

$$\therefore \frac{Q_2}{Q_1} = \frac{T_2}{T_1}$$

$$\text{or} \quad \frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \text{constant.}$$

The heat transfer from a heat reservoir is proportional to its temperature. This is called *Carnot principle*.

2. Carnot efficiency is independent of working fluid.
3. It is a function of absolute temperature of heat reservoirs.
4. Carnot efficiency increases with the decrease of sink temperature  $T_2$ .  
If  $T_2 = 0$ ,  $\eta$  is 100%. It violates Kelvin-Planck statement as no cold reservoir is required.
5. The sink temperature  $T_2$  cannot be lower than atmospheric temperature or temperature of sea or river or lake which can act as heat sink.
6. Carnot efficiency increases with the increase of source temperature  $T_1$ . This requires high temperature resisting materials which are very costly and not easily available. Therefore,  $T_1$  is restricted by metallurgical considerations and is called *metallurgical limitation*.
7. Carnot efficiency depends upon temperature difference ( $T_1 - T_2$ ). Higher the temperature difference between source and sink, higher the Carnot efficiency.
8. For same degree increase of source temperature or decrease in sink temperature, Carnot efficiency is more sensitive to change in sink temperature.

### 2.12.4 Impracticability of Carnot Engine (Carnot Cycle)

Although Carnot cycle is most efficient, it is not practical and Carnot engine is only a hypothetical device. The main reasons for impracticability of Carnot cycle are listed below:

1. All the processes are taken as reversible in a Carnot cycle. Practically processes are not reversible due to the presence of internal fluid friction of working substance and mechanical friction between piston and cylinder.
2. There is also irreversibility due to heat absorption and rejection taking place with finite temperature difference between reservoir and working substance.

$$\therefore \frac{W_A}{Q_1} > \frac{W_B}{Q_1}$$

$$\text{or } W_A > W_B$$

### 2. Reverse Engine $E_B$

The engine  $E_B$  is reversible. Reverse the engine  $B$  so that all input and output energy values are same but directions are reversed. Since  $W_A$  is more than  $W_B$ , therefore, part of  $W_A$  (equal to  $W_B$ ) may be used to drive the reversed heat engine  $\exists_B$ .

The heat  $Q_{1B}$  discharged by  $\exists_B$  may be supplied to  $E_A$ . The source can be eliminated.

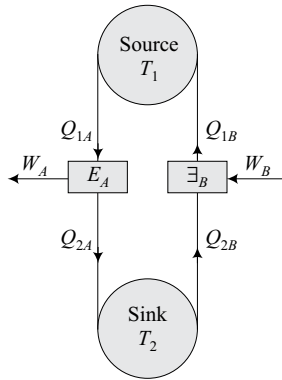


Fig. 2.28 Engine  $E_B$  is reversed

### 3. Combined Engines ( $E_A + \exists_B$ )

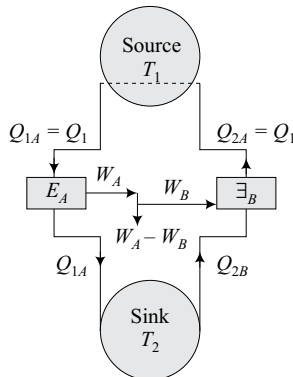


Fig. 2.29 Combined operation of  $E_A + \exists_B$

The combined engines  $E_A + \exists_B$  constitute a heat engine system operating on a single reservoir ( $T_2$ ) and still produces work equal to  $W_A - W_B$ . This system violates the Kelvin-Planck statement of second law of thermodynamics.

Therefore, our assumption that  $\eta_A > \eta_B$  is wrong.

$$\therefore \eta_B \geq \eta_A$$

The efficiency of an irreversible engine cannot be greater than that of the reversible engine if both operate between same thermal reservoirs with fixed temperatures.

**Example 2.26:** A Carnot engine operates between source and sink temperature of  $260^\circ\text{C}$  and  $-17.8^\circ\text{C}$ . If the system receives 100 kJ from the source, find the net work transfer, heat rejected to sink and efficiency of the system.

**Solution:** The schematic diagram of the Carnot engine is shown.

$$T_1 = 260^\circ\text{C} + 273 = 533 \text{ K}$$

$$T_2 = -17.8 + 273 = 255.2 \text{ K}$$

$$Q_1 = 100 \text{ kJ}$$

From Carnot principle,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\therefore Q_2 = \frac{T_2}{T_1} Q_1$$

$$= \frac{255.2}{533} \times 100 = \mathbf{47.88 \text{ kJ}} \quad \mathbf{Ans.}$$

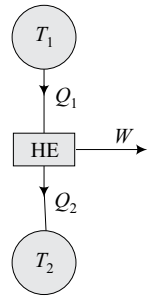
The net work transfer,

$$W = Q_1 - Q_2 = 100 - 47.88 = 52.12 \text{ kJ}$$

The Carnot efficiency,

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{255.2}{533} = 0.5212$$

$$= \mathbf{52.12\%} \quad \mathbf{Ans.}$$



Carnot Engine

**Example 2.27:** A Carnot engine operates between temperatures 500 K and 100 K. Its efficiency may be increased by increasing the source temperature to 600 K. Determine the reduction in sink temperature necessary to affect the same improvement in efficiency holding the source temperature at 500 K. On the basis of these calculations, which method appears more effective for increasing the efficiency of Carnot cycle.

**Solution:** The schematic diagram of Carnot engine is shown.

$$T_1 = 500 \text{ K}$$

$$T_2 = 100 \text{ K}$$

$$\eta_c = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{500} = 80\%$$

**Case I.**

$$T_2 = 100 \text{ K}$$

$$T_1 = 600 \text{ K}$$

$\therefore$

$$\eta_c = 1 - \frac{T_2}{T_1} = 1 - \frac{100}{600} = 83.33\%$$

**Case II.**

$$T_1 = 500 \text{ K}$$

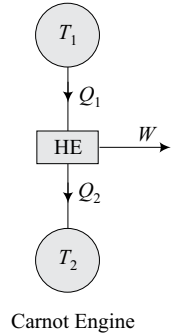
$$\eta_c = 0.8333$$

$$\eta_c = 1 - \frac{T_2}{T_1}$$

$\therefore$

$$T_2 = (1 - \eta_c)T_1 = (1 - 0.8333)500 = 83.35 \text{ K}$$

In order to achieve an increase in efficiency to 83.33%, either increase the source temperature by 100 K or decrease the sink temperature by 16.65 K, i.e., (100 – 83.35) K. Therefore, decreasing the sink temperature is more effective to increase the Carnot efficiency as we have to decrease it only by 16.65 K against 100 K increase in source temperature.



## 2.14 CLAUSIUS INEQUALITY

The clausius theorem states: “Whenever a closed system undergoes a cyclic process, the cyclic integral  $\oint \frac{dQ}{T}$  is less than zero (i.e., negative) for an irreversible cyclic process and equal to zero for a reversible cyclic process”.

Mathematically

$$\oint \frac{dQ}{T} < 0 \text{ for an irreversible process}$$

$$\oint \frac{dQ}{T} = 0 \text{ for reversible process}$$

Combining the equations for reversible and irreversible processes

$$\oint \frac{dQ}{T} \geq 0$$



**Solution:** The schematic diagram of the engine is shown,

$$T_1 = 650^\circ\text{C} + 273 = 923 \text{ K}$$

$$T_2 = 100^\circ\text{C} + 273 = 373 \text{ K}$$

$$Q_1 = 2512 \text{ kJ/min}$$

(i)  $Q_2 = 867 \text{ kJ/min}$

Applying Clausius inequality,

$$\frac{Q_1}{T_1} - \frac{Q_2}{T_2} = \frac{2512}{923} - \frac{867}{373} = 0.34770 > 0$$

The cycle is not possible.

(ii)  $Q_2 = 1015 \text{ kJ/min}$

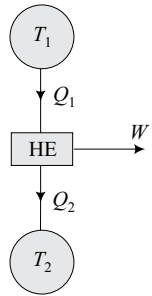
$$\frac{2512}{923} - \frac{1015}{373} = 0$$

The cycle is reversible.

(iii)  $Q_2 = 1494 \text{ kJ/min}$

$$\frac{2512}{923} - \frac{1494}{373} = -1.284 < 0$$

The cycle is irreversible and possible.



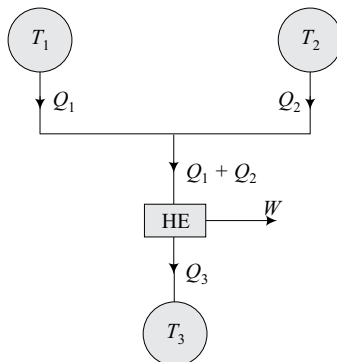
**Example 2.29:** A reversible heat engine receives heat from two thermal reservoirs at 750 K and 500 K. The engine develops 100 kW and rejects 3600 kJ/min of heat to the sink at 250 K. Determine thermal efficiency of the engine and heat supplied by each thermal reservoir.

**Solution:** The schematic diagram of the engine is shown,

$$T_1 = 750 \text{ K}$$

$$T_2 = 500 \text{ K}$$

$$T_3 = 250 \text{ K}$$



$$W = 100 \text{ kW} = 100 \times 60 = 6000 \text{ kJ/min}$$

$$Q_3 = 3600 \text{ kJ/min}$$

As per 1st law of thermodynamics,

$$\begin{aligned} Q_1 + Q_2 &= W + Q_3 = 6000 + 3600 \\ &= 9600 \text{ kJ/min} \end{aligned}$$

$$Q_2 = 9600 - Q_1$$

As per Clausius theorem,

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} - \frac{Q_3}{T_3} = 0 \quad (\text{for a reversible heat engine})$$

$$\frac{Q_1}{750} + \frac{(9600 - Q_1)}{500} - \frac{3600}{250} = 0$$

$$Q_1 + 14400 - 1.5Q_1 - 10800 = 0$$

$$Q_1 = 7200 \text{ kJ/min}$$

$$Q_2 = 9600 - 7200 = 2400 \text{ kJ/min}$$

**Example 2.30:** A heat engine working on Carnot cycle absorbs heat from three reservoirs at 1000 K, 800 K and 600 K. The engine does 10 kW of net work and rejects 400 kJ/min of heat to a heat sink at 300 K. If heat supplied by the reservoir at 1000 K is 60% of the heat supplied by reservoir at 600 K. Calculate the quantity of heat absorbed from each reservoir.

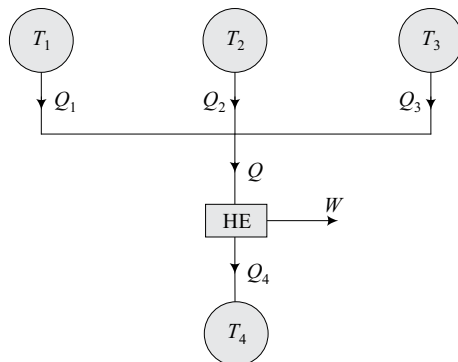
**Solution:** The schematic diagram of engine is shown,

$$T_1 = 600 \text{ K}$$

$$T_2 = 800 \text{ K}$$

$$T_3 = 1000 \text{ K}$$

$$T_4 = 300 \text{ K}$$



$$Q_4 = 400 \text{ kJ/min}$$

$$W = 10 \text{ kW} = 10 \times 60 = 600 \text{ kJ/min}$$

$$\begin{aligned} Q &= Q_1 + Q_2 + Q_3 = W + Q_4 \\ &= 600 + 400 \\ &= 1000 \text{ kJ/min} \end{aligned}$$

$$Q_3 = 0.6Q_1$$

$$\begin{aligned} Q_2 &= Q - (Q_1 + Q_3) = 1000 - Q_1 - 0.6Q_1 \\ &= 1000 - 1.6Q_1 \end{aligned}$$

Apply Clausius theorem,

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} + \frac{Q_3}{T_3} - \frac{Q_4}{T_4} = 0$$

$$\frac{Q_1}{600} + \frac{(1000 - 1.6Q_1)}{800} + \frac{0.6Q_1}{1000} - \frac{400}{300} = 0$$

$$Q_1 = \mathbf{312.5 \text{ kJ/min}}$$

$$Q_2 = 1000 - 1.6(312.5) = \mathbf{500 \text{ kJ/min}}$$

$$Q_3 = 0.6(312.5) = \mathbf{187.5 \text{ kJ/min}} \quad \text{Ans.}$$

## 2.15 CONCEPT OF ENTROPY

Entropy is a thermodynamic property of a working substance which increases with the addition of heat and decreases with its removal. Heat absorbed by the working substance can be expressed as:

$$dQ = TdS$$

where,  $T$  = Absolute temperature

$dS$  = Change in entropy.

The units of entropy are kJ/kg-K. It is an extensive property.

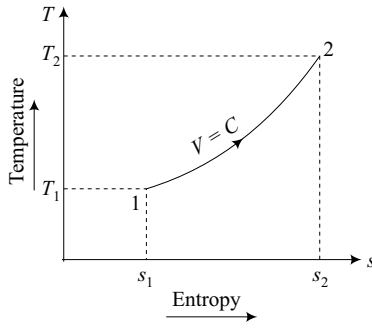
The datum for zero entropy is OK.

### 2.15.1 Entropy: A Thermodynamic Property

Entropy is a point function and not a path function. Therefore, it is a property of the system.

In Fig. 2.31, 1A2B1 is a reversible cyclic process consisting of two reversible processes 1A2 and 2B1. As per Clausius theorem,

$$\int_{1A}^{2A} \frac{dQ}{T} + \int_{2B}^{1B} \frac{dQ}{T} = 0 \quad \dots(1)$$



**Fig. 2.33** Constant volume process

Integrating between limits 1 and 2,

$$\int_{s_1}^{s_2} ds = mCv \int_{T_1}^{T_2} \frac{dT}{T}$$

$$\therefore S_2 - S_1 = mCv \ln \frac{T_2}{T_1}$$

For  $V = C$ ,

$$\frac{T_2}{T_1} = \frac{p_2}{p_1}$$

$$\therefore S_2 - S_1 = mCv \ln \left( \frac{p_2}{p_1} \right)$$

Again,

$$dS = mCv \frac{dT}{T}$$

$$\therefore \frac{dT}{dS} = \frac{T}{mCv}$$

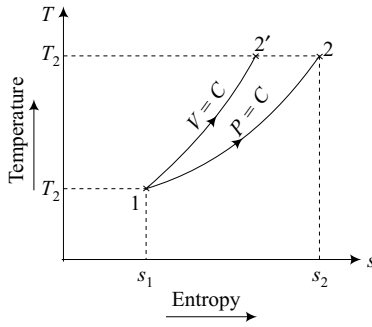
This equation gives the shape (or slope) of process 1–2 on  $T$ - $s$  diagram. Equations are valid for both reversible and irreversible processes.

## 2. Constant pressure process

A constant pressure process is plotted on  $T$ - $s$  diagram. For a constant pressure process,

$$dQ = mCp dT$$

$$\therefore \frac{dQ}{T} = mCp \frac{dT}{T}$$


**Fig. 2.34** Constant pressure process

$$dS = m C_p \left( \frac{dT}{T} \right)$$

$$\int_{s_1}^{s_2} dS = m C_p \int_{T_1}^{T_2} \frac{dT}{T}$$

$$S_2 - S_1 = m C_p \ln \frac{T_2}{T_1}$$

For  $p = C$ ,

$$\frac{T_2}{T_1} = \frac{V_2}{V_1}$$

$$\therefore S_2 - S_1 = m C_p \ln \left( \frac{V_2}{V_1} \right)$$

This equation is valid for both reversible and irreversible processes.

Again,

$$dS = m C_p \frac{dT}{T}$$

$$\therefore \frac{dT}{dS} = \frac{T}{C_p} = \text{slope of process line 1-2}$$

For constant volume process,

$$\frac{dT}{dS} = \frac{T}{C_v}$$

$$\therefore C_v < C_p$$

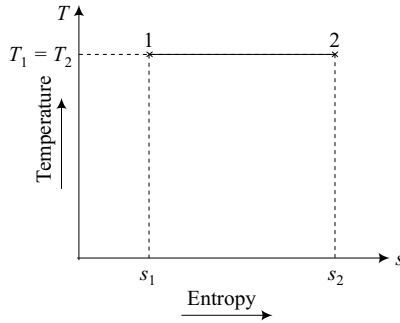
$$\therefore \frac{1}{C_v} > \frac{1}{C_p}$$

$$\therefore \frac{T}{C_v} > \frac{T}{C_p}$$

The slope of constant volume process is higher than that for constant pressure process.

### 3. Isothermal process

An isothermal process 1–2 is shown on  $T$ - $s$  diagram. For isothermal process,



**Fig. 2.35** Isothermal process

$$Q_{1-2} = W_{1-2} = mRT \ln \left( \frac{V_2}{V_1} \right)$$

$$\begin{aligned} \therefore S_2 - S_1 &= \frac{mRT}{T} \ln \left( \frac{V_2}{V_1} \right) \\ &= mR \ln \left( \frac{V_2}{V_1} \right) \\ &= m(C_p - C_v) \ln \left( \frac{V_2}{V_1} \right) \end{aligned}$$

For  $T = C$ ,

$$\frac{V_2}{V_1} = \frac{p_1}{p_2}$$

$$\therefore S_2 - S_1 = mR \ln \left( \frac{p_1}{p_2} \right)$$

or

$$S_2 - S_1 = m(C_p - C_v) \ln \left( \frac{p_1}{p_2} \right)$$

The equation is valid for reversible and irreversible processes.

Work done during cycle,

$$W = Q_1 - Q_2 = (T_1 - T_2)(s_2 - s_1)$$

Thermal efficiency,

$$\eta = \frac{W}{Q_1} = \frac{(T_1 - T_2)(s_2 - s_1)}{T_1(s_2 - s_1)} = \frac{T_1 - T_2}{T_1}$$

### 2.15.3 Entropy Change for Ideal Gases

Changes of entropy of a certain quantity,  $m$  of a perfect gas during heating by any thermodynamic process from state 1 ( $p_1V_1T_1$ ) to state 2 ( $p_2V_2T_2$ ) can be found out as follows:

1. *In terms of volume and temperature*

General energy equation of 1st law of thermodynamics:

$$dQ = dU + dW = mC_v dT + pdV$$

$$\therefore \frac{dQ}{T} = mC_v \frac{dT}{T} + \frac{pdV}{T}$$

From equation of state,  $pV = mRT$

$$\frac{p}{T} = \frac{mR}{V}$$

$$\therefore \frac{dQ}{T} = dS = mC_v \frac{dT}{T} + mR \frac{dV}{V}$$

$$\int_{s_1}^{s_2} dS = mC_v \int_{T_1}^{T_2} \frac{dT}{T} + mR \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\begin{aligned} \therefore S_2 - S_1 &= mC_v \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right) \\ &= m \left[ C_v \ln \frac{T_2}{T_1} + (C_p - C_v) \ln \left( \frac{V_2}{V_1} \right) \right] \end{aligned}$$

2. *In terms of pressure and temperature*

From equation of state,

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\therefore \frac{V_2}{V_1} = \frac{p_1 T_2}{p_2 T_1}$$

$$\begin{aligned} S_2 - S_1 &= mCv \ln\left(\frac{T_2}{T_1}\right) + mR \left(\frac{p_1}{p_2} \times \frac{T_2}{T_1}\right) \\ &= mCv \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{p_1}{p_2}\right) + mR \ln\left(\frac{T_2}{T_1}\right) \\ &= m(Cv + R) \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{p_1}{p_2}\right) \end{aligned}$$

Now,  $R = Cp - Cv$

$$Cv + R = Cp$$

$$\begin{aligned} \therefore S_2 - S_1 &= mCp \ln\left(\frac{T_2}{T_1}\right) + m(Cp - Cv) \ln\left(\frac{p_1}{p_2}\right) \\ &= m \left[ Cp \ln\left(\frac{T_2}{T_1}\right) + (Cp - Cv) \ln\left(\frac{p_1}{p_2}\right) \right] \end{aligned}$$

### 3. In terms of pressure and volume

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2}$$

$$\therefore \frac{T_2}{T_1} = \frac{p_2}{p_1} \times \frac{V_2}{V_1}$$

$$\begin{aligned} S_2 - S_1 &= mCv \ln\left(\frac{T_2}{T_1}\right) + mR \ln\left(\frac{V_2}{V_1}\right) \\ &= mCv \ln\left(\frac{p_2}{p_1} \times \frac{V_2}{V_1}\right) + m(Cp - Cv) \ln\left(\frac{V_2}{V_1}\right) \\ &= mCv \ln\left(\frac{p_2}{p_1}\right) + mCv \ln\left(\frac{V_2}{V_1}\right) + m(Cp - Cv) \ln\left(\frac{V_2}{V_1}\right) \\ &= mCv \ln\left(\frac{p_2}{p_1}\right) + mCv \ln\left(\frac{V_2}{V_1}\right) + mCp \ln\left(\frac{V_2}{V_1}\right) - mCv \ln\left(\frac{V_2}{V_1}\right) \end{aligned}$$



$$S_2 - S_1 = m \left[ C_v \ln \frac{p_2}{p_1} + C_p \ln \frac{V_2}{V_1} \right]$$

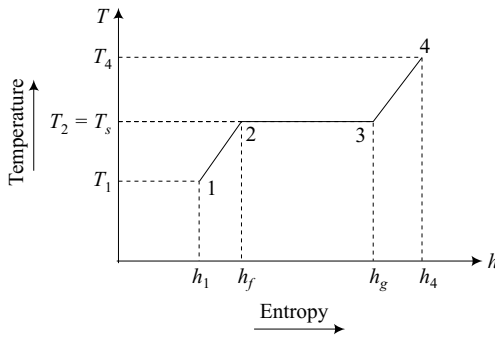
### 2.15.4 Entropy Change during Steam Generation

- (i) 1 kg of water at temperature  $T_1$  is heated to boiling temperature  $T_2$ . Heat supplied,

$$Q_{1-2} = h_f - h_1 = C_w(T_2 - T_1)$$

where,  $h_f$  = enthalpy of saturated water,

$$\begin{aligned} C_w &= \text{specific heat of water} \\ &= 4.186 \text{ kJ/kg-K} \end{aligned}$$



**Fig. 2.39**  $T$ - $h$  diagram

Change of entropy,

$$S_2 - S_1 = \int_{T_1}^{T_2} \frac{dQ}{T} = \int_{T_1}^{T_2} \frac{C_w dT}{T} = C_w \ln \left( \frac{T_2}{T_1} \right)$$

- (ii) During process 2–3 there is evaporation of water. The temperature is constant at saturation temperature  $T_s$ .

$$S_3 - S_2 = \int_2^3 \frac{dQ}{T} = \frac{Q_{2-3}}{T_s} = \frac{h_{fg}}{T_s}$$

- (iii) During process 3–4, the steam is superheated. The heat added

$$dQ = m C_s dT$$

where,

$$\begin{aligned} C_s &= \text{specific heat of steam} \\ &= 2.1 \text{ kJ/kg-K} \end{aligned}$$

### 2.15.9 Important Mathematical Relations of Entropy

1.  $dS = \frac{dQ}{T}$
2.  $dQ = TdS$
3.  $TdS = dU + pdV$
4. Unavailable heat energy,  $UHE = T_{\infty}dS$ .  
where,  $T_{\infty}$  = Ambient temperature
5. For reversible process,  $dS = \frac{dQ}{T}$
6. For irreversible process,  $dS > \frac{dQ}{T}$
7. For universe (isolated system),  $ds \geq 0$
8. For reversible cyclic process,  $ds = 0$
9. For irreversible cyclic process,  $ds > 0$
10. For a constant pressure process,

$$S_2 - S_1 = mC_p \ln \frac{V_2}{V_1} = mC_p \ln \frac{T_2}{T_1}$$

**Example 2.31:**  $10 \text{ m}^3$  of air at  $175^\circ\text{C}$  and 5 bar is expanded to a pressure of 1 bar while temperature is  $36^\circ\text{C}$ . Calculate the entropy change for the process.

[U.P.T.U. II Sem., 2003-04]

**Solution:** *Initial conditions,*

$$V_1 = 10 \text{ m}^3$$

$$T_1 = 175^\circ\text{C} + 273 = 448 \text{ K}$$

$$p_1 = 5 \text{ bar} = 500 \text{ kPa}$$

Assume  $R$  for air =  $0.287 \text{ kJ/kg-K}$

$$p_1 V_1 = mRT_1$$

$$\therefore m = \frac{p_1 V_1}{RT_1} = \frac{500 \times 10}{0.287 \times 448} = 38.8875 \text{ kg}$$

*Final conditions,*

$$p_2 = 1 \text{ bar} = 100 \text{ kPa}$$

$$T_2 = 36^\circ\text{C} + 273 = 309 \text{ K}$$

$$\begin{aligned}
 \text{Total heat transfer, } Q_{1-2} &= m C_i (T_S - T_1) + m h_{\text{fusion}} \\
 &= 5 \times 2.093 (273 - 263) + 5 \times 335 \\
 &= 104.65 + 1675 \\
 &= 1779.65 \text{ kJ}
 \end{aligned}$$

Change of entropy of the surrounding at constant temperature 303 K,

$$(\Delta S)_{\text{surrounding}} = -\frac{Q_{1-2}}{303} = -\frac{1779.65}{303} = -5.8734 \text{ kJ}$$

Change of entropy of the system,

$$\begin{aligned}
 &= m C_i \ln \left( \frac{T_S}{T_1} \right) + \frac{m h_{\text{fusion}}}{T_S} \\
 &= 5 \times 2.093 \ln \left( \frac{273}{263} \right) + \frac{5 \times 335}{273} \\
 &= 0.39 + 6.1355 \\
 &= 6.5255 \text{ kJ}
 \end{aligned}$$

$$\begin{aligned}
 (\Delta S)_{\text{universe}} &= (\Delta S)_{\text{surrounding}} + (\Delta S)_{\text{system}} \\
 &= -5.8734 + 6.5255 = \mathbf{0.652 \text{ kJ} \quad \text{Ans.}}
 \end{aligned}$$

**Example 2.34:** 0.25 kg/s of water is heated from 30°C to 60°C by hot gases that enter at 180° and leave at 80°C. Calculate the mass flow rate of gases when its  $C_p = 1.08 \text{ kJ/kg-K}$ . Find the entropy change of water and of hot gases. Take the specific heat of water as 4.186 kJ/kg-K. Also find the increase of unavailable energy if the ambient temperature is 27°C. [U.P.T.U. II Sem., 2002-03]

**Solution:**

*Data on water,*

$$\begin{aligned}
 \dot{m}_w &= 0.25 \text{ kg/s} \\
 T_{w1} &= 30^\circ\text{C} + 273 = 303 \text{ K} \\
 T_{w2} &= 60^\circ\text{C} + 273 = 333 \text{ K} \\
 C_w &= 4.186 \text{ kJ/kg-K}
 \end{aligned}$$

*Data on gas*

$$\begin{aligned}
 T_{g1} &= 180^\circ\text{C} + 273 = 453 \text{ K} \\
 T_{g2} &= 80^\circ\text{C} + 273 = 353 \text{ K} \\
 C_g &= 1.08 \text{ kJ/kg-K} \\
 T_\infty &= 27^\circ\text{C} + 273 = 300 \text{ K}
 \end{aligned}$$

9. Classify different types of thermodynamic processes. Compare the work done in non-flow and flow processes.
10. Prove that work done for a steady flow adiabatic process is given by:

$$W_{1-2} = \frac{\gamma}{\gamma - 1} (p_1 V_1 - p_2 V_2)$$

11. Estimate the work done for an isothermal process.
12. Define engine efficiency and COP of a heat pump and a refrigerator. Prove that COP of heat pump is more than COP of refrigerator.
13. State and explain the second law of thermodynamics.
14. Write short note on "Equivalence of Kelvin-Planck and Clausius statement of second law of thermodynamics".
15. Explain Carnot cycle on  $p$ - $V$  and  $T$ - $s$  diagrams.
16. State and prove Carnot theorem.
17. Explain available and unavailable energy.
18. Show that the entropy change in a process when a perfect gas changes from state 1 to state 2 is given by:

$$s_2 - s_1 = Cp \ln \frac{T_2}{T_1} + R \ln \frac{p_2}{p_1} .$$

19. Are the following statements true or false?
  - (i) The process that violates the second law also violates the first law of thermodynamics.
  - (ii) No process in a closed system is possible in which it exchanges heat with a single reservoir and produces an equal amount of work.
  - (iii) The change in entropy of a closed system must be greater than or equal to zero.
  - (iv) The entropy of an isolated system always increases.
  - (v) Entropy increases in the process of mixing hot water with cold water.
20. Show that the work,  $W$  in non-flow processes for polytropic process, i.e.,  $pV^n = \text{constant}$ , is given by:

$$W = (p_1 V_1 - p_2 V_2) / n - 1 .$$