CHAPTER 2

Basic Terminology and Interrelations

2.1 PHASES IN SOILS

If any volume of soil mass is considered, it comprises in general of three phases, viz., solids (mineral particles), air (gas) and liquid (water), as shown in Fig. 2.1.

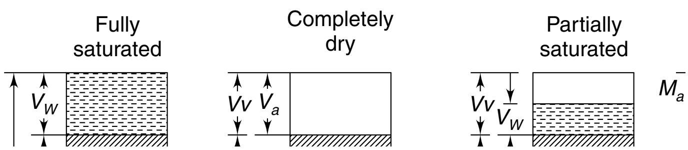

Fig. 2.1 *Multiphase soil model.*

A soil mass, if fully saturated, will be a two-phase system of solid soil grains and pore water. If completely dry, it will also be a two-phased system comprising solid soil grains and pore air. A partially saturated soil will, however, be three-phase system, comprising solid soil grains, water and air. Typical two- and three-phase diagrams are shown in Fig. 2.2.

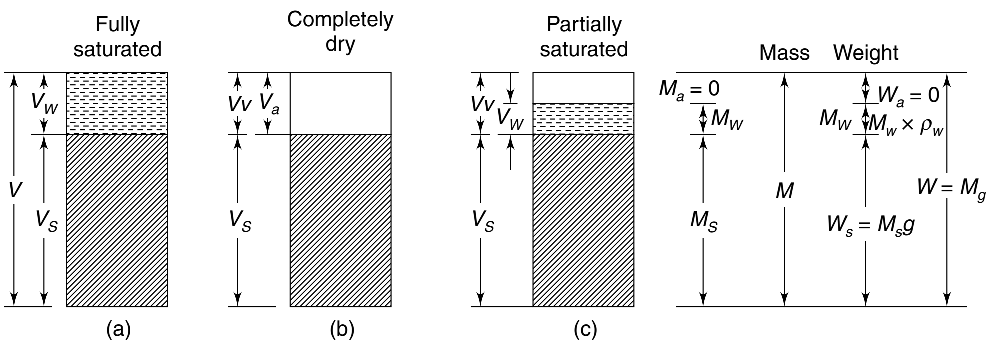

Fig. 2.2 *Two-phase diagrams (a) and (b) for fully saturated and completely dry soils (c) three-phase diagram for partially saturated soil.*

$$e = \frac{\eta}{1-\eta} \text{ also } \eta = \frac{e}{1+e}$$

$$(M_C)_w = \frac{W_W}{W_S}$$

and **Unit Weight** $\gamma = \frac{W}{V} = \frac{W_s(1+w)}{V}$

dry unit weight $\gamma_d = \frac{W_S}{V} = \frac{\gamma}{1+w}$

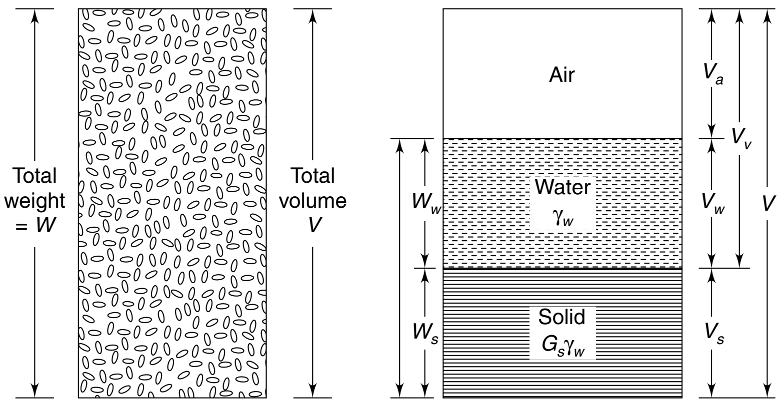

Fig. 2.3

2.3 UNIT WT., VOID RATIO, MC AND SP. GRAVITY RELATIONSHIP

Wt. of soil solids, $W_S = G_s\gamma_w$ where G_S = Sp. gravity of soil solids

Wt. of water = $w\ W_S = wG_s\gamma_w$ w = Moisture content

γ_w = unit wt. of water

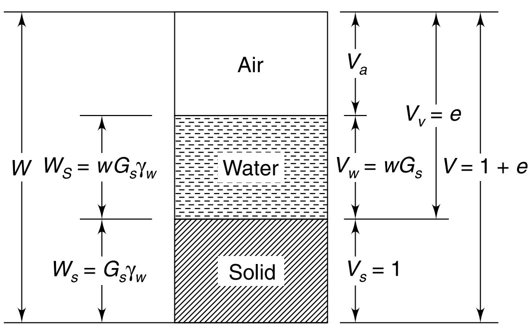

Fig. 2.4

ILLUSTRATIVE EXAMPLES

Example 2.1 The bulk density of a soil sample is 1.90 gm/cc. The moisture content is 15%. The specific gravity of the soil particles is 2.61. Calculate the dry unit weight, dry density, void ratio, porosity and degree of saturation.

Solution: Bulk density $\rho = \dfrac{\text{Total mass}}{\text{Total volume}} = \dfrac{M}{V} = 1.9$ gm/cc

Bulk unit weight $\gamma = \dfrac{W_w + W_s}{V};\ w = \dfrac{W_w}{W_s} \times 100$

$$\therefore \quad \gamma = \frac{W_s\left(1 + \frac{w}{100}\right)}{V} \text{ or } W_s = \frac{V \times \gamma}{\left(1 + \frac{w}{100}\right)}$$

Dry unit weight, $\gamma_d = \dfrac{W_s}{V} = \dfrac{V \times \gamma}{\left(1 + \frac{w}{100}\right) V} = \dfrac{\gamma}{1 + \frac{w}{100}} = \dfrac{\rho \times g}{1 + \frac{w}{100}} = \dfrac{1.9 \times 9.8}{1 + \frac{15}{100}} = 16.2 \text{ kN/m}^3$

Dry density $\rho_d = M_s/V;\ M/V = 1.9,\ M = 1.9\text{ V},\ M_s = \dfrac{V \times \rho}{\left(1 + \frac{w}{100}\right)V}$

$$= \frac{1.9\ V}{\left(1 + \frac{w}{100}\right)V} = \frac{1.9}{1.15} = 1.65 \text{ gm/cc}$$

Consider 1 cc of soil ($V = 1$), $V_s = 1.65/G_s = 1.65/2.61 = 0.63$ cc

$$V_v = 1 - 0.63 = 0.37 \text{ cc}$$

$$c = V_v/V_s = 0.37/0.63 = 0.59$$

$$n = \frac{V_v}{V} = 0.37/1.00 = 0.37$$

Mass of water in 1 cc of soil = 1.90 – 1.65 = 0.25 gm

Degree of saturation = $V_w/V_v = 100 \times \dfrac{0.25}{0.37} = 67.6\%$.

Example 2.2 If a soil has a void ratio of 0.7 and a specific gravity of 2.72, calculate the following:

(a) Dry unit weight (b) Saturated unit weight

(c) Submerged unit weight

Also compute the unit weight and water content at a degree of saturation of 75%.

Solution: $e = 0.7; \quad G_s = 2.72$

(a) Dry unit weight $\gamma_d = \dfrac{G_s \gamma_w}{1+e} = \dfrac{2.72 \times 9.8}{1+0.7} = 15.68 \text{ kN/m}^3$

(b) Saturated unit weight $\gamma_{sat} = \dfrac{G_s + e}{1+e}\gamma_w; \; S = 1$

$$\gamma_{sat} = \frac{2.72 + 0.7}{1 + 0.7} \times 9.8 = 19.7 \text{ kN/m}^3$$

(c) Submerged (or buoyant) unit weight $\gamma = \dfrac{G_s - 1}{1+e} \times \gamma_w$

$$\gamma = \frac{2.72 - 1}{1 + 0.7} \times 9.8 = 9.91 \text{ kN/m}^3$$

$$\gamma = \frac{G_s + S_r \times e}{1+e}\gamma_w$$

At $S_r = 0.75$, $\gamma = \dfrac{2.72 + (0.75 \times 0.70)}{1 + 0.70} \times 9.8 = \dfrac{2.72 + 0.525}{1.70} \times 9.8$

$$= \frac{3.245 \times 9.8}{1.70} = 18.7 \text{ kN/m}^3$$

Since $wG_s = S_r e$

$$w \times 2.72 = 0.75 \times 0.70 \quad \therefore\; w = \frac{0.75 \times 0.70}{2.72} = 19.3\%$$

Example 2.3 A cylindrical soil specimen having a volume of 86.15 cm^3 weighs 168.0 gm in its natural condition. When dried completely in an oven, the specimen weighs 130.5 gm. The value of G_s is 2.73. What is the degree of saturation of the specimen?

Solution: $W = 168.0$ gm, $W_s = 130.5$ gm

Water content $w = W_w/W_s = (168.0 - 130.5)/130.5 = 37.5/130.5 = 0.287$ or 28.7%

$G_s = W_s/V_s$ i.e., $2.73 = 130.5/V_s$ or $V_s = 47.8 \text{ cm}^3$

$V_v = V - V_s$

$V_v = 86.1 - 47.8 = 38.3 \text{ cm}^3$

Void ratio $e = V_v/V_s = 38.3/47.8 = 0.80$

Now $S_r e = wG_s$

$S_r \times 0.8 = 0.287 \times 2.73$

or $S_r = 0.979$

i.e., degree of saturation of the soil sample = 97.9%

Example 2.4 An attempt was made to determine the water content of a given moist soil of known specific gravity, using a pycnometer. The usual laboratory procedure for specific gravity determination of dry soil is done on the wet soil.

Following are the observations:

Mass of pycnometer (M_1) = 545 g

Mass of pycnometer with moist soil (M_2) = 790 g

Mass of pycnometer with soil and water (M_3) = 1540 g

Mass of pycnometer and water (M_4) = 1415 g

Specific gravity of soil grains = 2.67

Determine the water content of the soil from the first principles

Solution: Consider the two-phase diagrams shown in Fig. 2.5 representing the observations.

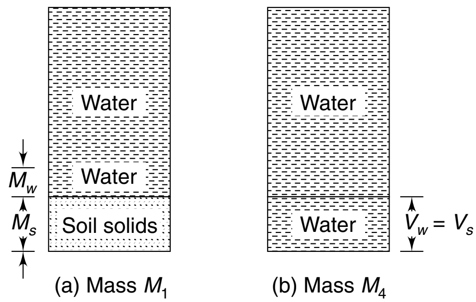

Fig. 2.5

Thus,

$$M_3 - M_4 = M_s - \text{(mass of an equal volume of water)}$$

$$= M_s - \left(\frac{M_s}{G_{\rho w}}\right) \rho w \qquad \left(\because V_s = \frac{M_s}{\rho_s} = V_w\right)$$

$$= M_s \left(\frac{G-1}{G}\right)$$

$\therefore$

$$M_s = (M_3 - M_4)\left(\frac{G}{G-1}\right)$$

and

$$w = \frac{(M_2 - M_1) - M_s}{M_s} \times 100$$

Substituting the respective values

$$M_s = (1540 - 1415)\,\frac{2.67}{(2.67-1)} = 199.85 \text{ g}$$

and $$w = \frac{(790 - 545) - 199.85}{199.85} \times 100 = 22.6\%$$

Example 2.5 The bulk unit weight of a soil is 19.10 kN/m^3, the water content is 12.5% and the specific gravity of soil solids 2.67. Determine the dry unit weight, void ratio, porosity and degree of saturation.

Solution: $$\gamma_d = \frac{\gamma}{1 + w} = \frac{19.1}{1 + \frac{12.5}{100}} = 16.98 \text{ kN/m}^3$$

$$\gamma_d = \frac{G\gamma_w}{1 + e}$$

∴ $$e = \frac{G\gamma_w}{\gamma_d} - 1 = \frac{2.67 \times 9.81}{16.98} - 1 = 0.54$$

∴ $$n = \frac{e}{1 + e} \times 100 = \frac{0.54}{1 + 0.54} \times 100 = 35.07\%$$

and $$S_r = \frac{wG}{e} \times 100 = \frac{12.5}{100} \times \frac{2.67}{0.54} \times 100 = 61.8\%$$

Example 2.6 How many cubic metres of fill can be constructed at a void ratio of 0.65 from 2,21,000 m^3 of borrow material that has a void ratio of 1.25?

Solution: Let e_b and e_f be the void ratios of the borrow material and the fill, respectively. Also let V_{vb} and V_{vf} be the volume of voids in the borrow and the fill, respectively. As the same number of soil grains obtained from the borrow are used in the fill, the volume of soil solids is same in both the cases.

From Fig. 2.6

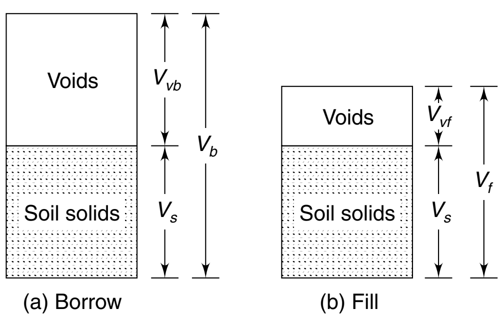

Fig. 2.6

$$e_b = \frac{V_{vb}}{V_s} \quad \text{and} \quad e_f = \frac{V_{vf}}{V_s}$$

∴ $$V_{vb} = e_b V_s \quad \text{and} \quad V_{vf} = e_f V_s$$

Total volume of soil in the borrow = $V_b = V_{vb} + V_s$

i.e., $$V_b = e_b V_s + V_s = (1 + e_b)\, V_s$$

$$V_s = \frac{V_b}{1 + e_b}$$

Total volume of soil in the fill = $V_f = (1 + e_f)\, V_s$

Substituting for V_s

$$V_f = (1 + e_f)\,\frac{V_b}{1 + e_b} = \frac{1 + 0.65}{1 + 1.25} \times 221000 = 162066.7 \text{ m}^3$$

Example 2.7 For a stable packing of regular spheres at the minimum density, find the void ratio and the dry unit weight. Unit weight of soil solids is 25 kN/m^3.

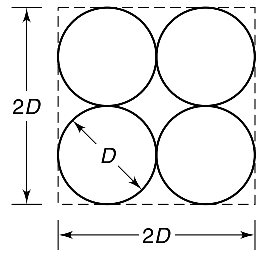

Fig. 2.7

Solution: Let D be the diameter of each sphere.

Volume of each sphere = $\dfrac{\pi D^3}{6}$

For the arrangement in Fig. 2.7 the density will be minimum

Total volume = $2D \times 2D \times D = 4\, D^3$

$$\therefore \quad e = \frac{4D^3 - 4 \times \pi D^3/6}{4 \times \pi D^3/6} = \frac{1 - \pi/6}{\pi/6} = \frac{6 - \pi}{\pi} = 0.91$$

Also $$\gamma_d = \frac{M_s}{V}\, g = \frac{V_s \gamma_s}{V} = \frac{4 \times \pi D^3/6 \times \gamma_s}{4D^3} = \frac{\pi \gamma_s}{6} = \frac{\pi \times 25}{6}$$

or $$\gamma_d = 13.09 \text{ kN/m}^3$$

Example 2.8 In order to determine the in-place density of a highway subgrade, the sand bottle method was adopted. The mass of soil extracted from a hole at the surface was 4.87 kg. The hole was then filled with sand from the sand bottle and found to have a mass of 3.86 kg. While calibrating the sand bottle, to fill a container of volume 0.0048 m^3 a mass of 6.82 kg of sand was needed. In moisture content determination, 28.26 g of the moist soil weighed 22.2 g after oven drying. If the specific gravity of the soil was 2.67, determine the bulk and dry densities of the soil and the degree of saturation of the soil.

Solution: Density of sand in sand bottle = $\dfrac{6.82}{0.0048} = 1420.8 \text{ kg/m}^3$

$$= 1.42 \text{ Mg/m}^3$$

Volume of the hole $$= \frac{3.86}{1420.8} = 0.00272 \text{ m}^3$$

Bulk density $= \dfrac{4.87}{0.00272} = 1790.4\ \text{kg/m}^3 = 1.79\ \text{mg/m}^3$

Water content $= \dfrac{28.26 - 22.2}{22.2} \times 100 = 27.3\%$

Dry density $= \dfrac{1790.4}{1 + \dfrac{27.3}{100}} = 1406.4\ \text{kg/m}^3 = 1.41\ \text{mg/m}^3$

$$e = \frac{G\rho_w}{\rho_d} - 1 = \frac{2.67 \times 1000}{1406.4} - 1 = 0.899$$

$$S_r = \frac{wG}{e} \times 100 = \frac{27.3}{100} \times \frac{2.67}{0.899} \times 100 = 81.08\%$$

Example 2.9 A fully saturated soil sample was extracted during an oil well drilling. The wet mass of the sample was 3.15 kg and the volume of the sampling tube was 0.001664 m^3. After analysis the soil sample was found to contain 28.2% of the liquid as kerosene and the dry mass as 2.67 kg. The specific gravity of soil grains was 2.68. Determine the bulk density, void ratio, and water content of the sample.

Solution: Bulk density $= \dfrac{3.15}{0.001664} = 1893\ \text{kg/m}^3 = 1.89\ \text{mg/m}^3$

Volume of soil grains $= \dfrac{2.67}{2.68 \times 1000} = 0.000996\ \text{m}^3$

Volume of voids $= 0.001664 - 0.000996 = 0.000668\ \text{m}^3$

$\therefore$ $e = \dfrac{0.000668}{0.000996} = 0.67$

As the soil was fully saturated,

Volume of liquid = volume of voids = 0.000668 m^3

Volume of water $= (1 - 0.282) \times 0.000668 = 0.00048\ \text{m}^3$

Mass of water = 0.48 kg

Water content $= \dfrac{0.48}{2.67} \times 100 = 17.98\%$

Exercise

1. A soil sample whose water content is 20% has a bulk density of 2.16 gm/cc. The sample undergoes air drying with significant change in void ratio. What is the water content of this sample when its bulk density is reduced to 2.0 gm/cc. **(Ans. = 11.11%)**