2 Bridge Measurement

2.1 INTRODUCTION

Bridges are often used for the precision measurement of component values, like resistance, inductance, capacitance, etc. The simplest form of a bridge circuit consists of a network of four resistance arms forming a closed circuit as shown in Fig. 2.1. A source of current is applied to two opposite junctions and a current detector is connected to other two junctions. The bridge circuit operates on null detection principle and uses the principle of comparison measurement methods. It compares the value of an unknown component with that of an accurately known standard component. Thus, the accuracy of measurement depends on the bridge and not on the null detector. When no current flows through the null detector, the bridge is said to be balanced. The relationship between the component values of the four arms of the bridge at the balancing is called balancing condition or balancing equation. Balancing equation gives up the value of the unknown component.

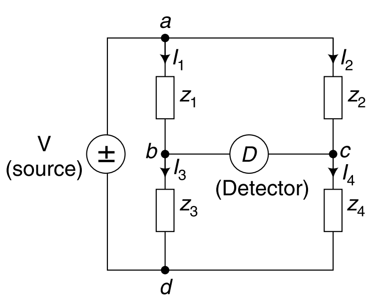


Fig. 2.1 General form of a bridge circuit

2.1.1 Advantages of Bridge Circuit

The various advantages of a bridge circuit are:

1. The balance equation is independent of the source voltage magnitude and its impedance.

Wheatstone Bridge under Small Unbalance

As discussed in previous section different galvanometers have different current/voltage sensitivities. Hence, in order to determine whether the galvanometer has the required sensitivity to detect an unbalance condition the bridge circuit can be solved for a small unbalance by converting the Wheatstone bridge into its equivalent Thevenin's circuit.

Let us consider the bridge is balanced, when the branch resistances are R_1, R_2, R_3 and R_X, so that at balance

$$R_X = \frac{R_1 R_3}{R_2}$$

Let the resistance R_X be changed to ΔR_X creating an unbalance. Due to this an emf e^1 has appeared across the galvanometer. To compute this emf let us use the Thevenin's method. With galvanometer branch open, the open circuit voltage drop between points b and d is V_{oc}. At balance

$$E_{ab} = I_1 R_1,\ I_1 = I_2 = \frac{E}{R_1 + R_2}$$

$$\Rightarrow \qquad E_{ab} = \frac{ER_1}{R_1 + R_2} \tag{2.9}$$

$$E_{ad} = I_3(R_X + \Delta R_X), \qquad I_3 = I_4 = \frac{E}{R_3 + R_X + \Delta R_X}$$

$$\Rightarrow \qquad E_{ad} = \frac{E(R_X + \Delta R_X)}{R_3 + R_X + \Delta R_X} \tag{2.10}$$

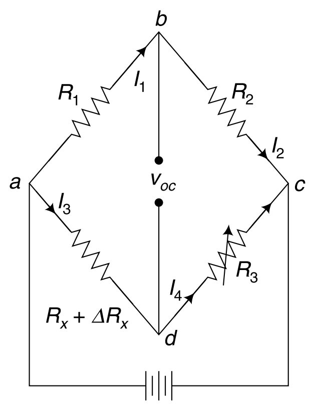


Fig. 2.3 Wheatstone bridge under unbalance condition

$$V_{oc} = V_{ad} = V_{TH} = E_{ad} - E_{ab}$$

$$= E\left[\frac{R_X + \Delta R_X}{R_3 + R_X + \Delta R_X} - \frac{R_1}{R_1 + R_2}\right] \tag{2.11}$$

$$\Rightarrow \qquad \boxed{S_B = \frac{S_v E}{\frac{R_3}{R_X} + \frac{R_X}{R_3} + 2}} \tag{2.13}$$

Thus, the bridge sensitivity depends on the bridge parameter—the supply voltage and the voltage sensitivity of the galvanometer.

Thevenin Equivalent Circuit (Under Balance Case)

The Thevenin equivalent resistance can be found by looking back into the terminal *b-d* and replacing the source by its internal resistance (if internal resistance is very low the source is replaced by short circuiting the terminal *a-c*) as shown in Fig. (2.4).

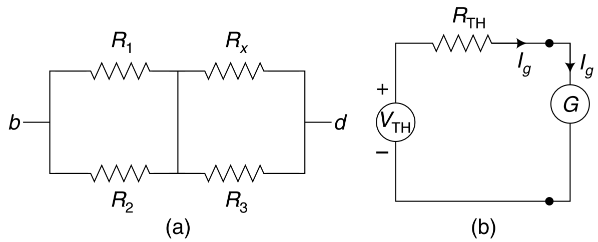


Fig. 2.4 (a) Equivalent resistance (R_{TH}) between *b* and *d*, (b) Thevenin equivalent circuit for Wheatstone bridge

$$R_{TH} = (R_1 \| R_2) + (R_3 \| R_X) = \frac{R_1}{R_1 + R_2} + \frac{R_3}{R_3 + R_X} \tag{2.14}$$

and $\quad V_{TH} = V_{oc} = E_{ad} - E_{ab} \qquad$ (R_X is not changed by ΔR_X)

$$= I_2 R_X - I_1 R_1 = \frac{E R_X}{R_3 + R_X} - \frac{E}{R_1 + R_2} R_1$$

$$V_{TH} = E\left[\frac{R_X}{R_3 + R_X} - \frac{R_1}{R_1 + R_2}\right]$$

If R_g is the galvanometer resistance and I_g is the galvanometer current, then we have

$$\boxed{I_g = \frac{V_{TH}}{R_{TH} + R_g}} \tag{2.15}$$

Galvanometer Current (Unbalance Case)

Let the resistance R_X be changed by ΔR_X, such that the arm resistance is $R_X + \Delta R_X$ and creating an unbalance. From Eqn. (2.12)

$$V_{TH} = \frac{ER_3 \Delta R_X}{(R_3 + R_X)^2}$$

Now equivalent resistance looking back into the terminals b and d.

$$R_{eq} = R_{TH} = (R_1 \| R_2) + (R_3 \| R_X + \Delta R_X)$$

$$= \frac{R_3(R_X + \Delta R_X)}{R_3 + R_X + \Delta R_X} + \frac{R_1 R_2}{R_1 + R_2}$$

Since $\Delta R_X << R_3, R_4$, neglecting ΔR_X in the above equation

$$R_{TH} = \frac{R_3 R_X}{R_3 + R_X} + \frac{R_1 R_2}{R_1 + R_2} \tag{2.16}$$

Since $$I_g = \frac{V_{TH}}{R_{TH} + R_g} \tag{2.17}$$

Now, if $R_X = R_1 = R_2 = R_3 = R$, we have from Eqns. (2.15) and (2.16)

$$I_g = \frac{ER\Delta R_X}{\frac{4R^2}{R + R_g}}$$

$$\boxed{I_g = \frac{E\Delta R_X}{4R\,(R + R_g)}} \tag{2.18}$$

Bridge Sensitivity in Terms of Current Sensitivity of Galvanometer

Let a bridge be unbalanced by changing R_X to $R_X + \Delta R_X$. Due to this an emf e' has appeared across galvanometer which is basically equal to the Thevenin equivalent voltage between the galvanometer nodes. Hence, the deflection of the galvanometer

$$\theta = S_V\, e' = S_V\, V_{TH} = \frac{S_V E R_3\, \Delta R_X}{(R_3 + R_X)^2} \tag{2.19}$$

The current sensitivity of galvanometer is expressed as

$$S_V = \frac{S_i}{R_{TH} + R_g}$$

Hence, from equation

$$\theta = \frac{S_i E R_3 \Delta R_X}{(R_{TH} + R_g)\,(R_3 + R_X)^2} \tag{2.20}$$

errors. The consideration of effect of contact and lead resistances is the basic purpose of Kelvin bridge. So that it provides increased accuracy for measurement of low value resistances Fig. 2.5 shows the basic circuit of Kelvin bridge.

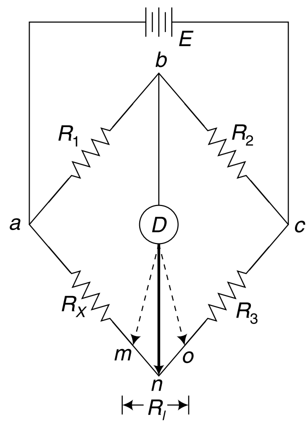


Fig. 2.5 Kelvin bridge

The resistance R_l represents the resistance of connecting leads from R_X to R_3. The R_X is unknown resistance to be measured.

Two possible galvanometer connections are indicated by dotted lines either to point *m* or to point *o*. When the galvanometer is connected to point *m*, the lead resistance R_1 is added to the unknown resistance value resulting in high value of unknown resistance R_X. On the other hand, if the galvanometer is connected to point *o*, the lead resistance R_l is added to standard resistance R_3, resulting too low value of unknown resistance R_X. If the galvanometer is connected to an intermediate point say *n* (as shown by dark line), such that the ratio of the lead resistance from point *m* to *n* and that from *o* to *n* is equal to the ratio of R_1 and R_2, i.e.,

$$\frac{R_{mn}}{R_{on}} = \frac{R_1}{R_2} \tag{2.22}$$

The Bridge balance equation is given as

$$(R_X + R_{mn})\, R_2 = R_1 (R_3 + R_{on}) \tag{2.23}$$

Since R_3 and R_X are changed to $R_3 + R_{on}$ and $R_X + R_{mn}$ respectively, from Eqn. (2.22), we get

$$\frac{R_{mn}}{R_{on}} + 1 = \frac{R_1}{R_2} + 1$$

$$= \frac{R_{mn} + R_{on}}{R_{on}} = \frac{R_1 + R_2}{R_2} \tag{2.24}$$

Since $R_l = R_{mn} + R_{on}$

Hence, from Eqn. (2.24), we get

$$\frac{R_l}{R_{on}} = \frac{R_1 + R_2}{R_2}$$

$$\Rightarrow \qquad R_{on} = \frac{R_2}{R_1 + R_2} \cdot R_l \qquad (2.25)$$

Hence,
$$R_{mn} = R_l - R_{on} = R_l\left(\frac{R_1}{R_1 + R_2}\right) \qquad (2.26)$$

Substituting the value of R_{mn} and R_{on} in Eqn. (2.23), we get

$$\left(R_X + \frac{R_l R_1}{R_1 + R_2}\right) R_2 = R_1\left(\frac{R_3 + R_2\, R_l}{R_1 + R_2}\right)$$

$$R_X + \frac{R_l\, R_1}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_2}{R_1 + \mathrm{R}_2}$$

$$R_X = \frac{R_1 R_3}{R_2} \qquad (2.27)$$

Equation (2.27) represents the balance equation for Wheatstone bridge and hence eliminating the resistance of connecting leads from point m to o. However, practically it is very difficult to determine the intermediate point n. The modified form of Kelvin bridge is termed Kelvin double bridge.

2.3.3 Kelvin Double Bridge

The modified form of Kelvin bridge is shown in Fig. 2.6. The two actual resistances of correct ratio are connected between point m and o. The galvanometer is connected at the junction of these resistances. Due to the introduction of second set of ratio arms, this bridge is termed Kelvin double bridge.

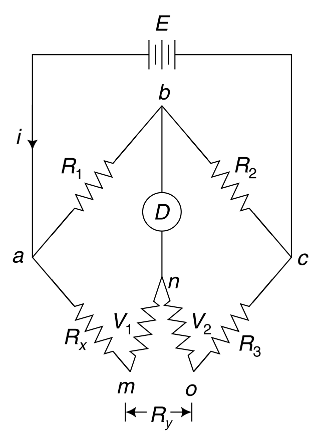


Fig. 2.6 Kelvin double bridge

The first set of ratio arm is R_1 and R_2 and the second set of ratio arm is r_1 and r_2. The galvanometer is connected to point n such as to eliminate the effect of connecting lead of resistance R_l between the unknown resistance R_X and standard resistance R_3. The ratio of r_1/r_2 is made equal to R_1/R_2. At bridge balance condition the current through galvanometer is zero, i.e., the voltage drop between point a to b (E_{ab}) is equal to the voltage drop between the point a-m-n i.e. E_{amn}

$$E_{ab} = \frac{R_1}{R_1 + R_2} \cdot E$$

$$= \frac{R_1}{R_1 + R_2}\, i[R_X + R_y \| (r_1 + r_2) + R_3]$$

$$= \frac{R_1 i}{R_1 + R_2}\left[R_X + \frac{R_y(r_1 + r_2)}{R_y + r_1 + r_2} + R_3\right] \tag{2.28}$$

and

$$E_{amn} = E_{am} + E_{r_1}$$

$$= \left[iR_X + \frac{i\, r_1}{(r_1 + r_2)}\left[\frac{(r_1 + r_2)R_y}{r_1 + r_2 + R_y}\right]\right]$$

$$= iR_X + \frac{i r_1 R_y}{r_1 + r_2 + R_y}$$

$$= i\left[R_X + \frac{r_1\, R_X}{R_y + r_1 + r_2}\right] \tag{2.29}$$

At bridge balance

$$E_{ab} = E_{amn}$$

$$\frac{R_1}{R_1 + R_2}\left[R_X + R_3 + \frac{R_y(r_1 + r_2)}{(R_y + r_1 + r_2)}\right] = R_X + \frac{r_1 R_Y}{R_y + r_1 + r_2}$$

$$R_X\left(\frac{R_2}{R_1 + R_2}\right) = \frac{R_1 R_3}{(R_1 + R_2)} + \frac{r_2 R_y}{R_y + r_1 + r_2}\left[\frac{R_1 r_2 - R_2 r_1}{(R_1 + R_2) r_2}\right]$$

$$\Rightarrow \qquad R_X = \frac{R_1 R_3}{R_2} + \frac{r_2 R_y}{R_y + r_1 + r_2}\left[\frac{R_1}{R_2} - \frac{r_1}{r_2}\right] \tag{2.30}$$

Since $$\frac{R_1}{R_2} = \frac{r_1}{r_2}$$

$$\Rightarrow \qquad R_X = \frac{R_1 R_3}{R_2} \tag{2.31}$$

The above equation represents the standard Wheatstone bridge balance equation. Since the resistance r_1, r_2, and R_y are not present in this equation. Hence, the effect of lead and contact resistance is completely eliminated. In typical Kelvin's bridge the range of resistance lies between 0.1 μΩ and 1.0 Ω, with accuracy of ± 0.05% to ± 0.2%.

Example 2.2 Kelvin double bridge uses:

Standard resistance = 100 μΩ

Inner ratio arms = 15 Ω and 30 Ω

Outer ratio arms = 40 Ω and 60 Ω

If the resistance of the connecting leads from standard to unknown resistance is 800 μΩ. Calculate the unknown resistance under this condition.

Solution Given

$$R_1 = 40\ \Omega,\ R_2 = 60\ \Omega,\ R_3 = 100\ \mu\Omega$$

$$r_1 = 15\ \Omega,\ r_2 = 30\ \Omega,\ R_y = 800\ \mu\Omega$$

From equation 2.30

$$R_X = \frac{R_1 R_3}{R_2} + \frac{r_2 R_y}{R_y + r_1 + r_2}\left[\frac{R_1}{R_2} - \frac{r_1}{r_2}\right]$$

$$= \frac{40 \times 100 \times 10^{-6}}{60} + \frac{30 \times 800 \times 10^{-6}}{(800 \times 10^{-6} + 15 + 30)}\left[\frac{40}{60} - \frac{15}{30}\right]$$

$$= 0.66 \times 10^{-4} + 5.33 \times 10^{-4}\ [0.16]$$

$$= 1.5128 \times 10^{-4}\ \Omega$$

$$= 151.28\ \mu\Omega$$

2.4 HIGH RESISTANCE MEASUREMENT BRIDGE

The measurement of high resistance of the order of 10^{10} Ω or more is often required in electrical equipment. For example, (i) insulation resistance of components like machines, cables, (ii) leakage resistance of capacitors, (iii) resistance of high circuit elements like vacuum tube circuits, etc. Because of very high resistance, current, flowing through the measuring circuit is low, which is very difficult to sense. Normal Wheatstone bridge used for measurement of resistances is not suitable for this purpose due to the leakage current around the test specimen becomes of the same order as through the specimen itself or quite high compared to it.

For high resistance measurement, high voltage source is used to obtain sufficient current and galvanometer deflection. As a result leakage current is also increased. In order to avoid the leakage current effect in the measuring circuit, guard circuits are generally used. The high resistance to be measured is mounted on two insulating posts which in turn are mounted on a metal platform known as guard point. The guard point is electrically connected to the junction of the ratio arms of the bridge circuit. Due to this insulating posts become electrically parallel to the resistances of the ratio arm resistance. Hence, leakage resistance R_{la} is parallel to R_1 and R_{lb} to R_2. Effectively the unknown resistance R_X to be measured has three terminals marked (a), (b) and (c) as shown in Fig. 2.7. With R_{la} and R_{lb} very large, resistances R_1 and R_2, whom they are paralleling, are not much affected as R_1 and R_2 are not made very large, like R_3 which again is selectable. The unknown resistance R_X is obtained with null balance method and an amplifier is used to drive the display meter which is basically a null detection.

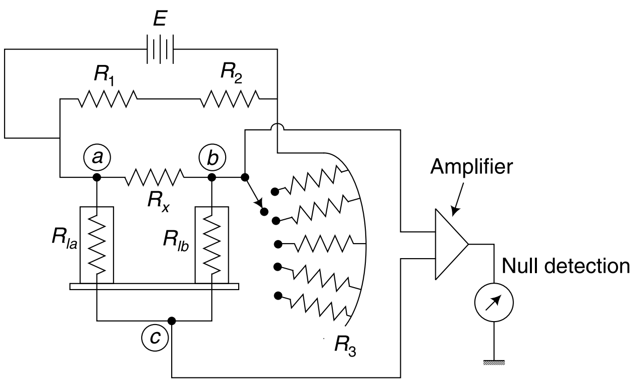


Fig. 2.7 High resistance measurement bridge

Example 2.3 (a) What would be the error in measuring a high resistance of $10^9\ \Omega$ if the leakage resistances R_a and R_b are $10^{10}\ \Omega$ each but the guard point arrangement is not used?

(b) If the leakage resistances are the same as the unknown resistance, what would be the error.

Solution (a) In the absence of the guard point arrangement, two $10^{10}\ \Omega$ resistances are in series and become parallel to the $10^9\ \Omega$ resistance. Hence, the effective unknown resistance

$$R_X = \frac{10^9 \times 2 \times 10^{10}}{10^9\ (1 + 20)} = 0.95 \times 10^9\ \Omega$$

$$\text{The error} = \frac{(10^9 - 0.95 \times 10^9)}{10^9} \times 100 = 5\%$$

(b) In this case, the effective unknown resistance

$$R_X = \frac{10^9 \times 2 \times 10^9}{10^9\,(1+2)} = 0.67 \times 10^9\ \Omega$$

$$\text{The error} = \frac{(10^9 - 0.67 \times 10^9)}{10^9} \times 100$$

$$= 33.3\%$$

2.5 AC BRIDGES AND THEIR APPLICATIONS

An ac bridge similar to dc bridges consists of four arms, an ac source of excitation at the desired frequency and a null detector. For measurements at low frequencies the power line may be used as a source of excitation whereas at higher frequencies generally, an oscillator is used as a source. The operating frequencies of these oscillators are constant and easily adjustable. A typical oscillator has a frequency range of 40 Hz to 125 kHz with a power output of 7W headphones, vibrational galvanometer and tuneable amplifier circuits are generally used as a null detector for ac bridges. The headphones are used as a detector at the frequency of 250 Hz to 3-4 kHz. While working with single frequency a tuned detector is the most sensitive detector. Vibrational galvanometers are useful for low audio frequency range from 5 Hz to 1 kHz, but are commonly used below 200 Hz. Tuneable amplifier detectors are used for frequency range of 10 Hz to 100 Hz.

2.5.1 General Equation for Bridge Balance

Let us consider a general form of an ac bridge as shown in Fig. 2.8. The bridge circuit consists of a network of four impedance arms z_1, z_2, z_3 and z_4 respectively, forming a closed circuit. For bridge balance, the potential of point *b* must be same as the potential of point *d*. These potentials must be equal in terms of amplitude as well as phase. Thus, the voltage drop from *a* to *b* must be equal to voltage drop across *a* to *d*, in both magnitude and phase for the bridge balance, i.e.

$$E_1 = E_3$$

$$\Rightarrow \qquad i_1 Z_1 = i_3 Z_3 \qquad \text{(i)}$$

Also at balance

$$i_1 = i_2 = \frac{E}{Z_1 + Z_2}$$

and

$$i_3 = i_4 = \frac{E}{Z_3 + Z_4}$$

Hence, from Eqn. (i), we have

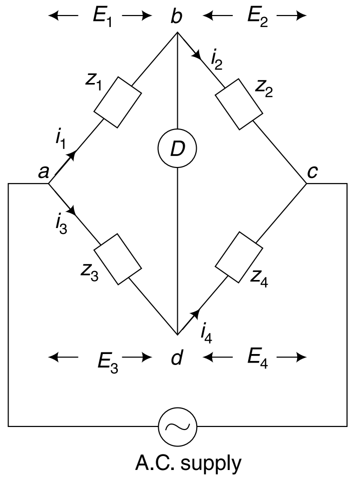


Fig. 2.8 Basic ac bridge circuit

$$\frac{EZ_1}{Z_1 + Z_2} = \frac{EZ_3}{Z_3 + Z_4}$$

$$\Rightarrow \qquad Z_1 Z_4 = Z_2 Z_3 \tag{2.32}$$

Equation (2.32) is the equation for balance of ac bridge in the impedance form. The balance equation in the admittance (reciprocal of impedance) form can be expressed as

$$\frac{1}{Y_1\, Y_4} = \frac{1}{Y_2 Y_3}$$

$$\Rightarrow \qquad Y_1\, Y_4 = Y_2\, Y_3 \tag{2.33}$$

In the polar form the impedance Z can be written as

$$Z = Z \angle \theta$$

where Z represents the impedance and θ represents the phase angle of complex impedance Z. Hence, the bridge arm impedances in polar form can be expressed as

$$Z_1 = Z_1 \angle \theta_1$$

$$Z_2 = Z_2 \angle \theta_2$$

$$Z_3 = Z_3 \angle \theta_3$$

$$Z_4 = Z_4 \angle \theta_4$$

where Z_1, Z_2, Z_3 and Z_4 are the magnitudes and θ_1, θ_2, θ_3, and θ_4 are the phase angles. Hence, the balance equation in polar form representation will be

$$(Z_1 \angle \theta_1) \cdot (Z_4 \angle \theta_4) = (Z_2 \angle \theta_2)\,(Z_3 \angle \theta_3) \tag{2.34}$$

Since, in complex number multiplication the magnitudes are multiplied and the phase angles are added, the equation can be written as

$$Z_1 Z_4 \left(\underline{/\theta_1 + \theta_4}\right) = Z_2\, Z_3 \left(\underline{/\theta_2 + \theta_3}\right) \tag{2.35}$$

i.e. ⇒ $$Z_1 Z_4 = Z_2 Z_3$$

and $$\angle \theta_1 + \angle \theta_4 = \angle \theta_2 + \angle \theta_3$$

Hence, from above equation (2.35), the two conditions must be satisfied for bridge balance.

(i) The product of the magnitudes of the opposite arms must be equal.

(ii) The sum of phase angles of the opposite arms must be equal.

The value of phase angles depends on the type of components of individual impedance. For inductive impedance the phase angles are positive and for capacitive impedance the phase angles are negative, i.e.,

$$Z_L = R + jX_L = |Z_L| \angle +\theta$$

$$Z_C = R - jX_C = |Z_C| \angle -\theta$$

where

$$X_L = 2\pi f L\ \Omega$$

$$X_C = \frac{1}{2\pi f C}\ \Omega$$

f = operating frequency

Example 2.4 The four impedances of an ac bridge as shown in Fig. 2.8 are $Z_1 = 500 \angle 40^\circ\ \Omega$, $Z_2 = 100 \angle -90^\circ\ \Omega$, $Z_3 = 45 \angle 20^\circ\ \Omega$ $Z_4 = 30 \angle 30^\circ\ \Omega$. Find out whether the bridge is balanced or not.

Solution As we know in polar co-ordinate system representation, the bridge balance conditions are

$$Z_1 Z_4 = Z_2\, Z_3 \quad \text{and}$$

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

Thus,

$$Z_1 Z_4 = 500 \times 30 = 15 \times 10^3\ \Omega$$

$$Z_2 Z_3 = 100 \times 45 = 4.5 \times 10^3\ \Omega$$

also

$$\angle\theta_1 + \angle\theta_4 = 40^\circ + 30^\circ = 70^\circ$$

$$\angle\theta_2 + \angle\theta_3 = 90^\circ + 20^\circ = 70^\circ$$

This indicates that phase relationship is satisfied whereas magnitude condition is not satisfied. Hence, bridge is unbalanced.

2.5.2 Schering Bridge (Measurement of Capacitance)

This is the most common bridge used for measurement of unknown capacitance, dielectric loss, relative permittivity and power factor. Figure 2.9 shows the basic circuit arrangement of the bridge and its phasor diagram under balance conditions.

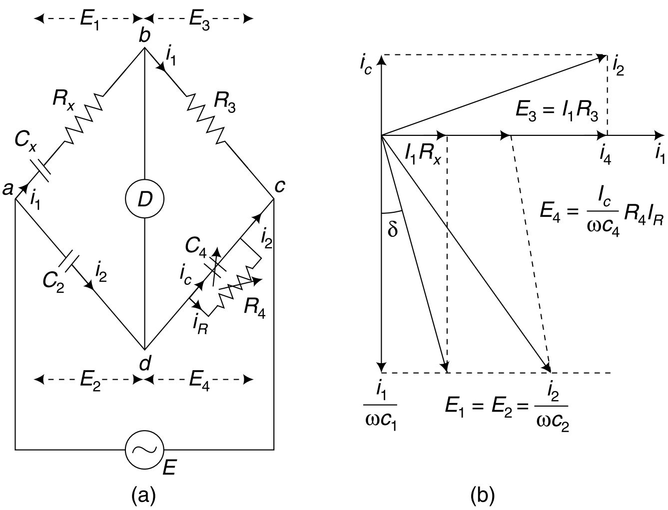


Fig. 2.9 (a) Schering bridge for measurement of capacitance (b) Phasor diagram

Two branches consist of non-inductive resistance R_3 and a standard capacitor C_2. The standard capacitor is usually a high-quality mica capacitor (low-loss) for general measurement or an air capacitor (having very stable value and a very small elastic field) for insulation measurement. One of the arms consists of a variable capacitor connected in parallel with a variable non-inductive resistance R_4. The remaining arm consists of unknown capacitor C_X whose capacitance is to be determined. Connected in series with a resistance R_X to represent loss in the capacitance C_X, the impedance of four arms are

$$Z_1 = \left(R_x + \frac{1}{j\omega c_x}\right)$$

$$Z_2 = \left(\frac{1}{j\omega c_1}\right)$$

$$Z_3 = R_3$$

we obtain that the dissipation factor is the reciprocal of the quality factor Q and therefore

$$Q = \frac{1}{D} \tag{2.41}$$

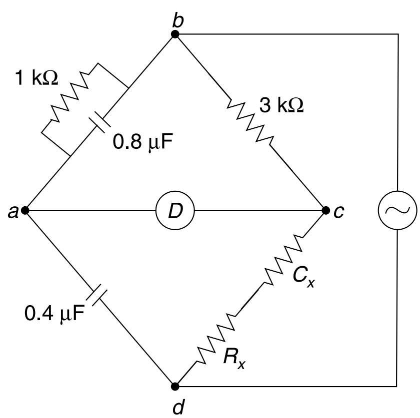
Hence, the dissipation factor tells us about the quality of the capacitor, i.e., how close the phase angle of the capacitor is to the ideal value 90º. Substituting the value of C_x and R_x in Eqn. (2.39), we have

$$D = \omega \frac{R_4 C_2}{R_3} \cdot \frac{C_4 R_3}{C_2} \Rightarrow D = \omega R_4 C_4 \tag{2.42}$$

If the frequency and resistor R_4 in Schering bridge is fixed, the capacitor C_4 can be calibrated to read the dissipation factor directly.

Example 2.5 An ac bridge was made up as follows: arm *ab*, a capacitor of 0.8 μ*F* in parallel with 1 kΩ resistance, *bc* a resistance of 3 kΩ, arm *cd* an unknown capacitor C_x and R_x in series, arm *da* a capacitance of 0.4 μF. The supply at 1 kHz is connected across *bd* and a detector across ac. Determine the value of unknown capacitance C_x, unknown series resistance R_x and dissipation factor.

Solution If we draw the sketch of the given problem we find the given ac bridge is Schering bridge as shown below. Hence, R_3 = 3 kΩ, C_2 = 0.4 μF, C_4 = 0.8 μ*F*, R_4 = 1 kΩ, ω = 1 kHz



$$R_x = \frac{C_4 R_3}{C_2}$$

$$= \frac{3 \times 10^3 \times 0.8 \times 10^{-6}}{0.4 \times 10^{-6}}$$

$$= 6 \text{ k}\Omega$$

$$C_x = \frac{R_4 C_2}{R_3}$$

$$= 1 \times 10^3 \times 0.4 \times 10^{-6} \times 10^3 = 0.133 \times 10^{-6} \text{ F}$$

Dissipation factor $\quad D = \omega C_x R_x = 2\pi f\, C_x R_x$

$$= 2 \times 3.14 \times 10^3 \times 0.133 \times 10^{-6} \times 6 \times 10^3$$

$$= 5.011$$

2.5.3 High Voltage Schering Bridge

The Schering bridge is widely used for measurement of small capacitance and dissipation factor, and is then usually supplied from a high voltage or a high frequency source. Figure 2.10 shows the circuit arrangement for high voltage Schering bridge. The bridge is connected to a high voltage supply through transformer usually at 50 Hz. The vibrational galvanometer is used as a detector.

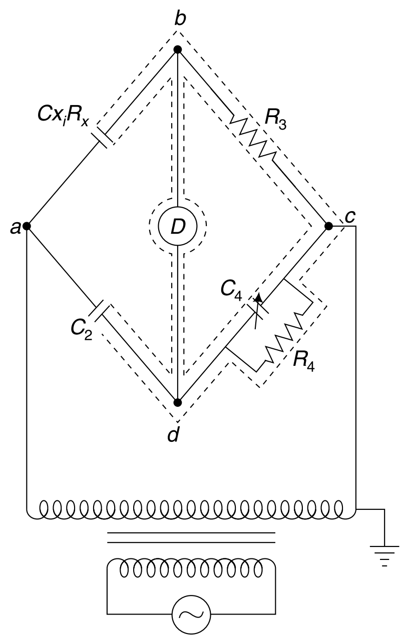


Fig 2.10 High voltage Schering bridge

The capacitors designed for high voltage are connected in arms *ab* and *ad*. The impedance of these two arms is kept very high in comparison to the other two arms *bc* and *cd*, so that major portion of the potential drop will be in the arms *ab* and *ad* and very small potential drop occurs in the arms *bc* and *cd*. In order to maintain this, the point *c* is earthed. Hence, for the safety of the operator it is advantageous to locate the controls in arms *bc* and *cd*. These controls should be and are at low potential with respect to earth. For the same reason detector is also at low potential.

A spark gap (set to breakdown about 100 V) is connected across arms *bc* and *cd* in order to prevent high voltage appearing across arms *bc* and *cd* in the case of breakdown of either of the high voltage capacitor.

Earth screens are provided in order to avoid errors caused due to inter-capacitance between high and low impedance arms of the bridge.

2.5.4 Wien Bridge (Measurement of Frequency)

The Wien's bridge is generally known as a frequency determining bridge. Wien's bridge finds its application in various useful circuit. For example, it is used in audio and high frequency oscillators as the frequency determining device. It is used as notch filter in harmonic distortion analyzer. It can also be used for the measurement of an unknown capacitor with great accuracy. Figure (2.11) shows the circuit arrangement of Wien bridge under balance condition. Two branches (*bc* and *cd*) consist of non-inductive resistances R_3 and R_4. One of the arms (*ab*) consists of a capacitor C_1 connected in parallel to the non-inductive resistor R_1. The adjoining arm (*bc*) has a series combination of capacitor C_2 and resistor R_2. A source of current is applied to two opposite junctions across *a* and *c* and null detector is connected to other two junctions *b* and *d*.

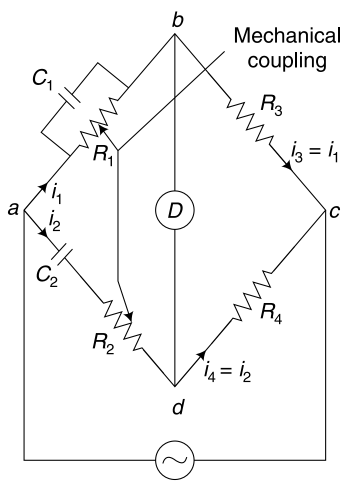


Fig. 2.11 Wien bridge

Hence, the impedances of four arms are

$$Z_1 = \left(R_1 \parallel \frac{1}{j\omega C_1}\right) = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$Z_2 = R_2 + \frac{1}{j\omega C_2}$$

Example 2.6 The arms of a four-arm bridge *a*, *b*, *c* and *d* supplied with sinusoidal voltage have the following values.

arm *ab*: A resistance of 800 Ω in parallel with a capacitance of 2 μF

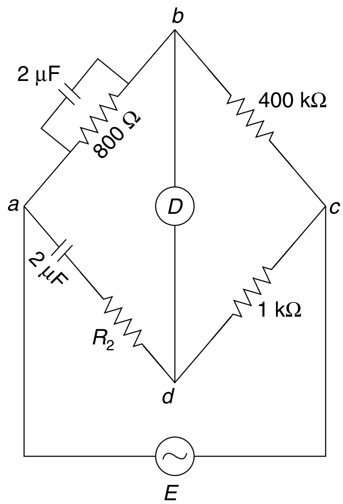
arm *bc*: 400 Ω resistance

arm *cd*: 1 kΩ resistance

arm *da*: A resistance R_2 in series with 2 μF capacitance

Determine the value of R_2 and frequency at which the bridge will balance.

Solution If we draw the sketch of the given problem, we find that given ac bridge is Wien bridge as shown below.



Hence, from balance equation

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

$$\Rightarrow \qquad R_2 = \left(\frac{R_4}{R_3} - \frac{C_1}{C_2}\right) R_1$$

$$= \left(\frac{1000}{400} - \frac{2 \times 10^{-6}}{2 \times 10^{-6}}\right) \times 800$$

$$= 1.2 \text{ k}\Omega$$

and

$$f = \frac{1}{2\pi\sqrt{R_1 R_2\, C_1 C_2}} = \frac{1}{2\pi\sqrt{1200 \times 1000 \times 2 \times 10^{-6} \times 2 \times 10^{-6}}}$$

$$= 72.6 \text{ Hz}$$

2.5.5 Maxwell Bridge (Measurement of Inductance)

Maxwell bridge can be used to measure inductance by comparison either with a variable standard self-inductance or with a standard variable capacitance. These two measurement methods can be done by using two different Maxwell bridge forms.

Maxwell Inductance Bridge

In this bridge arrangement the value of unknown inductance is measured by comparison with a variable standard self-inductance. Figure 2.12 shows the circuit arrangement and phasor diagram for Maxwell's inductance bridge under balance condition. Two branches *bc* and *cd* consist of non-inductive resistance R_3 and R_4. One of the arms *ad* consists of variable inductance L_2 of fixed internal resistance r_2 connected in series with variable resistance R_2. The remaining arm *ab* consists of unknown inductance L_x of resistance R_x. A source of current is applied to two opposite junctions across *ac* and a null detector is connected to the other two junctions *b* and *d*.

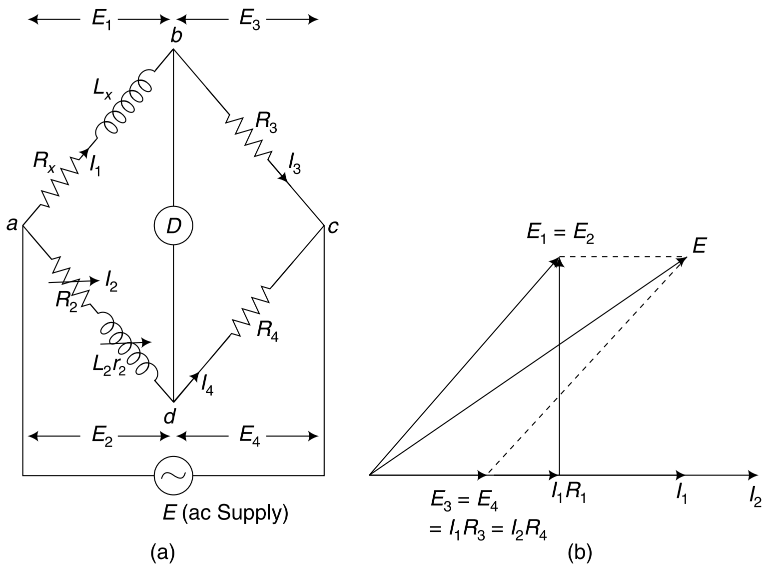


Fig. 2.12 (a) Maxwell inductance bridge (b) Phasor diagram

Hence, the impedances of four arms are

$$Z_1 = R_x + j\omega L_x$$

$$Z_2 = R_2 + r_2 + j\omega L_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4$$

At balance we get

$$Z_1Z_4 = Z_2Z_3$$

$$\Rightarrow \qquad (R_x + j\omega L_x)\, R_4 = (R_2 + r_2 + j\omega L_2)\, R_3$$

$$R_xR_4 + j\omega L_x\, R_4 = (R_2 + r_2)\, R_3 + j\omega L_2\, R_3 \tag{2.49}$$

Equating real and imaginary term in the above equation, we have

$$R_x = \frac{R_3}{R_4}(R_2 + r_2) \tag{2.50}$$

$$L_x = \frac{R_3}{R_4} L_2 \tag{2.51}$$

We observe that the two conditions for bridge balance Eqn. (2.50) and Eqn. (2.51) result in an expression determining the unknown inductance value by comparison with variable inductance L_2 of fixed resistance r_2. The resistance r_2 is a decade resistance box.

2.5.6 Maxwell Inductance Capacitance Bridge

In this bridge arrangement, the value of unknown inductance is measured by comparison with a variable standard capacitor. Figure 2.13 shows its circuit arrangement and phasor diagram. Two arms *bc* and *ad* consist of non-inductive resistance R_2 and R_3. One of the arms *ac* consists of variable standard capacitor C_4 connected in parallel to a non-inductive resistance R_4. The remaining arm *ab* consists of unknown inductance L_x of effective resistance R_x. A source of current is applied to two opposite junctions across *ac* and a null detector is connected to the other two junctions *b* and *d*. Hence, the impedances of four arms are

$$Z_1 = R_x + j\omega L_x$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4 \parallel \frac{1}{j\omega C_4} = \frac{R_4}{1 + j\omega C_4 R_4}$$

At balance we get

$$Z_1Z_4 = Z_2Z_3$$

$$(R_x + j\omega L_x)\frac{R_4}{1 + jwC_4R_4} = R_2R_3$$

$$= R_xR_4 + jwL_xR_4 = R_2R_3 + jwC_4\, R_2R_3R_4 \tag{2.52}$$

Equating real and imaginary terms in Eqn. (2.52), we have

$$R_x = \frac{R_2 R_3}{R_4} \tag{2.53}$$

$$L_x = R_2 R_3 C_4 \tag{2.54}$$

Hence, from the above condition for bridge balance the unknown inductance value can be determined by comparison with variable standard capacitor. The quality factor of the coil is given by

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega R_2 R_3 C_4}{\left(\frac{R_2 R_3}{R_4}\right)}$$

$$\boxed{Q = \omega R_4 C_4}$$

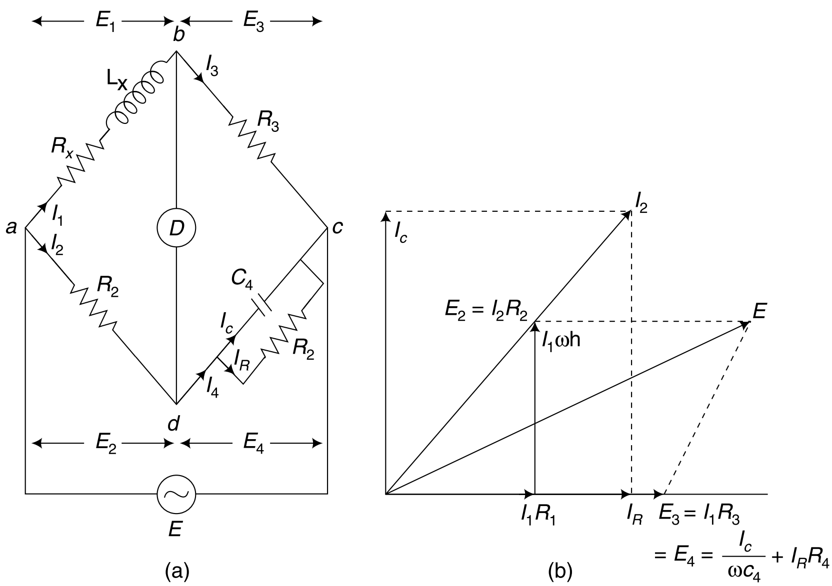


Fig. 2.13 (a) Maxwell inductance capacitance bridge

Advantages

1. The balance equation is independent of losses associated with inductance.
2. The frequency does not appear in any of the two balance equations.
3. The scale of resistance can be calibrated to read inductance directly.
4. It is very useful for measurement of wide range of inductances at power and audio frequency.

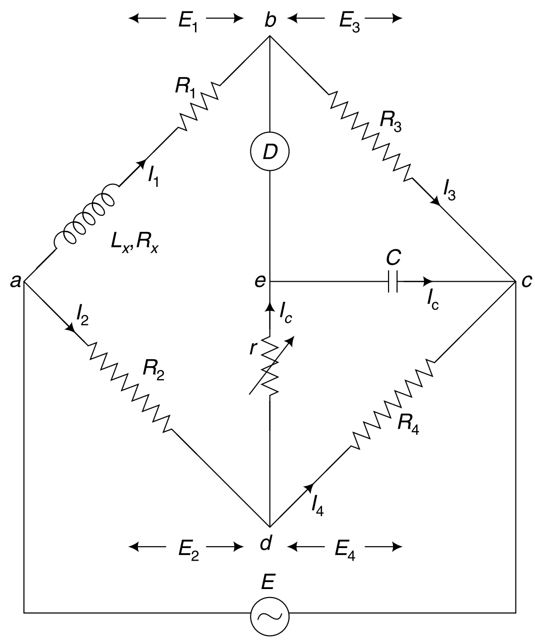


Fig. 2.14 (a) Anderson bridge

Hence, the impedance of four arms are

$$Z_1 = R_x + R_L + j\omega L_x$$

$$Z_2 = R_2$$

$$Z_3 = R_3$$

$$Z_4 = R_4$$

and impedance between dc

$$Z_{dc} = r + \frac{1}{j\omega c}$$

At balance, we have

$$I_1 = I_3$$

$$I_2 = I_4 + I_c$$

$$E_1 = E_2$$

Since the capacitor C is connected between point c and e. Hence, the voltage drop across the capacitor and R_3 will be equal, i.e.

$$I_3 R_3 = I_C \times \frac{1}{j\omega C}$$

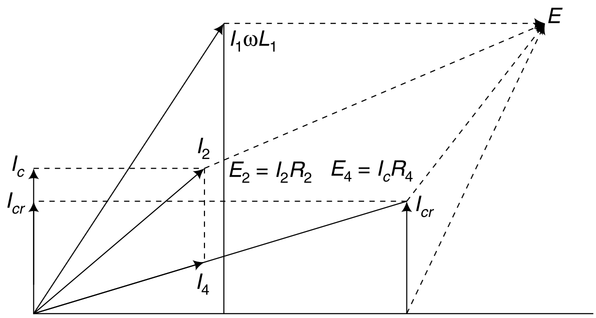


Fig. 2.14 (b) Phasor diagram

Advantages

1. It can be used for precise measurement of inductance over a wide range of values.
2. A fixed capacitor is used instead of variable capacitor as in the case of Maxwell bridge. Hence, it is cost effective.
3. It can also be used for accurate measurement of capacitance in terms of inductance.
4. The balance can be easily obtained in Anderson bridge than in Maxwell bridge for low Q-coils.

Disadvantages

1. The bridge circuit is more complicated than other bridges.
2. It requires more number of components.
3. Increased occuputational complexity.
4. Bridge cannot be easily shielded due to additional junction point.

EXERCISE

Short Answer Type Questions

1. What is a bridge circuit?
2. Explain the applications and advantages of bridge circuits?
3. What are the types of bridges?
4. Name a few ac bridges used and specify which bridge is used for what type of measurement.
5. Explain briefly how a 'Wheatstone bridge' is used for measurement of resistance.
6. Explain the sources of errors in Wheatstone bridge and its limitations.
7. Describe the sources and null detectors used for ac bridges.
8. Explain briefly the following bridges (i) Kelvin bridge (ii) Kelvin double bridge.

under balance $r_1 = 43.1\ \Omega$ and $r = 229.7\ \Omega$.

Multiple Choice Questions

1. A Wheatstone bridge cannot be used for precision measurements because errors are introduced into an account of
   (a) thermoelectric emf (b) contact resistance
   (c) resistance of connecting leads (d) all of above
2. High resistances are provided with a guard terminal. The guard terminal is used to
   (a) guard the resistance against stray electrostatic field
   (b) guard the resistance against overload
   (c) bypass the leakage current
   (d) none of above
3. Maxwell inductance capacitance bridge is used for measurement of inductance of
   (a) low Q coils (b) medium Q coils
   (c) high Q coils (d) low and medium Q coils
4. Frequency can be measured by using
   (a) Wien bridge (b) Schering bridge
   (c) Maxwell bridge (d) Heaviside Campbell bridge
5. The effective reactance of an inductive coil
   (a) increases because of stray capacitance as frequency increases
   (b) remains unchanged
   (c) decreases because of stray capacitance as frequency increases
   (d) None of above
6. A bridge circuit works at a frequency of 10 kHz. The following can be used as detector for detection of null condition.
   (a) Vibrational galvanometer and headphones
   (b) Headphones and tuneable amplifier
   (c) Vibrational galvanometer and tuneable amplifier
   (d) Vibrational galvanometer, tuneable amplifier, headphone.
7. Dissipation factor (D) and quality factor (Q) of a coil are related as
   (a) $D = Q$ (b) $D = \frac{1}{Q}$
   (c) $D = Q^2$ (d) none of above
8. Inductance can be measured by
   (a) Maxwell bridge (b) Hay bridge
   (c) Schering bridge (d) Wien bridge