2
CHAPTER

Modes of Controller Operation

In this chapter the different modes of controller operation are discussed. In any control system, where the dynamic variable has to be maintained at the desired set point value, it is the controller alone which enables to meet the requirements of control objective. The controller inputs the result of a measurement of the controlled or dynamic variable and determines an appropriate output to the final control element.

Usually final control elements understand certain range of signal transmission only. Hence the o/p of a controller must be translated to the range of possible values of final control element. This range is called as *controller parameter range.*

In terms of analog signal transmission this range of controller output is 4–20 mA. standard signal.

4 mA corresponds to 0% controller output i.e. minimum and 100% controller output i.e. maximum would be 20 MA respectively.

In discrete state process control systems, the output will range over all the states of the n-bit output. Generally, all 0's correspond to minimum output and all 1's represent maximum controller output. The controller output as percentage of full scale when the output varies between specified limits is given by the expression

$$\% P = \frac{\Delta p - P_{\text{min}}}{P_{\text{max}} - P_{\text{min}}} \times 100$$

where Δp = Actual controller output.

$P_{\text{max}}, P_{\text{min}} \Rightarrow$ Standard signal transmission range.

For example in 4–20 mA signal transmission, 4 mA corresponds to 0% controller 0/p and 20 mA, 100% controller 0/p. If we want to evaluate how much in terms of percentage would be 12 mA controller output, the above equation can be used.

One of the most common examples is in the operation of thermostat. The heater is turned ON or OFF whenever there is error i.e temperature above or below set point. When the temperature is at set point ($e < 0$), the controller's response is undefined.

Neutral Zone: Practically in designing of two position controller, whenever the controller output has to change over from 0% → 100% or vice versa, there is a differential gap known as Neutral Zone around Zero error point where virtually no controller output results.

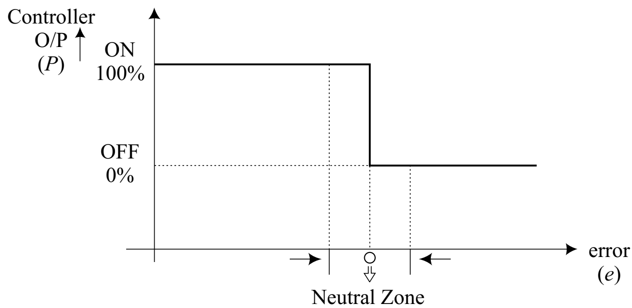


FIG. 2.1

This Neutral Zone is purposefully designed above a certain minimum quantity to prevent excessive cycling. It is an example of desirable hysteresis in a system. The process under two position control must allow continued oscillations in the controlled or dynamic variable when the error is zero and these oscillations are verily the function of Neutral Zone size.

Applications

The main application of two position control are:

(1) Room heating or air conditioning systems
(2) Liquid bath temperature control
(3) Level control in large volume tanks.

Multi position mode

It is the logical extension of two-position control where rather than only two settings of controller output, namely, 0% (OFF) and 100% (ON), several intermediate settings are provided.

The most common example is a three position controller

$$P = \begin{cases} 100\% & e > e_2 \\ 50\% & -e_1 < e < e_2 \\ 0\% & e < -e_1 \end{cases}$$

As long as the error is between e_2 and e_1 of the set point, the controller stays at a nominal setting of 50% if the error exceeds the set point by e_1 or more, the output is increased by 100%. If it is less than the set point by $-e_1$ or more the controller output is reduced to zero.

2.3.2 Floating Controller Mode

In this control action, the specific controller output is not uniquely determined by error unlike the previous mode. If the error is zero, the controller output

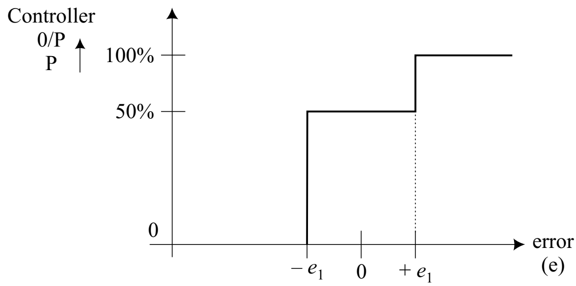


FIG. 2.2

floats at whatever setting it was when the error went to zero. And whenever error occurs the controller output again begins to change.

Single speed floating control mode

In this mode the controller output changes at a fixed rate whenever the error exceeds the neutral zone. The analytic expression is

$$\frac{dp}{dt} = \pm K_F \quad |e| > \text{Neutral Zone}$$

where $\frac{dp}{dt}$ = rate of change of controller output,

K_F = Rate constant [%/ sec]

The specific controller output is obtained by integrating the above equation.

$$P = \pm K_F t + P(0)$$

where $P(0)$ refers to initial setting of the controller when the error is zero. The characteristics of single speed floating controller mode is

Illustration of floating action

Consider the graph showing controller output with error, that exceeds. The Neutral Zone, after certain instants of time as shown in Figure (2.4).

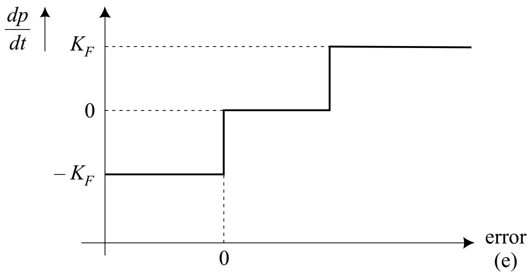


FIG. 2.3

Between the instants $0 - t_1$ the error is within the limits of Neutral Zone. Hence the controller output floats at whatever setting it was when the error went to zero i.e. $P(0)$. From t_1 to t_2 error exceeds the neutral zone and the controller output changes at a fixed rate, $\pm K_F\ t$, depending upon Nature of Process, i.e. reverse or direct action.

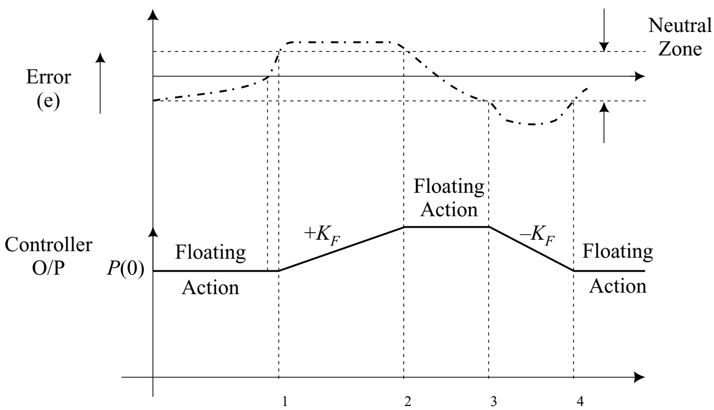


FIG. 2.4

Multiple speed floating controller mode

In single speed, the rate of change of controller output is fixed at certain levels of $\pm K_F$. In multiple speed control mode, there are several different rates of controller output depending on the nature of deviation or error.

The analytic expression can thus be modified as

$$\frac{dp}{dt} = \pm K_{Fi} \quad e > e_i$$

The above equation implies, whenever error is in the range of certain preset values there will be corresponding rate of change of controller output.

Application

The single speed or multiple speed floating action can be employed with

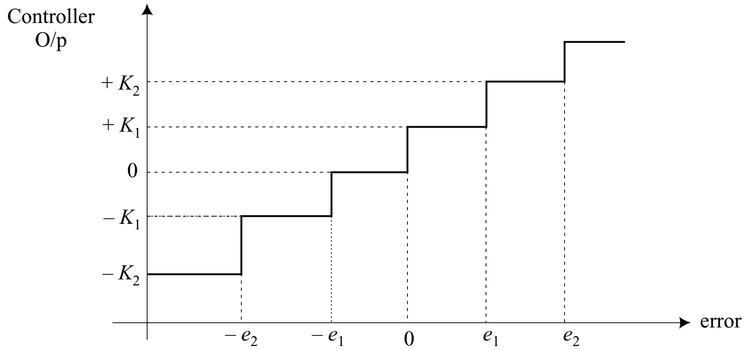


FIG. 2.5

general flow control system or pressure control system with small capacities and having self-regulation to some extent with small dead time or Lag.

2.4 CONTINUOUS CONTROLLER MODES

Unlike the discontinuous modes of operation, in continuous modes there is a smooth relation existing between controller output and error, whenever there is a deviation of the controlled variable from set point. The controller responds in a smooth fashion to achieve the control objective.

Some of the continuous modes are natural extension of discontinuous modes itself. We will discuss about these modes in detail in this section.

2.4.1 Proportional Controller Mode

It is a natural extension of two position mode. In this mode there is a smooth relation between controller output and error before the output saturates at 0% [OFF state] or 100% [ON state]. Between these two saturation levels, there is a band of errors, where every value of error has a unique value of controller output i.e. there is a one-to-one correspondence existing between controller output and error.

This range of errors to cover 0% to100% controller output is known as **"PROPORTIONAL BAND".**

The analytic expression is written within this proportional band which describes the operation of proportional control action.

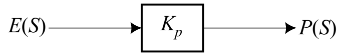
$$P = K_p e + P_0 \qquad ...(1)$$

K_p = proportional gain,

P_0 = Initial value of controller o/p.

The transfer function representation is obtained by applying Laplace transform to equation (1),

$$P(s) = K_p . E(s) \text{ [Initial value is assumed to be zero]}$$

∴ 

CHARACTERISTICS OF PROPORTIONAL MODE

Figure (2.6) represents the characteristics of *P*-control action. The proportional band ($P . B$) is related to the gain of proportional controller K_p and is defined

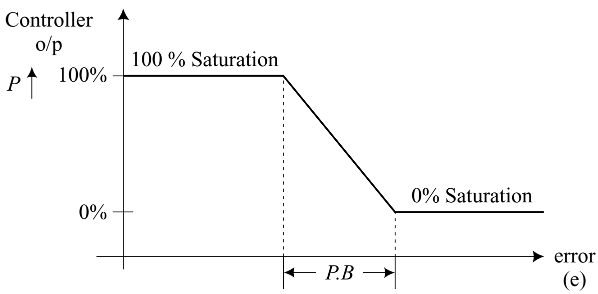


FIG. 2.6

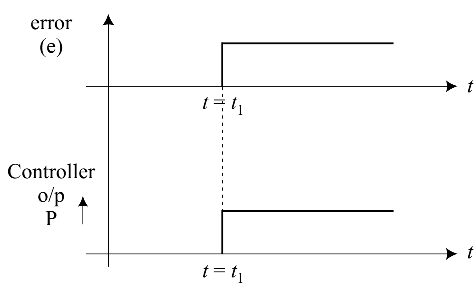


FIG. 2.7

as $P . B = \dfrac{100}{K_p}$. Thus by proper choosing of 'K_p' the $P . B$ can be changed

In Figure 2.7, assume a step change in deviation,

Let $\quad e = A$ [step change].

$$P = K_p\, e,$$

$$\Rightarrow \quad P = K_p\, A.$$

Thus if there is error, for every 1% of error a correction of K_p % is added or subtracted from initial value of controller output 'P_0' depending on the nature of error. If the error is zero which implies if the controlled variable is at set point value the controller output is constant at P_0.

Offset

One of the main disadvantage of proportional control action is that it produces a permanent residual error in the operating point of the controlled variable when a process load change results in error. This residual error is known as *OFFSET.*

Illustration of offset

Consider a first order process having process equation or transfer function

$$\frac{C(s)}{R(s)} = \frac{1}{1+TS}.$$

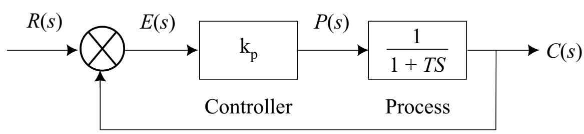


FIG. 2.8

If we assume a sudden change in the input causing a sudden error. The proportional controller adjusts the final control element proportionately affecting the dynamics of the process to bring back the output to the set point value. This is possible if and only if the steady state error is zero.

From the definition of steady state error

$$e_{ss} = \underset{s \to 0}{\text{Lt}} \frac{S.R(s)}{1+G(s)H(s)}$$

Let $$R(s) = \frac{A}{S} \text{[step change]}$$

$$e_{ss} = \underset{s \to 0}{\text{Lt}} \frac{\$ \frac{A}{\$}}{1+\frac{K_p}{1+TS}}$$

$$\boxed{e_{ss} = \frac{A}{1+K_p}} \qquad ...(2)$$

This steady state error is not zero but inversely proportional to gain K_p. Thus, this control action fails to make the error zero whenever the o/p changes due to changes in input or Load (process) variables. This is known as offset. From eqn. (2),

$$\text{offset} \propto \frac{1}{K_p} \quad ...(3)$$

Therefore both $P.B$ and offset are inversely proportional to gain K_p. To reduce offset K_p should be increased but it reduces the width of P.B and the control mode changes to simple ON/OFF mode. Hence offset constitutes the inherent disadvantage of proportional controller mode.

2.4.2 Integral Controller Mode

This mode is an extension of floating control mode. Unlike the previous discontinuous mode, the rate of change of controller output is not constant at $\pm K_F$ but is directly proportional to error.

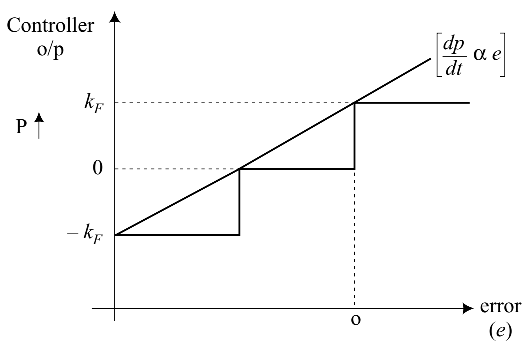


FIG. 2.9

The analytic expression may be written as

$$\frac{dp}{dt} \propto e$$

$$\frac{dp}{dt} = K_I \,.\, e \quad ...(4)$$

where K_I = Integral scaling.

From equation (4), the rate of change of controller o/p is proportional to error. Thus when error occurs, the controller responds by sending an output at a rate that depends upon the size of the error and integral scaling K_I.

For particular error, the output will begin to increase or decrease at a rate K_I %/second for every 1% of error.

The specific controller output is obtained by integrating eqn. (4).

$$P = K_I \int e\,dt + P(0) \qquad \text{...(5)}$$

where $P(0)$ = Initial value of controller o/p when error is zero.

The integral scaling K_I is often expressed in terms of integral time "T_i"

$$T_i = \frac{1}{K_I} \qquad \text{...(6)}$$

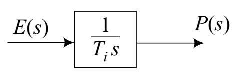
From (5) and (6),

$$\boxed{P = \frac{1}{T_i}\int e\,dt + P(0)} \qquad \text{...(7)}$$

The units of integral time is seconds or minutes.

The transfer function of integral controller mode is obtained by applying $L.T$ to equation (7)

$$P(s) = \frac{1}{T_i\,s} \,.\, E(s).$$



Characteristics of integral action

Consider a step change in deviation

$$e = A \text{ [step charge]}$$

$$P = \frac{1}{T_i} \int A.dt$$

$$\Rightarrow \qquad P = \left[\frac{A}{T_i}\right] t$$

From Figure (2.10), the main disadvantage of integral action is that, its response to error is slow or sluggish. But however, due to its basic nature of rate of change of output it eliminates the error and brings back the controlled variable to the set point value.

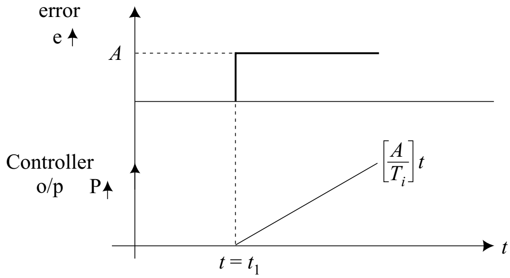


FIG. 2.10

Since the rate of change of controller output can be reset at any instant of time t by changing "T_i". This mode is often referred to as *"RESET CONTROLLER MODE"*. The integral time "T_i" is also known as *Reset time*. Let us see the effect of integral control action on error by taking a first order process.

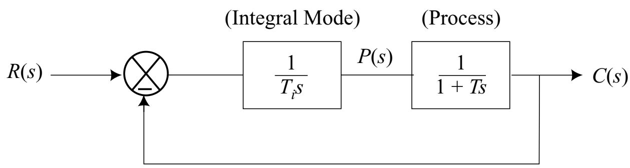


FIG. 2.11

$$e_{ss} = \underset{s \to 0}{\text{Lt}} \frac{S.R(s)}{1 + G(s)H(s)}$$

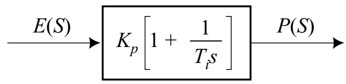
Let $\quad R(s) = \dfrac{A}{S}$ [causing step change in deviation],

$$e_{ss} \text{ [error]} = \underset{s \to 0}{\text{Lt}} \frac{\$ \dfrac{A}{\$}}{1 + \dfrac{1}{T_i\, s(1 + T\, s)}}$$

$$e_{ss} \text{ [error]} = \frac{A}{1 + \infty} = 0.$$

Thus it can be established that integral control action successfully eliminates the error, but in the process its response is *sluggish*. The integral action, due to its basic nature, might introduce oscillations in the controlled variable about the set point value, whenever it deviates from the set point value.

Thus, a $P+I$ controller has a transfer function representation as shown in the block diagram.



Characteristics of $P+I$ controller mode

To understand the working of proportional action and integral action in this combination, consider a step change in deviation (error), as shown in Figure (2.13).

Consider a step change in error at an instant of time $t = t_1$. Initially the proportional action responds to error. It gives an output that is proportional to error but it does not make the error zero and floats at the offset condition which is its inherent characteristic. At this point the integral action takes over and gives out a smooth rate of change of controller output about the offset thereby gradually reducing it to zero. Mathematically, the characteristics of $P+I$ action shown in Figure (2.13) can be represented as follows.

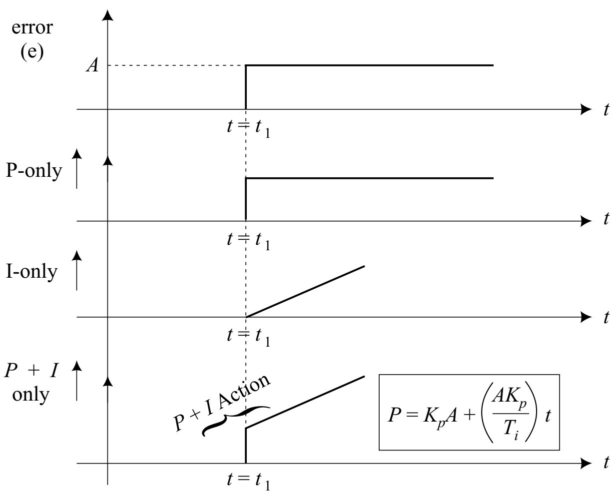


FIG. 2.13

$$e = A \text{ [step change]}$$

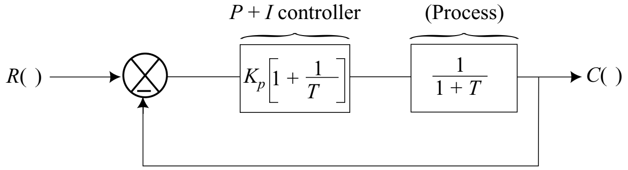
$$P = K_p e + \frac{K_p}{T_i}\int e\,dt.$$

$$P = K_p A + \frac{K_p}{T_i}\int A\,dt.$$

$$P = \underbrace{K_p A}_{P-Action} + \underbrace{\left(\frac{A K_p}{T_i}\right) t}_{I-Action}$$

Therefore, it may be stated that the integral action eliminates the inherent offset of proportional action. To Illustrate consider a first order process whose transfer function is

$$\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$$



The steady state error is given by the expression

$$e_{ss} = \underset{S\to 0}{\text{Lt}} \frac{S \,.\, R(s)}{1+G(s)\,H(s)}$$

Since $$R(s) = \frac{A}{S}$$

$$G(s)H(s) = \left[K_p\left(1+\frac{1}{T_i\, s}\right)\right]\left[\left(\frac{1}{1+T_s}\right)\right]$$

$$e_{ss} = \underset{S\to 0}{\text{Lt}} \frac{\$\,\frac{A}{\$}}{1+K_p\left(1+\frac{1}{T_i\, s}\right)\left(\frac{1}{1+Ts}\right)}$$

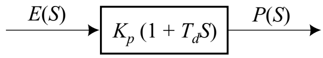
$$e_{ss} = \frac{A}{1+\left(K_p+\infty\right)(1)} = \frac{A}{\infty} = 0$$

Thus $\boxed{e_{ss} = 0}$. Hence it can be mentioned here that a $P + I$ controller mode is capable of improving the steady state response i.e. minimising the steady state error in a process. From Figure (2.13) it appear that this control action also has good speed of response in dealing with the error when ever it occurs, thus effecting the transient state behaviour of the process. Whether a particular composite controller mode affects the transient state or speed of response of the process can be very well verified by considering sinusoidal deviation.

Applying $L \, . \, T$ to get transfer function representation

$$P(s) = K_p E(s) + K_p T_d s \, . \, E(s)$$

$$P(s) = [K_p (1 + T_d s)] E(s)$$



Characteristics of P + D controller

We cannot consider a step change in deviation because the derivative action does not respond to such errors. Hence considering rate of change of error.

At an instant of time $t = t_1$, the error is changing at a continuous rate. As shown in the figure, the derivative action first sends its output in its in imitable anticipatory mode which is followed by proportional action. Such a response would lead to large instability in a process which may result in error increasing beyond proportions rather than becoming zero. If we consider this control action for level control system discussed in earlier chapters, when the liquid level in the tank starts rising beyond the actual set point resulting in an error shown in Fig (2.14), the derivative mode opens the final control element to its maximum capacity causing large quantity of outflow, followed by proportional action further opening the control valve resulting in outflow greater than inflow. This condition results in large error and High instability. Since this mode is incapable of improving the steady state behaviour i.e. eliminate steady state error and inherently exhibits good speed of response, we can use it to processes having slow response characteristics, to effectively increase their speed of response. Referring to the figure below

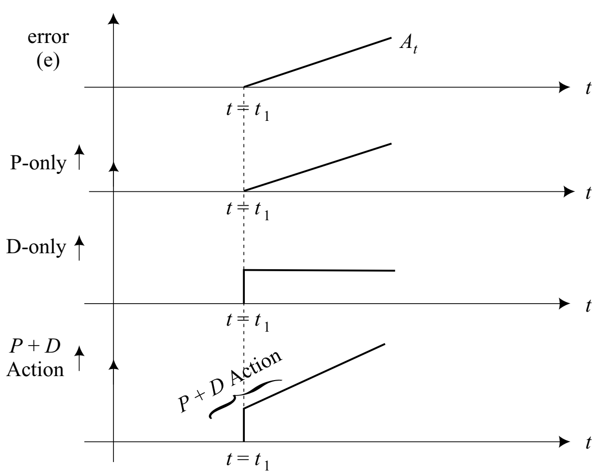
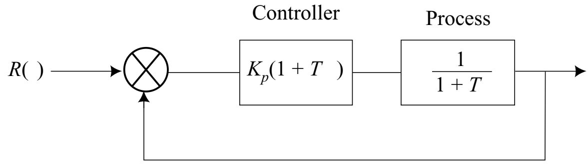


FIG. 2.14



let $R(s) = \frac{A}{S^2}$ [Ramp change in deviation].

$$e_{ss} = \underset{S \to 0}{\text{Lt}} \frac{\cancel{s}\,\frac{A}{\cancel{s}^2}}{1 + \frac{K_p\left(1 + T_d\, s\right)}{\left(1 + T\, s\right)}}$$

$$= \underset{S \to 0}{\text{Lt}} \frac{A}{s + \frac{s\, K_p\left(1 + T_d\, s\right)}{\left(1 + T\, s\right)}}$$

$$\therefore \qquad e_{ss} = \frac{A}{0} = \infty$$

Therefore this mode has no effect on the steady state response. Whether or not it has any effect on transient state behaviour can be established by considering sinusoidal deviation and respective analysis

Let $e = \sin w\, t$

$$P = K_p \sin w\, t + K_p\, T_d \frac{d}{d\, t} \sin w\, t.$$

$$P = K_p \sin w\, t + w K_p\, T_d \cos w\, t.$$

$$P = \sqrt{K_{p^2} + \left(w K_p\, T_d\right)^2}\ .\sin\,[wt + \text{Tan}^{-1} w T_d]$$

$$\text{P} = \sqrt{K_p^2 + \left(w K_p\, T_d\right)^2}\ .\sin\left[w\, t + \text{Tan}^{-1}\, w\, T_d\right] \qquad ...(12)$$

As seen from equation (12), for sinusoidal deviation the phase of the controlled variable leads by $\text{Tan}^{-1}\, wTd$. Hence a P+D control action can be employed with processes having slow response characteristics, which can be conveniently improvised by this mode of control action.

Since the phase of controller output is leading by $\text{Tan}^{-1}\, wT_d$ for sinusoidal deviation its characteristics are similar to Lead Compensator and High Pass Filter.

2.5.3 Proportional + Integral + Derivative Controller Mode [*P* + *I* + *D* Mode]

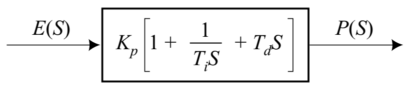
It is the most versatile controller mode which can be used for treating any type of errors affecting the processes. It is the combination of all the three modes and is capable of improving both steady state and transient state responses i.e. it successfully eliminates the error and also has good speed of response.

The analytic equation can be obtained as follows

$$P = K_p e + \frac{K_p}{T_i}\int e\,dt + K_p T_d + P(0) \qquad ...(13)$$

The transfer function representation can be obtained by applying Laplace transform to equation (13).

$$P(s) = \left\{K_p\left[1 + \frac{1}{T_i S} + T_d S\right]\right\} E(s)$$



Characteristics of *P* + *I* + *D* controller

To understand the working of all the there modes, considering a ramp change in deviation. Let us assume in the level control system described in chapter-1. The level of the liquid in the tank is increasing at a constant rate beyond set point value as the inlet flow rate is also changing at a constant rate. Since the controller is *PID* controller, initially when the error begins to occur the derivative mode opens the control element to its maximum capacity and the level drastically falls below the set point, Again the level of the liquid starts increasing from the new value from where it has fallen below set point because inlet flow rate is changing at a constant rate. At this stage both proportional and integral actions take over and manipulate the final control element in such a way that gradually the outlet flow rate and inlet flow rate are made equal. Thus the error is completely eliminated and the level is made to float at the set point value. It is important to define the term "Repeats per minute" as applied to P + I action, which occurs after the derivative action as shown in Fig. 2.15. The term derives from the observation the integral gain K_I, has the effect of cousing the controller output to change every unit time by proportional mode amount.

Graphically,

Let $e = At$

$$P = K_p At + \frac{K_p}{T_i}\int At\,dt + K_p T_d \frac{d}{dt}(At)$$

$$P = (K_p A)\,t + \left[\frac{A K_p}{2T_i}\right] t^2 + [A\,Kp\,T_d] \qquad ...(14)$$

$$e_{ss} = \frac{AT_i}{K_p} \quad \text{since } T_i << K_p$$

$$\boxed{e_{ss} = 0}$$

The effect of this mode on speed of response or transient state behaviour can be obtained by considering sinusoidal deviation

let $\qquad e = \sin wt$

$$P = K_p \sin wt + \frac{K_p}{T_i}\int \sin wt\, dt + K_p\, T_d \frac{d}{dt} \sin wt$$

$$P = K_p \sin wt + \left(\frac{-K_p}{wT_i}\right) \cos wt + w\, K_p\, T_d \cos wt$$

$$P = K_p \sin wt + \left[w\, K_p\, T_d - \frac{K_p}{wT_i}\right] \cos wt$$

$$P = \sqrt{K_p^{\,2} + \left(wK_p\, T_d - \frac{K_p}{wT_i}\right)^2} \sin\left[wt + \text{Tan}^{-1}\left(wT_d - \frac{1}{wT_i}\right)\right] \quad ...(15)$$

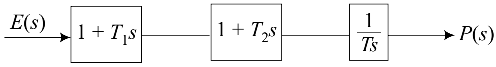
Therefore from eqn. (15), that phase of the controller output leads by $\text{Tan}^{-1}\left(wT_d - \frac{1}{wT_i}\right)$, which implies it is capable of improving the transient state behavior of the process.

Therefore the performance of composite controller modes may be summarised as follows:

(1) $P + I$ controller characteristics are similar to lag compensator or low pass filter. It is capable of improving only the steady state response characteristics i.e. the presence of integral action eliminates the offset of proportional action and also any type of error.

(2) $P + D$ controller characteristics are similar to lead compensator or high pass filter. It is capable of improving only transient state characteristics i.e. speed of response of the system. It is best suitable for processes having very slow response.

(3) $P + I + D$ controller characteristics are similar to LEAD-LAG compensator. From the pole-zero configuration of lead. Lag compensator it may be concluded that it exhibits band reject filter characteristics. It is capable of improving both transient and steady state response characteristics of the system.

3. 

$$P(s) = \left[\frac{T_1T_2s^{\cancel{2}}}{T\cancel{s}} + \frac{\cancel{s}[T_1+T_2]}{T\cancel{s}} + \frac{1}{Ts}\right]E(s)$$

$$P(s) = \left[\frac{T_1T_2}{T}.s + \frac{T_1+T_2}{T} + \frac{1}{Ts}\right]E(s)$$

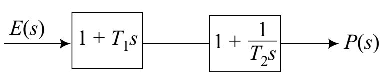
$$E(s).\left\{\frac{T_1+T_2}{T}\left[\frac{\cancel{T}}{T_1+T_2}.\frac{T_1T_2}{\cancel{T}}.s + 1 + \frac{\cancel{T}}{T_1+T_2}.\frac{1}{\cancel{T}s}\right]\right\}$$

$$\therefore \qquad P(s) = \left\{\frac{T_1+T_2}{T}\left[1+\left(\frac{T_1T_2}{T_1+T_2}\right)s + \frac{1}{(T_1+T_2)s}\right]\right\}E(s)$$

It is a *PID* controller comparing with

$$P(s) = \left\{K_p\left[1+\frac{1}{T_is}+T_ds\right]\right\}E(s)$$

$$K_p = \frac{T_1+T_2}{T} \qquad T_i = T_1 + T_2 \qquad T_d = \frac{T_1T_2}{T_1+T_2}$$

4. 

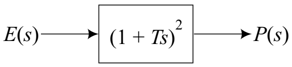
$$P(s) = \left[(1+T_1s)\left(1+\frac{1}{T_2s}\right)\right]E(s)$$

$$P(s) = \left[1+\frac{1}{T_2s}+T_1s+\frac{T_1\cancel{s}}{T_2\cancel{s}}\right]E(s)$$

$$P(s) = \left[\left(1+\frac{T_1}{T_2}\right)+\frac{1}{T_2s}+T_1s\right]E(s)$$

$$P(s) = \left[\frac{T_1+T_2}{T_2}+\frac{1}{T_2s}+ T_1s\right]E(s)$$

$$P(s) = \left\{\frac{T_1+T_2}{T_2}\left[1+\frac{\cancel{T_2}}{T_1+T_2}.\frac{1}{\cancel{T_2}s}+\frac{T_2}{T_1+T_2}.T_1s\right]\right\}E(s)$$

7. 

$$P(s) = [1 + T^2S^2 + 2Ts]E(s)$$

Put $\quad s = d/dt$

$$P = T^2\frac{d^2e}{dt^2} + 2T\frac{de}{dt} + e\,.$$

let $\quad \dfrac{de}{dt} = e_P.$

$$P = T^2\frac{de_P}{dt} + 2T\,.e_P + \int e_P dt\,.$$

Applying L. T (Laplace transform).

$$P(s) = T^2.sE_P(s) + 2TE_P(s) + \frac{E_P(s)}{s}$$

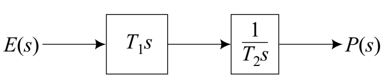
$$E_P(s)\left[T^2\,.s + 2T + \frac{1}{s}\right].$$

$$P(s) = \left\{2T\left[1 + \frac{T}{2}.s + \frac{1}{2Ts}\right]\right\}E_P(s)$$

It is a *PID* controller.

$\therefore \quad \boxed{K_p = 2T} \qquad \boxed{T_i = 2T}$

$$\boxed{T_d = \frac{T}{2}}$$

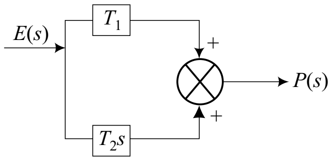
8. 

$$P(s) = \left[\frac{T_1s}{T_2s}\right]E(s) \Rightarrow P(s) = \left[\frac{T_1}{T_2}\right]E(s)$$

It is proportional controller

$$\boxed{K_p = T_1/T_2}$$

9.



$$P(s) = [T_1 + T_2]E(s).$$

$$P(s) = \left\{T_1\left[1 + \frac{T_2}{T_1}.s\right]\right\} \text{E(s)}$$

It is $P + D$ controller

$K_p = T_1$

$T_d = T_2/T_1$

10. $$P(s) = [1 + T_1 s + T_2^2 s^2]E(s).$$

Put $$s = \frac{d}{dt}$$

$$P = e + T_1 \frac{de}{dt} + T_2^2 \frac{de_p}{dt}$$

let $$\frac{de}{dt} = e_p.$$

$$P = \int e_p dt + T_1 e_p + T_2^2 \frac{de_p}{dt}$$

It is *PID* controller.
Applying laplace transform.

$$P(s) = \left[T_1 + \frac{1}{s} + T_2^2 s\right]E(s)$$

$$P(s) = \left\{T_1\left[1 + \frac{1}{T_1 s} + \frac{T_2^2}{T_1}.s\right]\right\}E(s)$$

$K_p = T_1$

$T_i = T_1$

$$T_d = \frac{T_2^2}{T_1}$$

Type-2 Problems

11. In an integral controller with reset time set to 0.6 minutes, what will be the phase of controller output for sinusoidal deviation?

Solution: $P = \frac{1}{T_i}\int e\,dt.$

Since $T_i = 0.6$ min. $\quad e = \sin wt$

$$P = \frac{1}{0.6}\int \sin wt.dt$$

$$P = \frac{-\cos wt}{w \times 0.6}$$

Since $\cos\theta = \sin[90 - \theta]$

$$-\cos\theta = \sin[\theta - 90°]$$

$$\therefore \quad P = \frac{\sin[wt - \pi/2]}{0.6w} \Rightarrow \frac{1}{0.6w}\sin(wt - \pi/2)]$$

The phase of the controller *o/p* Lays by $\pi/2$ degrees.

12. Determine the phase of the controller output of derivative controller for sinusoidal deviation?

Solution: $P = T_d\frac{de}{dt}$ Since $e = \sin wt$

$$p = T_d\frac{d}{dt}(\sin wt) = wT_d\cos wt$$

$$P = wT_d\sin(wt + \pi/2). \qquad [\because \sin(90° + \theta) = \cos\theta]$$

The phase of the controller *o/p* leads by $\pi/2$.

13. The input error signal for $P + I$ controller is sinusoidal in nature. Prove that the phase lag is a function of reset time?

Solution: $P = K_p e + \frac{K_p}{T_i}\int e\,dt$

$$e = \sin wt$$

$$p = K_p\sin wt + \frac{K_p}{T_i}\int \sin wt.dt$$

$$p = K_p\sin wt + \left[\frac{-K_p}{wT_i}\right]\cos wt$$

$$A \sin wt + B \cos wt = \sqrt{A^2 + B^2}\ \sin [wt + \tan^{-1} B/A]$$

$$\therefore \quad p = \sqrt{K_{p^2} + \left(\frac{-K_p}{wT_i}\right)^2}\ \sin\left[wt + \tan^{-1}\left(\frac{-K_p / wT_i}{K_p}\right)\right]$$

$$\therefore \quad P = \sqrt{K_{p^2} + \left(\frac{K_p}{wT_i}\right)^2}\ \sin\left[wt - \tan^{-1}\left(\frac{1}{wT_i}\right)\right]$$

Thus the phase lag $\theta = \tan^{-1}(1/wT_i)$ is a function of reset time (T_i).

14. A $P + I$ controller subject to sinusoidal deviation, is tuned to $T_i = 3$ min and $K_p = 10$. Determine its output when $f = 50$ Hz.

Solution: $$P = K_p e + \frac{K_p}{T_i}\int e\,dt.$$

Given $e = \sin wt$; $k_p = 10$; $T_i = 3$ min

$$P = 10e + \frac{10}{3}\int e\,dt$$

$$\Rightarrow \quad P = 10 \sin wt + \frac{(-3.33)}{w} \cos wt.$$

$$P = \sqrt{10^2 + \left(\frac{3.33}{w}\right)^2}\ .\sin\left[wt + \tan^{-1}\left(\frac{-3.33/w}{10}\right)\right]$$

$$P = \sqrt{10^2 + \left(\frac{11}{w^2}\right)}\ .\sin\left[wt - \tan^{-1}\frac{3.33}{10w}\right]$$

$$w = 2\pi f = 2 \times \pi \times 50 = 314.15 \text{ radians.}$$

$$P = \sqrt{100 + \frac{11}{(314.15)^2}}\ .\sin\left[314.15t - \tan^{-1}\frac{3.33}{10 \times 314.15}\right]$$

$$\boxed{P = 10 \sin [314.15 + (-0.06)]}$$

15. What is the phase of the controller output for sinusoidal error for $P+D$ controller?

Solution: $$P = K_p e + K_p T_d \frac{de}{dt}$$

$$e = \sin wt$$

$$P = K_p \sin wt + K_p T_d \frac{d}{dt} \sin wt$$

$$P = K_p \sin wt + wK_pT_d \cos wt$$

$$P = \sqrt{K_{p^2} + (wK_pT_d)^2}\ .\sin [wt + \tan^{-1} wTd]$$

The phase leads by $\tan^{-1}(wT_d)$ degrees.

16. A certain controller has a function

$$P = \frac{1}{T^2}\int\int edt + P(0).$$

Prove that the phase of the controller output lags by 180°.

Solution: Applying L.T [Assume Initial Condition to be zero $P(0) = 0$]

$$P(s) = \frac{1}{T^2}\frac{E(s)}{s^2}$$

Put $s = jw$

$$F(jw) = \frac{P(jw)}{E(jw)} = \frac{1}{(jw)^2.T^2} = \frac{1}{T^2}.\frac{1}{jw}.\frac{1}{jw}$$

$$F(jw) = \frac{1}{T^2}\frac{1+jw}{0+jw}.\frac{1+j0}{0+jw}$$

$$|F(jw)| = \frac{1}{T^2}.\frac{\sqrt{1^2+0^2}}{\sqrt{0^2+w^2}}.\frac{\sqrt{1^2+0^2}}{\sqrt{0^2+w^2}} = \frac{1}{w^2T^2}$$

$$\angle F(jw) = [0°]\frac{[\tan^{-1}0/1]}{[\tan^{-1}w/0]}.\frac{[\tan^{-1}0/1]}{[\tan^{-1}w/0]}$$

$$\Rightarrow \quad \frac{0°}{2\,\text{Tan}^{-1}\infty} = \frac{0°}{2\times 90°} = \mathbf{-180°}.$$

Therefore the phase of the controller 0/*p* lags by 180°.

17. Show that for a *PID* controller the phase of its output is a function of rate time and reset time for sinusoidal deviation?

Solution:

$$P = K_pe + \frac{K_p}{T_i}\int e\,dt + K_pT_d\frac{de}{dt}$$

$$e = \sin wt$$

$$P = K_p \sin wt + \frac{K_p}{T_i}\int \sin wt + K_pT_d\frac{d \sin wt}{dt}$$

$$P = K_p \sin wt + \left(\frac{-K_p}{wT_i}\right) \cos wt + wK_pT_d \cos wt.$$

$$P = K_p \sin wt + \left(wK_pT_d - \frac{K_p}{wT_i}\right) \cos wt$$

$$P = \sqrt{K_{p^2} + \left(wK_pT_d - \frac{K_p}{wT_i}\right)^2} .\sin\left[wt + \tan^{-1}\left(\frac{wK_pT_d - \frac{K_p}{wT_i}}{K_p}\right)\right]$$

$$P = \sqrt{K_{p^2} + \left(wK_pT_d - \frac{K_p}{wT_i}\right)^2} .\sin\left[wt + \tan^{-1}\left(wT_d - \frac{1}{wT_i}\right)\right]$$

$$\boxed{\phi = \tan^{-1}\left(wT_d - \frac{1}{wT_i}\right)}$$

The phase of the controller output is a function of rate time (T_d) and reset time (T_i).

18. A certain $P + D$ controller has $T_d = K_p = 1$. Find the phase of its 0/*p* when $w = \pi/3$?

Solution: From prob (15). The phase of $P + D$ controller leads by $\tan^{-1}(wT_d)$.

∴ Given $w = \pi/3$ $T_d = 1$

$$\tan^{-1}(\pi/3) = 89°.$$

Type-3 Problems

19. A pneumatic controller is employed to control the valve plug position between 0 – 10 mm. It the range of the controller output is 3 – 15 psi. What pressure will bring the valve\plug to a position of 8.5 mm.?

Solution: Since controller final control element operation is linearly dependent

$$Y = MX + C \qquad Y = \text{valve plug position}$$

$$0 = M.3 + C \qquad X = \text{controller } 0/p$$

$$10 = M.15 + C$$

Solving for "*M*" & "*C*"

$$\boxed{M = 0.833} \qquad \boxed{C = -2.5.}$$

∴ $$8.5 = 0.833x - 2.5$$

$$8.5 + 2.5 = 0.833x \qquad \boxed{x = 13.20 \text{ psi}}$$

20. In the above problem express 13.20 psi as percentage controller output in the range of 3 – 15 psi?

Solution: Using the formula

$$\%p = \frac{\Delta P - P_{\min}}{P_{\max} - P_{\min}} \times 100$$

$$= \frac{13.20 - 3}{15 - 3} \times 100$$

$$\boxed{\%P = 85\%}$$

21. In a 4 – 20 mA current range, it 0% corresponds to 4 mA and 100% corresponds to 20 mA, and what value of current indicates 42% of output?

Solution: Using the formula

$$\%p = \frac{\Delta P - P_{\min}}{P_{\max} - P_{\min}}$$

$$\frac{42}{100} = \frac{P - 4}{20 - 4}$$

$$\frac{42 \times 16}{100} + 4 = P.$$

$\therefore$ $$\boxed{P = 10.72 \text{ mA}}$$

10.27 mA corresponds to 42% of out put.

22. A controller outputs 4-20 mA signal to control motor speed from 140 rpm-600 rpm with Linear dependence

(a) Calculate current corresponding to 310 rpm.

(b) Express part (a) as % of controller output

Solution: (a) Using linear dependence relation

$$Y = MX + C.$$

Motor speed (y) = M[current (x)] + C

$\therefore$ $$140 = 4M + C$$

$$600 = 20M + C$$

Solving for M and C

$$\boxed{C = 25 \text{ rpm}} \qquad \boxed{M = 28.75 \text{ rpm/mA}}$$

$\therefore$ $$Y = 28.75(I) + 25$$

For 300 rpm.

$$300 = 28.75I + 25$$

$$\frac{300 - 25}{28.75} = I$$

$\therefore$ $$\boxed{I = 9.91 \text{ mA}}$$

(b) $$\% P = \frac{\Delta P - P_{\min}}{P_{\max} - P_{\min}} \times 100$$

$\therefore$ $$\% P = \frac{9.91 - 4}{20 - 4} \times 100 = 36.9\%$$

$\therefore$ $$\boxed{\% P = 36.9\%}$$

23. The controller output in percentage is 54.78%. Determine the corresponding pressure in the range of 3 – 15 psi?

Solution: $$\frac{54.78}{100} = \frac{\Delta P - P_{\min}}{P_{\max} - P_{\min}}$$

$\therefore$ $$\frac{54.78}{100} = \frac{\Delta P - 3}{15 - 3}$$

$\therefore$ $$\boxed{\Delta P = 9.57 \text{ psi}}$$

24. In a level control system, the controller output 3 – 15 psi signal to the final control element to monitor liquid level between 2.5 m – 22.5 m. What should be the output of controller to maintain level at 19.75 m?

Solution: Using linear dependence equation

$$Y[\text{level}] = M[X \text{ (pressure)}] + C$$

$$2.5 = 3M + C$$

$$22.5 = 15M + C$$

Solving for 'M' and C

$$M = 1.67 \text{ m/psi} \qquad C = 2.5 \text{ m}$$

$\therefore$ $$19.75 = 1.67x - 2.5$$

$\therefore$ $$x = \frac{19.75 + 2.5}{1.67} = 13.32 \text{ psi}$$

Therefore the rewired pressure is 13.32 psi

$$\% P = \frac{13.32 - 3}{15 - 3} \times 100$$

$$\boxed{\% P = 86\%}$$

25. A liquid level control system converts a displacement of 5 to 10 m into 4 – 20 mA control signal with linear dependence.

A relay, which serves as a two position controller opens or closes the inlet control valve. The relay closes at 12 mA and opens at 8 mA. What will be neutral zone in nut ran of the two position controller?

Solution: Using the relation

$$Y(\text{level}) = MX\,(\text{current}) + C$$

$$5 = M.4 + C$$

$$10 = M.20 + C$$

Solving for 'M' and 'C'

$$M = 0.3125 \text{ m/mA} \; \& \; C = 3.75 \text{ m.}$$

For 12 mA the relay closes

$$\therefore \quad Y_1 = 0.3125 \times 12 + 3.75$$

$$Y_1 = 7.5 \text{ m.}$$

The relay opens at 8 mA

$$Y_2 = 0.3125 \times 8 + 3.75$$

$$Y_2 = 6.25 \text{ m}$$

$\therefore$ The neutral zone $= Y_1 - Y_2 = 7.5 - 6.25$

$$\boxed{\text{Neutral zone} = 1.25 \text{ m}}$$

26. In a pressure control system a floating controller with single speed mode is employed to control pressure in a gas vessel. The controller output when the process error is within the limits of neutral zone is 35% If the rate constant $K_F = +2\%$ per second,

(a) What will be the controller output after 10 seconds.

(b) Find the time when the output saturates

Solution: (a) For floating controller

$$p = K_F t + P(0).$$

At $\quad t = 10$ seconds.

$$P = +2 \times 10 + 35\% = 55\%$$

(b) At $p = 100\%$ [output saturation]

$$100 = +2 \times t + 55$$

$$t = \frac{100 - 55}{2} = 22.5 \text{ seconds.}$$