2 Satellite Orbits

Learning Objectives

After completing this chapter you will be able to understand:

- Kepler's laws.
- Determination of orbital parameters.
- Eclipse of satellite.
- Effects of orbital perturbations on communications.
- Launching of satellites into the geostationary orbit.

The closer the satellite to the earth, the stronger is the effect of earth's gravitational pull. So, in low orbits, the satellite must travel faster to avoid falling back to the earth. The farther the satellite from the earth, the lower is its orbital speed. The lowest practical earth orbit is approximately 168.3 km. At this height, the satellite speed must be approximately 29,452.5 km/hr in order to stay in orbit. With this speed, the satellite orbits the earth in approximately one and half hours. Communication satellites are usually much farther from the earth, for example, 36,000 km. At this distance, a satellite need to travel only about 11,444.4 km/hr in order to stay in the orbit with a rotation speed of 24 hours.

A satellite revolves in an orbit that forms a plane, which passes through the centre of gravity of the earth or *geocenter.* The direction of the satellite's revolution may be either in same the direction as earth's rotation or against the direction of the earth's rotation. In the former case, the orbit is said to be *posigrade* and in the latter case, the *retrograde.* Most orbits are posigrade. In circular orbit, the speed of revolution is constant. However, in an elliptical orbit, the speed changes depending upon the height of the satellite above the earth.

In an elliptical orbit, the highest point is generally referred to as *apogee* and the lowest point is called *perigee.* These are measured typically from the geocentre of the earth and therefore include the earth's radius.

The time that it takes for a satellite to complete an orbit is called *sidereal period.* Some fixed or apparently motionless external object, such as the sun or a star, is used for reference in determining a sidereal period. The reason for this is that while the satellite is revolving around the earth, earth itself is rotating.

Another method of expressing the time for one orbit is *revolution* or *synodic period.* One revolution is a period of time that elapses between the successive passes of the satellite over a given meridian of earth's longitude. Synodic and sidereal periods differ from one another because of the earth's rotation. The time difference is determined by the height of the orbit, angle of plane of orbit and whether the satellite is posigrade or retrograde orbit.

The *angle of inclination* of a satellite orbit is the angle formed between the line that passes through the centre of the earth and the north pole which is also perpendicular to the orbital plane. It can be from 0° to 180°.

Another definition of inclination is the angle between equatorial plane and the satellite orbital plane as satellite enters northern hemisphere. When angle of inclination is 0° or 180°, the satellite will be directly above the equator. When it is 90°, it will pass over north and south poles. Orbits with 0° inclination are called *equatorial*, while orbits with 90° are referred to as *polar*.

The angle of *elevation* of a satellite is that angle which appears between the line from the earth station's antenna to the satellite and line between the earth station's antenna and the earth's horizon. If angle of elevation is too small, signals between the earth station and satellite have to pass through much more of the earth's atmosphere.

To use a satellite for communication relay or repeater purpose, the ground station's antenna must be able to follow or track the satellite as it passes overhead. Depending upon the height and speed of satellite, earth station will only be able to use it for communication purposes for that short period of time when it is visible. The earth station antenna will track the satellite from horizon, but at the same point the satellite will disappear around the other side of the earth.

One solution to this problem is launching a satellite with a very long elliptical orbit where the station can see the apogee. In this way, the satellite stays in view for a longer time and is useful. Eclipse of geostationary satellite occurs on the *autumnal* and *vernal (spring) equinoxes,* the forty-fourth day of fall and spring respectively and lasts from a minute to over an hour.

Orbital drift is caused by a variety of forces. The gravitational pull of sun and moon affects the satellite position. Earth's gravitational field is not perfectly consistent at all points on the earth. This is due to the fact that the earth is not a perfect sphere but an oblate spheroid. Due to this drift, the orbit of the satellite must be periodically adjusted. The adjustments is called *station keeping*.

Positioning of the satellite for optimum performance is called *attitude control.* Location of satellites is generally specified in terms of latitude and longitude. A point on the surface of the earth directly below the satellite specifies the its location. This point is known as *subsatellite point* (SSP).

Only geosynchronous satellites have a fixed SSP on earth. SSP of other satellites will move with respect to the given reference point on the earth. Their SSP traces a line on earth known as *subsatellite path* or *ground track.* The ground track for most satellites crosses the equator twice per orbit. The point where SSP crosses the equator headed northerly direction is called *ascending node*. The point where SSP crosses the equator headed in southerly direction is called *descending node*. With these two points known, the satellite path can be traced across the surface of the earth between them. Ascending node is sometime designated by EQX and is used as a reference point for locating and tracing a satellite.

Location of satellite at any given time is specified by SSP in terms of latitude and longitude. For the non-geostationary orbit satellite, exact position of the satellite is usually designated by the *orbit calendar*. This is a standard, usually consists of orbit number and occurrence of the ascending node EQX. Usually the number of orbits that a satellite makes is tracked from the very instant it is put into the orbit. By using various formulae involving height, speed and elliptical characteristics of an orbit, the time of occurrence of ascending node can be computed for each orbit. With orbital calendar, various maps and plotting devices, the ground track can be traced for each orbit. This allows satellite user to determine whether or not the satellite is within the useable range.

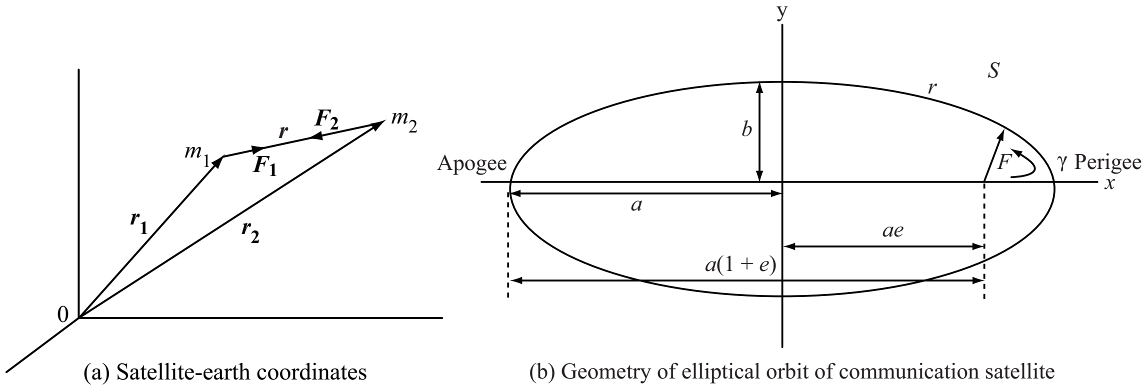

Figure 2.1 Geometry of communication satellite.

$$\frac{d^2\boldsymbol{r}}{dt^2} = -\mu\frac{\boldsymbol{r}}{r^3} \tag{2.4}$$

where $\mu = g(\boldsymbol{m_1}+\boldsymbol{m_2}) \approx gm_1$, since the mass of the satellite is negligible compared to that of earth. The value of gm_1 is 3.986013×10^5 km^3/s^2. The equation (2.4) is known as the *two-body equation of motion in relative form*. It describes the motion of a satellite orbiting the earth.

The first law of Kepler is stated as the polar equation of the ellipse with origin at the primary focus

$$p = r\,(1 + e\cos v) \tag{2.5}$$

where r is the distance of satellite from primary focus F which is the centre of the earth, v is the true anomaly, measured from primary focus F in the direction of motion from the perigee to the satellite position vector r, a is the semi major axis of the ellipse, b is the semi minor axis of the ellipse, e is eccentricity, p is semi parameter.

The second law of Kepler can be expressed as

$$\boldsymbol{r}\times\frac{d^2\boldsymbol{r}}{dt^2} = \boldsymbol{r}\times(-\mu\,\boldsymbol{r}/r^3) \tag{2.6}$$

with the help of first two Kepler's law, the *Kepler's Equation* can be derived as

$$M = E_a - e\sin E_a - \frac{\sqrt{\mu}}{a^{3/2}}\,(t - t_0) \tag{2.7}$$

M is called the *mean anomaly* and increases at a steady rate, N is known as *mean angular motion.*

$$N = \sqrt{\mu}\;/\;a^{3/2} \tag{2.8}$$

a certain region for a relatively long part of its period can minimize this handover problem. The Russian *Molniya* satellite has a highly inclined elliptical orbit with 63° inclination angle and an orbit period of 12h. The apogee is above the northern hemisphere. Communications are established when the satellite is in the apogee region where the orbital period is small and antenna tracking is slow. The satellite visibility for a station above 60° latitude with an antenna elevation greater than 20° is between 4.5 and 10.5h.

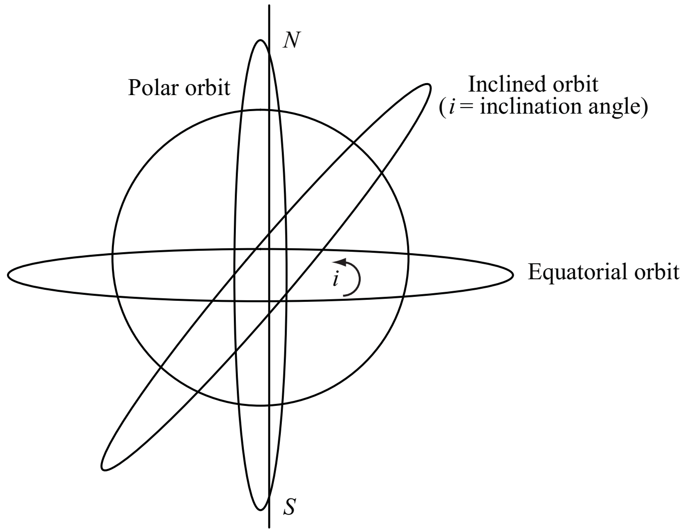

Figure 2.2 Inclined orbits.

Although a geostationary satellite appears to be stationary in its orbit, the gravitational attraction of the moon and to a lesser extent that of the sun cause it to drift from its stationary position and the satellite orbit tends to become inclined at a rate of about 1° per year. Also, the non-uniformity of the earth's gravitational field and the radiation pressure of the sun cause the satellite to drift in longitude. But this drift is several times the magnitude, smaller than that result from the attraction of the moon and the sun. Station keeping is required to maintain the satellite in its 'home' i.e., the orbital position of the satellite is to be monitored and in case of any error beyond tolerance limit, the attitude control system is to be activated. This eliminates the interference of adjacent satellites since the satellite is not allowed to move towards the other nearest satellite. North-south station keeping is required to prevent the drift caused in latitude. East-west station keeping takes care of the errors in the longitude direction.

2.4 EFFECTS OF ORBITAL INCLINATION

The maximum drift in the latitude and longitude due to *orbit inclination* is determined by using the Fig. 2.3. From the figure, for a non-rotating earth,

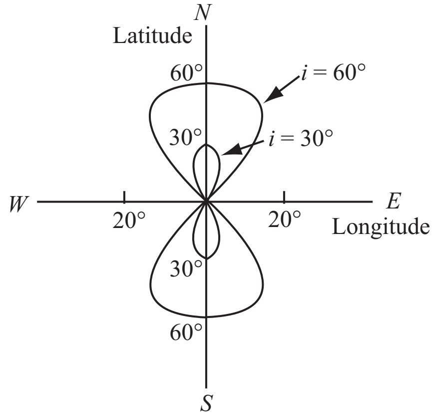

Figure 2.4 Apparent movement of a satellite in an inclined and synchronous orbit with respect to the ascending node.

where i is in degrees. The displacement in latitude is more pronounced than the displacement in longitude for a synchronous satellite with a small inclination. In this case, the displacement D_λ (corresponding to λ_{max}) and D_ψ (corresponding to ψ_{max}) can be written as

$$\frac{D_\lambda}{R_e \lambda_{max}} = \frac{a}{R_e} \tag{2.17}$$

where a is the orbital radius and R_e is the earth's radius. Thus,

$$D_\lambda = a\lambda_{max} = 735.9\, i \text{ (km)} \tag{2.18}$$

$$D_\psi = iD_\lambda/228 = 3.23\, i^2$$

To correct the orbital inclination, it is necessary to apply a velocity impulse perpendicular to the orbital plane when the satellite passes through the nodes as shown in Fig. 2.5. For the given i, the impulse amplitude is given by

$$\Delta V = V \tan i = \sqrt{(\mu/a)} \tan i \tag{2.19}$$

where $V = \sqrt{(\mu/a)} = 3074.7$ m/s is the orbital velocity.

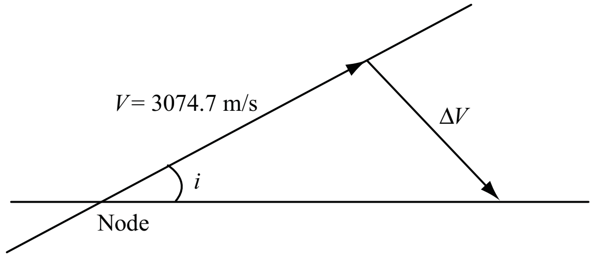

Figure 2.5 Correction of the inclination of a synchronous orbit.

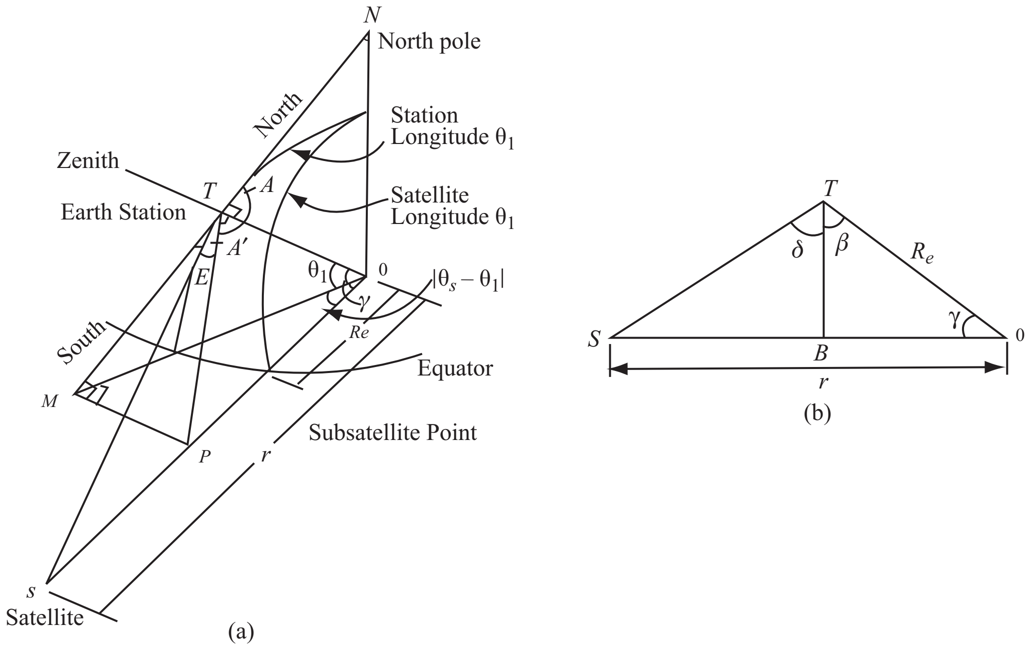

Figure 2.7 Solar and sidereal days–triangle to calculate elevation.

Earth station east to satellite: $A = 360^\circ - A'$.
where A' is as defined in Fig. 2.7 and is given as

$$A' = \tan^{-1}\left(\frac{\tan|\theta_s - \theta_L|}{\sin\theta_1}\right) \tag{2.20}$$

The elevation angle can be derived to be

$$E = \tan^{-1}\left(\frac{r - R_e\cos\theta_1\cos|\theta_s - \theta_L|}{R_e\sin[\cos^{-1}(\cos\theta_1\cos|\theta_s - \theta_L|)]}\right) - \cos^{-1}(\cos\theta_1\cos|\theta_s - \theta_L|) \tag{2.21}$$

2.6 COVERAGE ANGLE AND SLANT RANGE

Communication with a satellite is possible if the earth station is in the footprint of the satellite. In other words, the earth-satellite link is established only when the earth station falls in the beamwidth of the satellite antenna. This would be a function of time and the satellite is to be tracked in case of a non-geostationary satellite. But for a geostationary satellite once the link is established, the link is available throughout the lifetime of the satellite without any tracking. To have the communication between the earth station-satellite-earth station, both the antennas of the transmitting and receiving earth station are to be pointed towards the antenna of the spacecraft. With the help of look angle determination, this can be established. To locate the earth station in the footprint of the satellite, the information of *slant range* and *coverage area/angle* are required. To determine this information, Fig. 2.8 is considered.

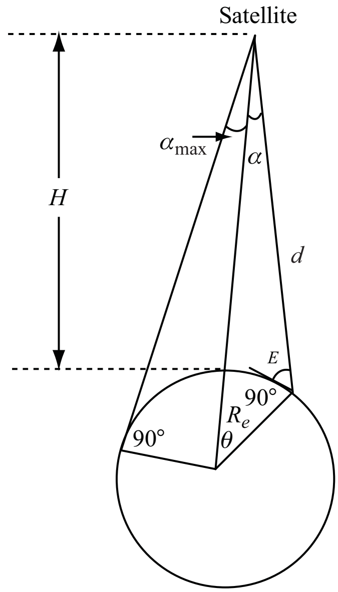

Figure 2.8 Coverage angle and slant range.

The *earth coverage angle*, $2\alpha_{max}$ is the total angle subtended by the earth as seen from the satellite. This angle is important in design of a global coverage antenna and depends on satellite altitude. The *communication coverage angle* 2α is similarly defined, except that the minimum elevation angle E_{min} of the earth station antenna must be taken into account. For an elevation angle E of the earth station antenna, the communication coverage angle 2α is given as

$$\frac{\sin\alpha}{R_e} = \frac{\sin(90^\circ + E)}{R_e + H} = \frac{\cos E}{R_e + H} \tag{2.22}$$

where R_e is the radius of the earth, assuming the earth to be spherical, H is the altitude of the satellite, which is a function of time for non-geostationary satellites. For geostationary satellite, $H = 35{,}786$ km. Thus,

$$2\alpha = 2\sin^{-1}\left[\frac{R_e}{R_e + H}\cos E\right] \tag{2.23}$$

and the earth coverage angle can be determined when $E = 0^\circ$

$$2\alpha_{max} = 2\sin^{-1}\left[\frac{R_e}{R_e + H}\right] \tag{2.24}$$

For a geostationary satellite orbit where R_e = 6378 km, the earth coverage angle is $2\alpha_{max}$ = 17.4°, the central angle, θ, which is the angular radius of the satellite footprint,

$$\theta = 180^\circ - (90^\circ + E + \alpha) = 90^\circ - E - \alpha \tag{2.25}$$

for geostationary orbit. For global coverage, when $2\alpha_{max} = 17.4°$, θ = 81.3°, if a minimum elevation angle of 5° is required for the above, these northern and southern latitudes of 76.3° will not be covered by the footprint of the satellite.

Besides the coverage angle, it is important to know the *slant range* from the earth station to the satellite, because this range determines the satellite roundtrip delay of the earth station. The slant range, *d* can be determined as

$$d^2 = (R_e + H)^2 + R_e^{\,2} - 2\,R_e(\,R_e + H\,)\sin\left[\mathrm{E} + \sin^{-1}\left(\frac{R_e}{R_e + H}\cos E\right)\right] \tag{2.26}$$

for a geostationary orbit and a minimum elevation angle of 5°, the maximum slant range is d = 41,127 km, yielding a satellite roundtrip delay of $2d/c$ = 0.274 s, where $c = 2.997925 \times 10^5$ km/s which is the speed of light.

2.7 ECLIPSE

A geostationary satellite utilizes solar energy to generate the required DC power to operate all the subsystems of the spacecraft. Solar energy is not available for a geostationary satellite when eclipse occurs. This occurs when the earth comes in between the sun and satellite in line and blocks the solar energy from reaching the solar panels of the satellite. This is a periodic feature and estimation of maximum duration of the eclipse period is very much required so as to determine the maximum capacity of the standby battery, which supplies the energy required for the subsystems during eclipse period.

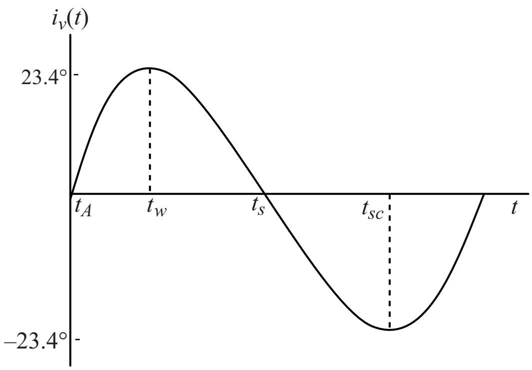

Figure 2.9(a) Sinusoidal variation of the earth's inclination angle.

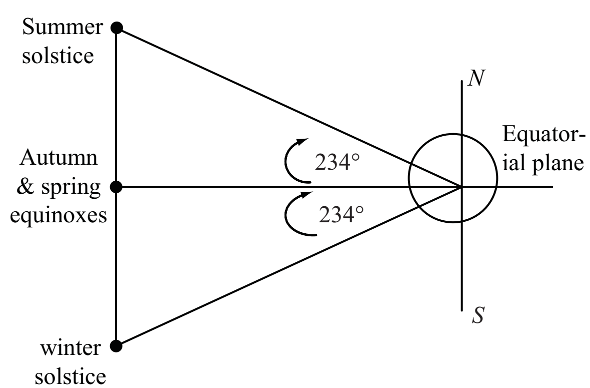

Figure 2.9(b) Apparent movement of the sun.

The earth's equatorial plane is inclined at an angle $i_e(t)$ with respect to the sun. The annual sinusoidal variation is given in degrees by

$$i_e(t) = 23.4 \sin (2\pi t/T) \tag{2.27}$$

where the annual period $T = 365$ days and maximum inclination is $i_{e,\max} = 23.4^\circ$. The time t_A and t_S when the inclination angle is zero are called the *autumn equinox* and *spring equinox* that occur on September 21 and March 21 respectively. The times t_W and t_{Su}, when the inclination angle is at its maximum are called *winter solstice* and the *summer solstice* that occur on December 21 and June 21 respectively.

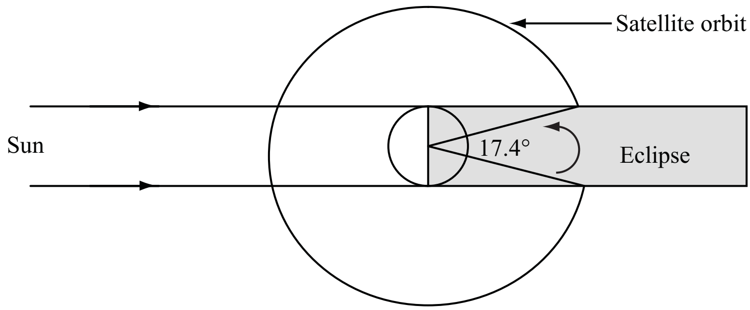

Figure 2.10 Eclipse when sun is at equinox.

Consider Fig. 2.10 to estimate the duration of eclipse. The diameter of the sun is neglected in these calculations (the sun is considered to be at infinity with respect to the earth and the rays of the sun are parallel) and hence the earth shadow is considered to be a cylinder of constant diameter. The maximum shadow angle occurs at the equinoxes and is given by

$$\varphi_{\max} = 180^\circ - 2 \cos^{-1} (R_e/a) = 17.4^\circ \tag{2.28}$$

Because the geostationary satellite period is 24 h, this maximum daily eclipse duration is

$$\tau_{\max} = 17.4^\circ \times 24 / 360^\circ = 1.16 \text{ h} \tag{2.29}$$

The first day of the eclipse before an equinox and the last day of eclipse after the equinox corresponds to the relative position of the sun such that the sun's rays fall tangent to the earth and passes through the satellite orbit. Thus, the inclination angle of the equatorial plane with respect to the direction of the sun in this case is

$$i_e = ½\, \varphi_{\max} = 8.7^\circ \tag{2.30}$$

Substituting this into equation (2. 27), it yields the time from the first day of eclipse to the equinox and also the time from the equinox to the last day of the eclipse.

$$t = (365/2\pi) \sin^{-1} (8.7/23.4) = 22.13 \text{ days} \tag{2.31}$$

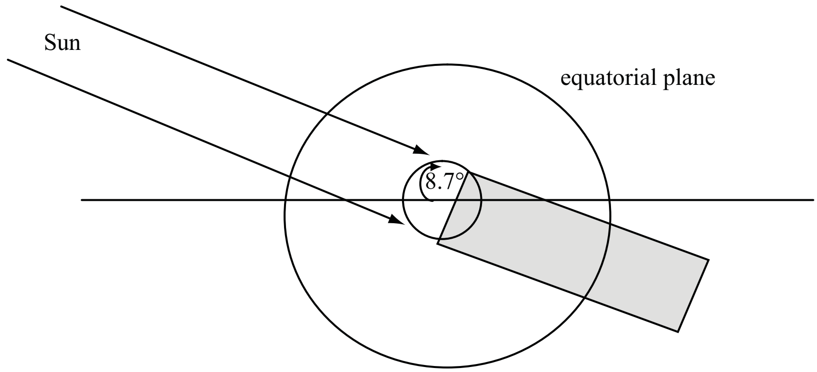

Figure 2.11 Earth inclination at first day of eclipse before equinox.

So, in total the satellite will be in eclipse for a period of 44.26 days around each equinox of the year. The period of equinox starts from zero per day from the starting day of the eclipse period i.e., 22.13 days before the equinox, gradually increases to 1.16 h per day on the day of equinox and then gradually reduces and becomes zero at the end of the eclipse period after 22.13 days after equinox. This process of eclipse occurs twice in an year.

2.8 ORBITAL PERTURBATIONS

Using the Kepler's laws, the equations for various parameters of orbit have been derived in the previous sections, i.e., it is assumed that the satellite orbit is ellipse. But this is not true ellipse due to various forces on the satellite tending to deviate from the perfect locus of ellipse. The factors, which influence this, are asymmetry of earth's gravitational field, the gravitational fields of sun and moon, solar radiation pressure, atmospheric drag, etc. If these interfering forces were left unchecked, the subsatellite point of the geostationary satellite would tend to move with time.

The approach to check and correct these effects of perturbations is first to derive an *osculating orbit* for some instant of time with orbital elements. The perturbations are assumed to cause the orbital elements to vary with time and the orbit and satellite location at any instant are taken from the osculating orbit calculated with orbital elements corresponding to that time. To visualize the process, assume the osculating orbital elements at time t_0 or a_0, e_0 etc. Then, assume that the orbital elements vary linearly with time at a constant rates given by da/dt, de/dt, etc. The satellite's position at any time t_1 is then calculated from a Keplerian orbit with elements as

$$a_0 + (t_1 - t_0)da/dt, \quad e_0 + (t_1 - t_0)\, de/dt$$

As the perturbed orbit is not an ellipse, some care must be taken in defining an orbital period. Since the satellite does not return to the same point in space once per revolution, the quantity most frequently specified is the *anomalistic period* defined as the time lapse between two successive perigee passages.

Effects of the earth's oblateness

Since the earth is not a perfect sphere with a symmetric distribution of mass, its gravitational potential does not have a simple (1/*r*) dependence assumed by the equations of previous sections. The earth's gravitational potential is represented more accurately by an expression in Legendre polynomials J_n in ascending powers of earth's radius/orbital radius. The effect of the dominant J_2 coefficient term is to cause an unconstrained geosynchronous satellite to drift towards and circulate around the nearer of two stable points. These correspond to subsatellite longitudes of 105 °W and 75 °E, locations called *graveyards* because they collect old satellites whose station-keeping fuel is exhausted.

Effects of sun and moon

Gravitational attractions by the sun and moon cause the orbital inclination of a geostationary satellite to change with time. If not countered by north-south station-keeping, these forces would increase the orbital inclination from an initial 0° to 14.67° 26.6 years later. Since, no satellite has such a long lifetime, the problem is not acute.

2.9 ORBITAL EFFECTS IN COMMUNICATIONS SYSTEM PERFORMANCE

Doppler shift

Doppler shift in frequency due to relative motion between the satellite and a point on the earth surface is the prime concern in the low orbit satellites and is to be taken care for establishment of perfect communication link. Doppler can be given as

$$\frac{f_R - f_T}{f_T} = \frac{\Delta f}{f_T} = \frac{V_T}{v_p} \tag{2.32}$$

f_R , f_T are transmitted and received frequencies respectively, V_T is the component of transmitter velocity directed towards the receiver and v_p is the phase velocity of light.

The Doppler is not available in the downlink, but it affects the uplink if unchecked. The Doppler is not available with geostationary satellite since there is no relative motion between the earth station and the satellite.

Range variations

With the best station-keeping systems available, the position of a geostationary satellite with respect to the earth exhibits cyclic variations daily. The resulting range variations has negligible effect on the power equations, an effect on the roundtrip delay. This adds unacceptably large guard times in the TDMA systems. So, the TDMA system continuously monitors the range and adjusts the burst timing accordingly.

Eclipse

A satellite is said to be in *eclipse* when the earth blocks the solar energy to solar panels of the satellite when the three come in line. As discussed in the previous sections, eclipse occurs twice in a year around equinox. Figure 2.12 shows the eclipse time per day during the period of eclipse.

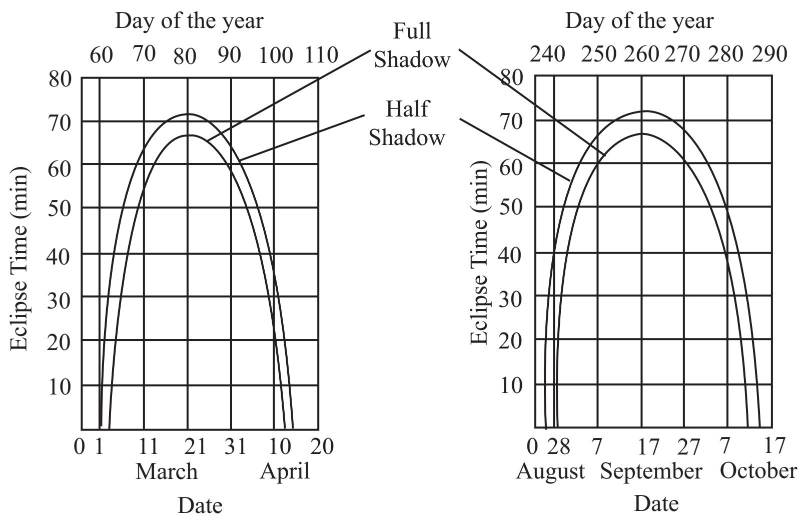

Figure 2.12 Dates and durations of eclipses.

The solar eclipse caused by moon to the geostationary satellite occurs when the moon moves to front of the sun. The eclipse occurs irregularly in time of duration and depth. In general, the eclipse may occur twice within a 24 hr period. Eclipse may range from a few minutes to over two hours within an average duration of about 40 minutes. Compared to earth-solar eclipse, the number of moon-solar eclipse range from zero to four with an average of two per year. It is worthwhile to note that if the moon-solar eclipse of long duration occurs just before or just after the earth-solar eclipse, the satellite has to face special problems in connection with battery recharging and spacecraft thermal reliability. In order to cope with the solar battery problems during eclipses an energy reserve is provided with the satellite.

During full eclipse a satellite receives no power from sun and it must operate entirely from batteries. This can reduce the available power significantly as the spacecraft nears the end of its life, and it may necessitate shutting down some of the transponders during the eclipse period. Spacecraft designers must guard harmful transients as solar power fluctuates sharply at the beginning and end of an eclipse. There is a possibility of having the primary power failure and so, the probability that a primary power supply failing is much more during eclipse rather than any other operations like deployment.

Sun-transit outage

The overall receiver noise will rise significantly to effect the communications when the sun passes through the beam of an earth station antenna. This effect is predictable and can cause outage for as much as 10 min a day for several days and for about 0.02% an average year. The receiving earth station has to wait until the sun moves out of the main lobe of the antenna. This occurs during the daytime, where the traffic is at its peak and forces the operator to hire some other alternative channels for uninterrupted communication link.

2.10 PLACEMENT OF SATELLITE INTO GEOSTATIONARY ORBIT

The placement of a satellite in a stationary orbit involves many complex sequences and is shown in Fig. 2.13. This type of satellite launching is known as *Hohmann Transfer.* First, the launch vehicle places the satellite in an elliptical transfer orbit whose apogee distance is equal to the radius of the geostationary orbit (42,164.2 km). The perigee distance of the elliptical transfer orbit is approximately 6678.2 km, about 300 km above the earth's surface. The satellite spin is stabilized in the transfer orbit so that ground control can communicate with the telemetry system. When the orbit and attitude of the satellite have been determined exactly and when the satellite is at the apogee of the transfer orbit, the apogee kick motor is fired to circularize the orbit. This circular orbit, with a radius of 42,164.2 km is a geostationary orbit if the launch is carried at 0° latitude, the equator. If the satellite is launched from any other latitude, the orbit would be geosynchronous with inclination i greater than or equal to the latitude θ_1 when the injection at the perigee is horizontal.

The velocity at the perigee and apogee can be determined as follows:

At perigee, r = 6678.2 km, a = 24,421.2 km and the velocity V_p = 10.15 km/s is given by equation (2.10) and

At apogee, r = 42,164.2 km, V_a = 1.61 km/s.

Velocity in the synchronous orbit is V_c = 3.07 km/s with $r = a$ = 42,164.2 km.

The incremental velocity required to circularize the orbit at the apogee of transfer orbit must be

$$\Delta V_c = V_c - V_a = 1.46 \text{ km/s} \tag{2.33}$$

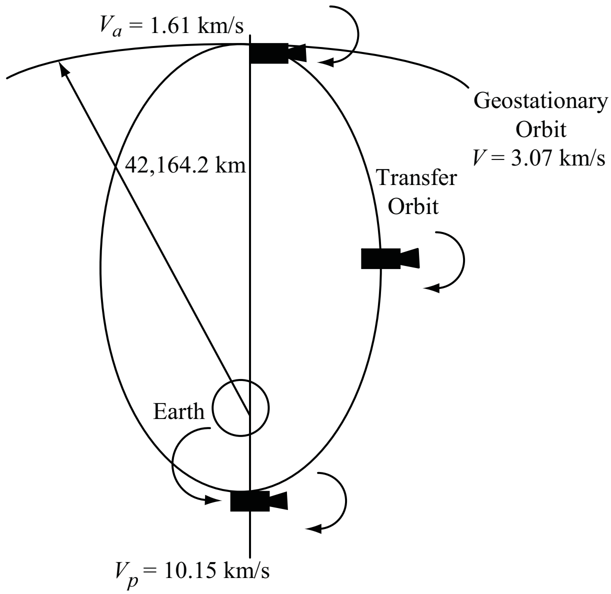

Figure 2.13 Placement of a satellite in a geostationary orbit.

Since the plane of transfer orbit is formed by the position vector ***r*** and the velocity vector ***V*** of the satellite at a given instant in time, the inclination correction can be made at the ascending or descending node where the orbit intersects the equatorial plane at an incremental velocity vector $\boldsymbol{\Delta V}$ in such a way that the sum of the node velocity vector $\boldsymbol{V_n}$ and the incremental velocity vector $\boldsymbol{\Delta V}$ is a vector ***V*** in the equatorial plane. The inclination correction is as shown in Fig. 2.14 and is given by

$$\Delta V = \sqrt{(V_n^2 + V_c^2 - 2\,V_n V_c \cos i)} \tag{2.34}$$

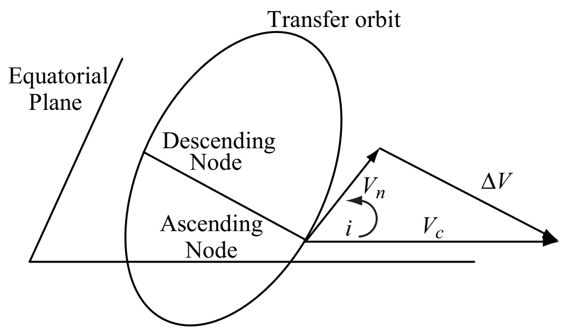

Figure 2.14 Simultaneous orbit circularization and inclination correction.

If the line connecting the apogee and the perigee is the node line, and the inclination correction is made at the apogee in conjunction with orbit circularization, then,

$$\Delta V = \sqrt{(V_a^2 + V_c^2 - 2\, V_a V_c \cos i)} \tag{2.35}$$

Solved Examples

2.1 A satellite is orbiting in a geosynchronous orbit of radius 42,000 km. Find the velocity and time period of the orbit. Also, determine the change in velocity required if the radius of the orbit is to be reduced to 36,500 km. Assume $g_0 = 398600.5$ km^3/s^2.

Solution

The gravitational coefficient, g_0 = 398600.5 km^3/s^2

Radius of the orbit = 42,000 km

Velocity in the orbit $v_s = \sqrt{\dfrac{g_0}{r_e + h}}$ = **3.08066 km/s**

Orbit period $T_s = \dfrac{2\pi d^{3/2}}{\sqrt{g_0}}$ = **85,661.34 s**

For $r_e + h$ = 36,500 km, $v_s = \sqrt{\dfrac{g_0}{r_e + h}}$ = 3.3046 km/s

Increase in velocity = 3.3046 – 3.08066 = **0.224 km/s**

2.2 Determine the slant range and viewing angle of a geostationary satellite orbiting at 42,500 km from the earth station making an elevation angle of 30°.

Solution

d = 42,500 km

Radius of earth = 6378 km

$$\gamma = \cos^{-1}\left[\frac{r_e \cos\xi}{d}\right] - \xi = 52.53^\circ$$

Slant range, $d_3 = d\left[1 + \left\{\dfrac{v_e}{d}\right\}^2 - \dfrac{2v_e}{d}\cos\gamma\right]^{1/2}$ = **38,950.3 km**

Viewing angle, $\delta = \sin^{-1}\dfrac{r_e}{r_e + h}\cos\xi$ = **7.4675°**

2.3 Calculate the duration of eclipse for a geostationary satellite orbiting 42,500 km and declination of sun rays is 2.8°. Determine the radius of the orbit so that no eclipse will ever occur.