

2

Arithmetic of Number Systems

INTRODUCTION

Arithmetic operations in number systems are usually done in binary because designing of logic networks is much easier than decimal.

In this chapter we will discuss arithmetic operations in binary, octal, and hexadecimal number systems. The 1's-complement and 9's-complements in the decimal system and the 2's-complement and 1's-complements in the binary system will also be discussed, which are the key elements for designing a logic circuit.

2.1 BINARY SYSTEM ARITHMETIC

The basic arithmetic in binary number system is binary addition. Binary subtraction is done by using 1's or 2's complements. Multiplication and division are discussed with shift registers in the later section.

The addition of numbers in any numbering system is accomplished as in decimal system, that is, the addition starts in the least significant position (rightmost position), and any carries are added in other positions to the left as it is done in the decimal system.

The binary number system has two bits 0 and 1 only, therefore, the possible binary additions are:

$$0 + 0 = 0 \quad 0 + 1 = 1 \quad 1 + 0 = 1 \quad 1 + 1 = 0 \text{ with a carry of } 1$$

Example 2.4 Subtract $(314)_8$ from $(713)_8$.

Solution

$$\begin{array}{r} 713 \\ 314 \text{ ---} \\ \hline 377 \end{array}$$

- Least significant digit of subtrahend is 4, locate it in the first row of Table 2.1. Go downwards in that column where 4 appears, choose 13 as the least significant digit of minuend is 3. Visualizing from the leftmost column the difference will be 7 with a borrow.
- Then, add the borrow to the next digit of the subtrahend, 1 in this case, hence we will subtract 2 from 1. Locate 2 in the first row of the Table and go downwards in the same column, choose 11 because the next digit of the minuend is 1, visualizing from the leftmost column the difference will be 7 with another borrow.
- Now, add that borrow to 3 and then subtract 4 from 7. Visualizing from the leftmost column the difference is 3 with no borrow.
- Hence, the result of subtraction will be 377, which can also be checked for correctness by converting the numbers to equivalent decimal numbers.

2.3 HEXADECIMAL SYSTEM ARITHMETIC

The process of hexadecimal addition and subtraction is similar to that of addition and subtraction of octal numbers except that we use Table 2.2. instead of Table 2.1. As hexadecimal numbers range from 0 to F, i.e. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F. The manipulation for 16 numbers is given below in Table 2.2 which will be used for addition and subtraction of hexadecimal numbers. The procedure is as follows.

For addition, locate the least significant digit of the first number (augend) in the upper row of the table, and then locate the least significant digit of the second number (addend) in the leftmost column of the Table. The point of intersection (number) of the augend with the addend gives the sum of the two numbers. Repeat the same procedure for all other digits from right to left.

TABLE 2.2 Table for addition and subtraction of hexadecimal numbers

0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	10
2	3	4	5	6	7	8	9	A	B	C	D	E	F	10	11
3	4	5	6	7	8	9	A	B	C	D	E	F	10	11	12
4	5	6	7	8	9	A	B	C	D	E	F	10	11	12	13
5	6	7	8	9	A	B	C	D	E	F	10	11	12	13	14
6	7	8	9	A	B	C	D	E	F	10	11	12	13	14	15
7	8	9	A	B	C	D	E	F	10	11	12	13	14	15	16
8	9	A	B	C	D	E	F	10	11	12	13	14	15	16	17
9	A	B	C	D	E	F	10	11	12	13	14	15	16	17	18
A	B	C	D	E	F	10	11	12	13	14	15	16	17	18	19
B	C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A
C	D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B
D	E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C
E	F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D
F	10	11	12	13	14	15	16	17	18	19	1A	1B	1C	1D	1E

Example 2.5 Perform a hexadecimal addition of $(C629)_{16}$ and $(9A52)_{16}$.

Solution

$$\begin{array}{r} C629 \\ 9A52 \quad + \\ \hline 1607B \end{array}$$

- Start with the least significant digit column in the example, add 2 with 9 and the Table shows B as intersection point (number) with no carry.
- Now, add 5 with 2 and from the Table and the intersection point is 7 with no carry.
- Add A with 6 and the intersection point is 0 with 1 carry.
- The sum can also be checked for correctness by converting the numbers to their equivalent decimal numbers.

Example 2.6 Perform the hexadecimal subtraction of $(8B6F)_{16}$ from $(E7AC)_{16}$.

Solution

$$\begin{array}{r} E7AC \\ 8B6F \quad - \\ \hline 5C3D \end{array}$$

- Locate the least significant digit of the subtrahend F in the example from the first row of Table 2.2, and go downwards in the same column to find C. Number C is available as 1 C at a point (number) D in the leftmost column of the table, which is the difference D with a borrow.
- Then, because of the borrow, reduce the digit A of the minuend to 9. Now subtract 6 from 9, with the same procedure as above. The difference will be 3 with no borrow.
- Now find the difference of B from 7 with the same procedure. The difference is C with another borrow.
- Because of the previous borrow, reduce E to D. Now subtract 8 from D, which produces the difference of 5 with no borrow.
- The difference can also be checked for correctness by converting the numbers to their equivalent decimal numbers.

2.4 COMPLEMENTS OF NUMBERS

To simplify the binary subtraction operation complement of numbers are used. In digital systems for each radix or base r , there are two types of complements, i.e. r 's-complement, and the $(r-1)$'s-complement. It simply means, for the decimal number system whose base is 10, we have 10's-complement and the 9's-complements. For octal number system with a base of 8 we have 8's-complement and 7's-complement. Similarly for binary number system with a base of 2 we have the 2's-complement and 1's-complement. And for the hexadecimal system with base of 16 we have the 16's-complement and the 15s-complement.

2.4.1 10's-Complement

The 10's-complement can be obtained by subtracting the least significant digit from 10 and all other digits from 9.

Example 2.7 Find the 10's-complement of 55274.

Solution

$$\begin{array}{r} 9999\ 10 \\ 5527\ 4\ - \\ \hline 4472\ 6 \end{array}$$

First subtract 4 (lsd) from 10 we obtain 6. The other digits of the given example, i.e. 7, 2, 5, and 5 will be subtracted from 9 and we obtain 2, 7, 4, and 4 respectively. Hence, the 10's-complement of 55274 is 44726.

Example 2.8 Find the 10's-complement of 0.6735.

Solution First subtract 5 (lsd) from 10 and all other digits from 9. Hence, the 10's-complement of 0.6735 is 0.3265.

Example 2.9 Find the 10's-complement of 46.653.

Solution First subtract 3 (lsd) from 10 and all other digits from 9. Hence, the 10's-complement of 46.653 is 53.347.

2.4.2 9's-Complement

The 9's-complement of a number can be obtained by subtracting every digit of the given number from 9.

Example 2.10 Find the 9's-complement of 55274.

Solution

$$\begin{array}{r} 99999 \\ 55274\ - \\ \hline 44725 \end{array}$$

Subtract every digit of the given number from 9 and the 9's-complement of 55274 is 44725. It is observed that this complement is one less than 44726 which was the 10's-complement of the same number. The 9's-complement of any number is always one less than the 10's-complement. The 10's-complement can also be obtained by adding 1 to the 9's-complement of the given number.

Example 2.11 Find the 9's-complement of 46.653.

Solution Subtract every digit of the given number from 9, the 9's-complement of 46.653 is 53.346.

2.4.3 2's-Complement

The 2's-complement can be found by unchanging all the least significant 0's and the least significant 1's and replacing all other 0's with 1's and all other 1's with 0's in the remaining number. 2's-complement of the number can also be obtained by adding 1 to the least significant digit in 1's-complement of the given number.

Example 2.12 Find the 2's-complement of 1011010.

Solution Looking from the right of the given number, the least significant one appears at 2nd digit, hence we will not alter 10 and for the remaining digits 10110, replace all ones with zeros and all zeros with ones to find the 2's-complement of a number. Hence, the 2's-complement of 1011010 is 0100110.

Example 2.13 Find the 2's-complement of 0.0101.

Solution Leave the lsd, i.e. 1, unchanged and replace all 1's with 0's and all 0's with 1's. Hence, the 2's-complement of 0.0101 is 0.1011. The first zero (0) to the left of the binary point, which separates the integer and the fractional parts remains unchanged.

Example 2.14 Find the 2's-complement of 1010.0011.

Solution Leave the lsd i.e., 1 unchanged and replace all 1's with 0's and all 0's with 1's. Hence, the 2's-complement of 1010.0011 is 0101.1101.

2.4.4 1's-Complement

To find 1's-complement of a number replace all 0's with 1's and all 1's with 0's. The 1's complement of a number is always 1 less than the 2's-complement of a number.

Example 2.15 Find the 1's-complement of 1011010.

Solution For the given example by replacing all 1's with 0's and all 0's with 1's, the 1's-complement of 1011010 is 0100101. It is also observed that 1's-complement of the given number is 1 less than 0100110 which was 2's-complement of the same number in the previous examples.

Example 2.16 Find the 1's-complement of 0.0101.

Solution Replace all 1's with 0's and all 0's with 1's, the 1's-complement of 0.0101 is 0.1010. The first zero (0) to the left of the binary point that separates the integer and the fractional part remains unchanged.

2.5 SUBTRACTION OF DIGITAL SYSTEMS

2.5.1 Subtraction with 10's-Complement and 2's-Complement

In the subtraction of digital systems it is assumed that both the numbers for the subtraction operation are positive. With the help of 10's-complement or 2's-complements the subtraction operation is performed in the following procedure:

- Obtain the 10's-complement or 2's-complement of the subtrahend and add it to the minuend.
- Verify the result (sum), and observe the carry. Discard the carry if it occurs at the end. Take the 10's-complement or 2's-complement of the result (sum) and place minus (–) sign before it if no carry occurs at the end. The examples of the both cases are given below:

Example 2.17 Find the subtraction $(51346 - 06934)_{10}$ using the 10's-complement method.

Solution

$$\begin{array}{rcl}
 \text{Minuend} & = & 51346 \\
 \text{Subtrahend} & = & 06938 \\
 & & \\
 \text{Minuend} & = & 51346 \\
 \text{10's-complement of subtrahend} & = & 93062 \quad + \\
 \hline
 & = & 1,44408
 \end{array}$$

Here, an end carry occurs, hence discard it.

The result of $(51346 - 06938)_{10}$ is $(44408)_{10}$.

Example 2.18 Find the subtraction $(06938 - 51346)_{10}$ using the 10's-complement method.

Solution

$$\begin{array}{rcl}
 \text{Minuend} & = & 06938 \\
 \text{Subtrahend} & = & 51346 \\
 & & \\
 \text{Minuend} & = & 06938 \\
 \text{10's-complement of subtrahend} & = & \underline{48654} \quad + \\
 & = & 55592
 \end{array}$$

In the given example it is observed that after performing the subtraction operation i.e. addition of 10's-complement of subtrahend with minuend, no carry occurs at the end.

Hence, the 10's-complement of the result 55592 is taken and we put a minus (-) sign before it resulting in -44408 . Hence, $(51346 - 06938)_{10} = (-44408)_{10}$.

Example 2.19 Find the subtraction $(1110101 - 1001101)_2$ using the 2's-complement method.

Solution

$$\begin{array}{rcl}
 \text{Minuend} & = & 1110101 \\
 \text{Subtrahend} & = & 1001101 \\
 & & \\
 \text{Minuend} & = & 1110101 \\
 \text{2's-complement of subtrahend} & = & \underline{0110011} \quad + \\
 & = & 1011000
 \end{array}$$

Here, an end carry occurs, hence discard it.

The result of $(1110101 - 1001101)_2$ is $(0101000)_2$.

Example 2.20 Find the subtraction $(1001101 - 1110101)_2$ using the 2's-complement method.

Solution

$$\begin{array}{rcl}
 \text{Minuend} & = & 1001101 \\
 \text{Subtrahend} & = & 1110101 \\
 & & \\
 \text{Minuend} & = & 1001101 \\
 \text{2's-complement of subtrahend} & = & \underline{0001011} \quad + \\
 & = & 1011000
 \end{array}$$

In the given example it is observed that after performing the subtraction operation i.e. addition of 2's-complement of subtrahend with minuend, no carry occurs at the end.

Hence, the 2's-complement of the result 1011000 is taken and we put a minus (-) sign before it resulting in -0101000 . Hence, $(1001101 - 1110101)_2 = (-0101000)_2$.

2.5.2 Subtraction with 9's-Complement and 1's-Complement

In the subtraction of digital systems it is assumed that both the numbers for the subtraction operation are positive. With the help of 9's-complement or 1's-complement the subtraction operation is performed in the following procedure:

- Obtain the 9's-complement or 1's-complement of the subtrahend and add it to the minuend.

- Verify the result (sum), and observe the carry. If there is an end carry, add 1 to lsd of the result, which is referred to as *end around carry*. If there is no end carry, obtain 9's-complement or 1's-complement of the result and put a minus (-) sign before it. The examples of both cases are given below.

Example 2.21 Find the subtraction $(51346 - 06938)_{10}$ using the 9's-complement method.

Solution

$$\begin{array}{r}
 \text{Minuend} \quad = 51346 \\
 \text{Subtrahend} \quad = 06938 \\
 \text{Minuend} \quad = 51346 \\
 \text{9's-complement of subtrahend} \quad = \underline{93061} \quad + \\
 \quad \quad \quad \quad \quad \quad \quad = 144407
 \end{array}$$

Here an end around carry occurs, hence add 1 to the lsd of the result

$$\begin{array}{r}
 44407 \\
 \text{End around carry} = \underline{\quad 1} \quad + \\
 \quad \quad \quad \quad \quad \quad \quad 44408
 \end{array}$$

Hence, the result of subtraction of $(51346 - 06938)_{10}$ is $(44408)_{10}$.

Example 2.22 Find the subtraction $(06938 - 51346)_{10}$ using the 9's-complement method.

Solution

$$\begin{array}{r}
 \text{Minuend} \quad = 06938 \\
 \text{Subtrahend} \quad = 51346 \\
 \text{Minuend} \quad = 03938 \\
 \text{9's-complement of subtrahend} \quad = \underline{48653} \quad + \\
 \quad \quad \quad \quad \quad \quad \quad = 55591
 \end{array}$$

Here no end around carry occurs, take the 9's-complement of the result (sum) i.e. 55591 and put a minus (-) sign before it.

Hence, the result of subtraction of $(51346 - 06938)_{10}$ is $(-44408)_{10}$.

Example 2.23 Find the subtraction $(1110101 - 1001101)_2$ using the 1's-complement method.

Solution

$$\begin{array}{r}
 \text{Minuend} \quad = 1110101 \\
 \text{Subtrahend} \quad = 1001101 \\
 \text{Minuend} \quad = 1110101 \\
 \text{1's-complement of subtrahend} \quad = \underline{0110010} \quad + \\
 \quad \quad \quad \quad \quad \quad \quad = 10100111
 \end{array}$$

Here an end around carry occurs, hence add 1 to the lsd of the result.

$$\begin{array}{r}
 0100111 \\
 \text{End around carry} = \underline{\quad 1} \quad + \\
 \quad \quad \quad \quad \quad \quad \quad 0101000
 \end{array}$$

Hence, the result of subtraction of $(1110101 - 1001101)_2$ is $(0101000)_2$.

Example 2.24 Find the subtraction of $(1001101 - 1110101)_2$ using the 2's-complement method.

Solution

$$\begin{array}{rcl}
 \text{Minuend} & = & 1001101 \\
 \text{Subtrahend} & = & 1110101 \\
 & & \\
 \text{Minuend} & = & 1001101 \\
 \text{1's-complement of subtrahend} & = & \underline{0001010} \quad + \\
 & = & 1010111
 \end{array}$$

Here no end around carry occurs, take the 1's-complement of the result (sum) i.e. 1010111 and put a minus (–) sign before it.

Hence, the result of subtraction of $(1001101 - 1110101)_2$ is $(-0101000)_2$.

2.6 SIGNED MAGNITUDE OF BINARY NUMBERS

Digital circuitry requires both positive and negative numbers. A signed binary number consists of a sign, either positive or negative and magnitude. In a signed magnitude representation of binary numbers, the most significant digit is zero for the representation of positive binary number and one for the representation of negative binary numbers. This msd (0 or 1) represents whether the number is positive or negative and the magnitude is the value of the numbers. There are three methods by which a signed number can be represented i.e. signed magnitude, 1's-complement and 2's-complement. Digital computers store negative binary numbers in the form of 2's-complement. Of the three methods mentioned above, for the representation of signed binary numbers, 2's-complement method is most widely used and sign magnitude is rarely used.

2.6.1 Sign Bit and Sign Magnitude

The most significant digit (msd) i.e. the leftmost bit in the signed binary number is known as signed bit. If this bit is zero (0), the number is positive and if this bit is one (1), the number is negative. When the signed binary number is represented in sign magnitude, the most significant digit, the leftmost is referred to as a sign bit and the remaining bits show the magnitude of that number. For example, the decimal number +28 is expressed in 8-bit signed binary number as 00011100, in which the most significant zero shows that the number is positive (+28). On the other hand, if it is required to express –28 in 8-bit signed binary number using sign magnitude, the binary equivalent will be 10011100, in which the most significant bit, leftmost bit, represents the number as negative.

Example 2.25 Express $(+33)_{10}$ and $(-33)_{10}$ into signed magnitude binary form.

Solution The binary equivalent of 33 is 100001. Its representation in 8-bit signed binary number will be (00100001). Hence, the representation of $(+33)$ will be (00100001) and (-33) will be (10100001). It is to be noted that there is a change in the most significant digit while representing +33 and –33 respectively.

Example 2.26 Express $(+56)_{10}$ and $(-56)_{10}$ into signed magnitude binary form.

Solution The binary equivalent of 56 is 111000. Its representation in 8-bit signed binary number will be (00111000). Hence, the representation of $(+56)$ will be (00111000) and (-56) will be (10111000). It is to be noted that there is a change in the most significant digit while representing +56 and –56 respectively.

In 2's-complement form (-19) is represented by taking 2's-complement of ($+19$).

$$\begin{array}{r} 1's\text{-complement of } (+19) = 11101100 \\ + \quad \quad \quad 1 \\ \hline 11101101 \end{array}$$

which is expressed as 11101101.

2.8 DECIMAL SIGNED NUMBERS

Decimal values of the positive and negative signed magnitude numbers can be determined by the summation of the weights of all the magnitude bits, where there are 1's and ignoring all other bits, where there are zeros (0).

Example 2.30 Express the decimal equivalent of signed binary number 10011100 expressed in its sign magnitude form.

Solution Here, there are seven magnitude bits and one sign bit. Separating sign bits and magnitude bits

sign bit = 1, which means that the magnitude of the number is negative.

magnitude bits = 0011100, assigning the weights to the bits, we get

$$\begin{array}{ccccccc} 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \end{array}$$

Summing the weights together where 1 exists and ignoring where 0 exists,

we get $2^4 + 2^3 + 2^2 = 16 + 8 + 4 = 28$.

Adding sign magnitude bit to the solution for the signed magnitude binary number (10011100) is (-28).

Example 2.31 Express the decimal equivalent of signed binary number 01011110 expressed in its sign magnitude form.

Solution Here, there are seven magnitude bits and one sign bit. Separating sign bits and magnitude bits,

Sign bit = 0, which means that the magnitude of number is positive.

Magnitude bits = 1011110, assigning the weights to the bits, we get

$$\begin{array}{ccccccc} 2^6 & 2^5 & 2^4 & 2^3 & 2^2 & 2^1 & 2^0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 0 \end{array}$$

Summing the weights together where 1 exists and ignoring where 0 exists,

we get $2^6 + 2^4 + 2^3 + 2^2 + 2^1 = 64 + 16 + 8 + 4 + 2 = 94$.

Adding sign magnitude bit to the solution for the signed magnitude binary number (11011110) is ($+94$)

2.9 BINARY MULTIPLICATION

2.9.1 Shift and Add Multiplication (Direct Addition) Method

Multiplication of binary numbers is performed similar as in the primary school by adding a list of shifted multiplicands, according to the digits of the multiplier. This method is used to obtain the product of two unsigned binary numbers. There are three operators used in multiplication operation i.e. multiplicand, the multiplier and the product. It is illustrated by the example given below:

Example 2.32 Multiply $(10010)_2$ and $(11001)_2$.

Solution Here, the multiplication will be performed as below:

	10010	Multiplicand	18
	× 11001	Multiplier	× 25
	<hr style="width: 100%;"/>		<hr style="width: 100%;"/>
	10010		90
	00000		36
Shift and add	00000	Shifted multiplicands	<hr style="width: 100%;"/>
	10010		450
	10010		
	<hr style="width: 100%;"/>		
	111000010	Product	

The multiplication of $(10010)_2$ and $(11001)_2$ is $(111000010)_2$.

2.9.2 Partial Product Method

In shift and add method it is sometimes difficult to list all the shifted multiplicands and then adding them in a digital system. Multiplication of binary numbers can also be performed by partial product method in which the shifted multiplicand is added to multiply the binary number.

This technique is more reliable than the previous one.

Example 2.33 Multiply $(10010)_2$ and $(11001)_2$ using the partial product method.

Solution Here, the multiplication will be performed as below:

	10010	Multiplicand	18
	× 11001	Multiplier	× 25
	<hr style="width: 100%;"/>		<hr style="width: 100%;"/>
	00000	Partial product	90
	10010	Shifted multiplicand	<hr style="width: 100%;"/>
	<hr style="width: 100%;"/>		<hr style="width: 100%;"/>
	010010	Partial product	450
	00000↓	Shifted multiplicand	
	<hr style="width: 100%;"/>		
	0010010	Partial product	
	00000↓↓	Shifted multiplicand	
	<hr style="width: 100%;"/>		
	0010010	Partial product	
	10010↓↓↓	Shifted multiplicand	
	<hr style="width: 100%;"/>		

2.11 SOLVED EXERCISES

Example 2.1 Add $(110100111)_2$ and $(1110101)_2$.

Solution

$$\begin{array}{r}
 11100111 \quad \text{Carries} \\
 110100111 \quad + \\
 \underline{1110101} \\
 1000011100
 \end{array}$$

Example 2.2 Add $(4712)_8$ and $(1624)_8$.

Solution

$$\begin{array}{r}
 101 \quad \text{Carries} \\
 4715 \quad + \\
 \underline{6624} \\
 13541
 \end{array}$$

Example 2.3 Subtract $(232)_8$ from $(417)_8$.

Solution

$$\begin{array}{r}
 417 \\
 \underline{232} \quad - \\
 165
 \end{array}$$

Example 2.4 Perform a hexadecimal addition of $(B49C)_{16}$ and $(4E2F)_{16}$.

Solution

$$\begin{array}{r}
 B49C \\
 \underline{4E2F} \quad + \\
 102CB
 \end{array}$$

Example 2.5 Perform the hexadecimal subtraction of $(C92D)_{16}$ from $(7F9E)_{16}$.

Solution

$$\begin{array}{r}
 C92D \\
 \underline{7F9E} \quad - \\
 498F
 \end{array}$$

Example 2.6 Find the 10's-complement of 63918.

Solution The 10's-complement of 63918 is 36082.

Example 2.7 Find the 10's-complement of 28.4592.

Solution The 10's-complement of 28.4592 is 71.5408.

Example 2.8 Find the 10's-complement of 0.5813.

Solution The 10's-complement of 0.5813 is 0.4187.

Example 2.9 Find the 9's-complement of 63918.

Solution The 9's-complement of 63918 is 36081.

Example 2.10 Find the 9's-complement of 28.4187.

Solution The 9's-complement of 28.4187 is 71.5812.

Example 2.11 Find the 2's-complement of 1011.11010000.

Solution The 2's-complement of 1011.11010000 is 0100.00110000.

Example 2.12 Find the 2's-complement of 111001100000.

Solution The 2's-complement of 111001100000 is 000110100000.

Example 2.13 Find the 2's-complement of 0.10001.

Solution The 2's-complement of 0.10001 is 0.01111.

Example 2.14 Find the 1's-complement of 101000011.

Solution For the given example by replacing all ones with zeros and all zeros with ones, the 1's-complement of 101000011 is 010111100.

Example 2.15 Find the 1's-complement of 1011.11010000

Solution The 1's-complement of 1011.11010000 is 0100.00101111.

Example 2.16 Find the subtraction $(96258 - 43271)_{10}$ using the 10's-complement method.

Solution

$$\begin{array}{rcl}
 \text{Minuend} & = & 96258 \\
 \text{Subtrahend} & = & 43271 \\
 & & \\
 \text{Minuend} & = & 96258 \\
 \text{10's-complement of subtrahend} & = & 56729 \quad + \\
 \hline
 & = & 1,52987
 \end{array}$$

Here, an end carry occurs, hence discard it.

The result of $(96258 - 43271)_{10}$ is $(52987)_{10}$.

Example 2.17 Find the subtraction $(128722 - 439811)_{10}$ using the 10's-complement method.

Solution

$$\begin{array}{rcl}
 \text{Minuend} & = & 128722 \\
 \text{Subtrahend} & = & 439811 \\
 & & \\
 \text{Minuend} & = & 128722 \\
 \text{10's-complement of subtrahend} & = & 560189 \quad + \\
 \hline
 & = & 688911
 \end{array}$$

In the given example it is observed that after performing the subtraction operation i.e. addition of 10's-complement of subtrahend with minuend, no carry occurs at the end.

Hence, the 10's-complement of the result 688911 is taken and we put a minus (-) sign before it resulting in -311089. Hence, $(128722 - 439811)_{10} = (-311089)_{10}$.

$$\begin{array}{rcl}
 \text{Minuend} & = & 43271 \\
 9\text{'s-complement of subtrahend} & = & \underline{03741} \quad + \\
 & = & 47012
 \end{array}$$

Here no end around carry occurs, take the 9's-complement of the result (sum) i.e. 47012 and put a minus (-) sign before it.

Hence, the result of subtraction of $(51346 - 06938)_{10}$ is $(-52987)_{10}$.

Example 2.22 Find the subtraction $(1011110 - 1001011)_2$ using the 1's-complement method.

Solution

$$\begin{array}{rcl}
 \text{Minuend} & = & 1011110 \\
 \text{Subtrahend} & = & 1001011 \\
 \text{Minuend} & = & 1011110 \\
 1\text{'s-complement of subtrahend} & = & \underline{0110100} \quad + \\
 & = & 10010010
 \end{array}$$

Here an end around carry occurs, hence add 1 to the lsd of the result.

$$\begin{array}{r}
 0010010 \\
 \text{End around carry} = \underline{\quad 1} \quad + \\
 0010011
 \end{array}$$

Hence, the result of subtraction of $(1011110 - 1001011)_2$ is $(0010011)_2$.

Example 2.23 Express $(+99)_{10}$ and $(-99)_{10}$ into signed magnitude binary form.

Solution The binary equivalent of 99 is 1100011. Its representation in 8-bit signed binary number will be (01100011). Hence, the representation of $(+99)$ will be (01100011) and of (-99) will be (11100011).

Example 2.24 Express $(+45)_{10}$ and $(-45)_{10}$ into signed magnitude binary form.

Solution The binary equivalent of 45 is 101101. Its representation in 8-bit signed binary number will be (00101101). Hence, the representation of $(+56)$ will be (00101101) and of (-56) will be (10101101).

Example 2.25 Express the decimal number (-61) in 8-bit signed magnitude, 1's-complement and 2's-complement form.

Solution The 8-bit binary representation of $(+61)$ is 00111101.

In signed magnitude form (-61) is represented by changing the leftmost bit to 1 and the remaining bits unchanged, which is expressed as 10111101.

In 1's-complement form (-61) is represented by taking 1's-complement of $(+28)$, which is expressed as 11000010.

In 2's-complement form (-61) is represented by taking 2's-complement of $(+61)$.

$$\begin{array}{r}
 1\text{'s-complement of } (+28) = 11000010 \\
 + \quad \quad \quad 1 \\
 \hline
 11000011
 \end{array}$$

which is expressed as 11000011.

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18. Perform $16.875_{10} \div 4.5_{10}$ using binary arithmetic.
19. Perform the following addition operations.
- (a) $(275 \cdot 75)_{10} + (37 \cdot 875)_{10}$
- (b) $(AB1 \cdot F3)_{16} + (FFF \cdot E)_{16}$.
20. Subtract $(1110 \cdot 011)_2$ from $(11011 \cdot 11)_2$ using the basic rules of binary subtraction.