Chapter **2**

PRESSURE AND HEAD

KEYWORDS AND TOPICS

- PASCAL'S LAW
- HYDROSTATIC LAW
- PRESSURE HEAD
- HYDROSTATIC PARADOX
- MERCURY BAROMETER
- ABSOLUTE PRESSURE
- GAUGE PRESSURE
- VACUUM PRESSURE
- PIEZOMETER
- U TUBE MANOMETER
- MICROMETER
- DIFFERENTIAL MANOMETER
- MICRO MANOMETER
- BOUDON TUBE GAUGE

INTRODUCTION

A fluid is a substance which is capable of flowing. If a certain mass of any fluid is held in static equilibrium by confining it within solid boundaries, then the fluid exerts forces against the boundary surfaces. The forces so exerted always act in the direction normal to the surface in contact. The reason for the forces having no tangential components is that the fluid at rest cannot sustain shear stress. The fluid pressure is therefore nothing but the normal force exerted by the fluid on the unit area of the surface. The fluid pressure and pressure force on any imaginary surface in the fluid remain exactly same as those acting on any real surface. The pressure at a point in a static fluid is same in all directions. The pressure inside the fluid increases as we go down in the fluid and the gradient of the pressure with respect to the depth of the fluid at any point is equal to its specific weight. The pressure exerted by a fluid is dependent on the vertical head and its specific weight.

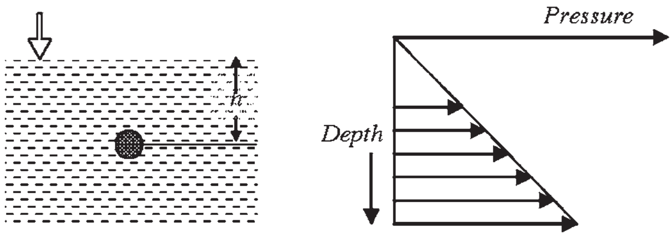
1. What are the forces acting on a fluid at rest?

When a fluid is at rest, there is no relative motion between the layers of the fluid. During rest as there is no velocity gradient, therefore tangential shear stress is zero. Hence at rest can write:

$$\frac{dP}{dz} = \rho \times g$$

or
$$dP = \rho \times gdz$$

Now to find pressure at any point a depth of h of the fluid as shown in the figure.



Pressure variation with depth

$$\int dP = \int_o^h \rho \times g \times dz$$

Since ρ = constant for incompressible fluid

$$\int dP = \rho \times g \int_o^h dz$$

$$P = \rho \times g \times h$$

$$\frac{P}{\rho g} = h = \text{pressure head}$$

The pressure exerted by a fluid is dependent on the vertical head of the fluid and its specific weight.

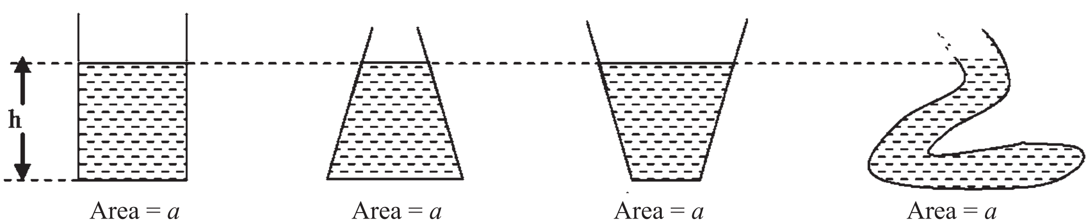
5. What is aerostatic law?

The density of incompressible fluid does not remain constant i.e. $\rho \neq$ constant. Now as per perfect gas equation:

$$PV = mRT$$

or
$$P = \frac{m}{V}RT \quad \text{but} \quad \frac{m}{V} = \text{density} = \rho$$

Now as per hydrostatic law, the pressure gradient is:



$$\frac{dP}{dz} = \rho g$$

$$= \frac{P}{RT} g \quad \text{as} \quad \rho = \frac{P}{RT}$$

$$\frac{dP}{P} = \frac{dz}{T} \times \frac{g}{R}$$

The above equation is called aerostatic law.

6. What do you understand from the hydrostatic paradox?

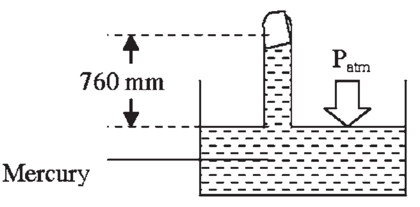
The pressure exerted by a fluid is dependent on the vertical head if its specific weight remains constant. In case we take four vessels as shown in the figure and all vessels have same fluid to the same height (h), then pressure at the button of each vessel will be same i.e. $P = \rho gh$. If each vessel has same area (a) at base, the volume and weight of the fluid in each vessel have to differ due to their varying shapes. However, the force acting on the base of each vessel is same as pressure (head is same) and base area are same for these vessels i.e. Force = pressure × base area. Inspite of different weights of the fluid in different vessels, the existence of constant force at base of these vessels looks like a hydrostatic paradox i.e. a statement seemingly absurd but really based on truth.

7. What do you understand from the atmosphere and atmospheric pressure?

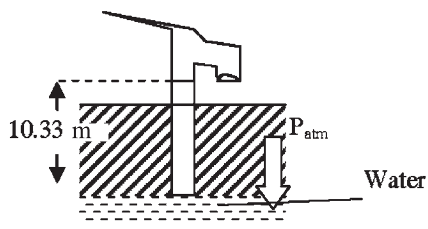
The mass of air surrounding the earth is called atmosphere. The atmospheric air exerts a normal pressure upon all surfaces with which the atmosphere comes in contact. This normal pressure is called atmospheric pressure.

8. What is the value of the atmospheric pressure?

Under normal atmospheric condition of 15°C, the atmospheric pressure is equal to the pressure generated by 760 mm of mercury or 10.33 m of water.



Atmospheric Pressure
(Barometric Height)



Atmospheric Pressure
(Water Lifting)

$$1 \text{ atm} = 760 \text{ mm of Hg or } 10.33 \text{ m of water}$$
$$= 101.104 \text{ kN/m}^2 \text{ or kPa}$$
$$= 1.01 \text{ bar}$$

The atmospheric pressure can lift 760 mm of Hg or 10.33 meter of water against vacuum.

9. What do you understand from manometery and manometers? What are the different types of manometers?

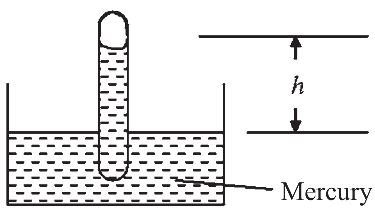
The technique of pressure measurement by employing hydrostatic law $\left(\frac{dP}{dz} = \rho g\right)$ is called manometery. The devices used employing this technique are called manometer. Hence manometer is device used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or other column of fluid. The manometer can be:

(a) Pressure tubes or piezometers
(b) U tube manometers
(c) Micro manometers
(d) Differential manometers

10. What is a mercury barometer?

A mercury barometer consists of a glass tube closed at one end and has diameter sufficient large to minimize the capillary affect. The tube is filled with mercury and a stopper is inserted at open end. The tube is immersed in the container having mercury so that open end with stopper is well beneath the mercury surface. The stopper is removed from the open end and the level of mercury (h) stabilised in the tube as per the atmospheric pressure. The atmospheric pressure as shown by the barometer is:

$$P_{atm} = \rho_{Hg} \times g \times h$$



Barometer

11. Differentiate between (1) absolute pressure (2) gauge pressure and (3) vacuum pressure.

Absolute Pressure: The pressure, which is measured with respect to absolute vacuum pressure or zero pressure, is called absolute pressure.

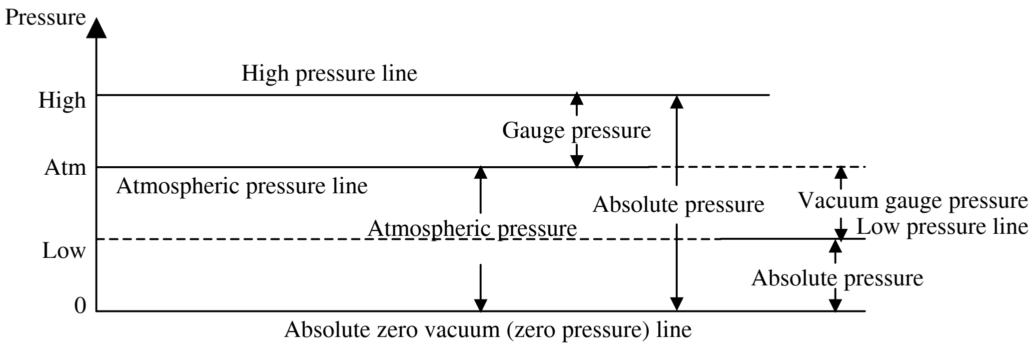
$$P_{absolute} = P_{gauge} + P_{atm}$$

Gauge Pressure: The pressure, which is measured above the atmospheric pressure, is called gauge pressure. Pressure measuring instruments provide gauge pressure.

$$P_{guage} = P_{absolute} - P_{atm}$$

Vacuum Pressure: The pressure, which is measured below the atmospheric pressure, is called vacuum or vacuum gauge pressure. The pressure in such system is above the absolute vacuum pressure (zero pressure) but it is less than atmospheric pressure.

$$P_{vacuum} = P_{atm} - P_{abs}$$



Different Pressures

12. Besides manometers, what are the other types of pressure measuring instruments?

Other types pressure measuring instruments are called mechanical gauges. The types of mechanical gauge are:

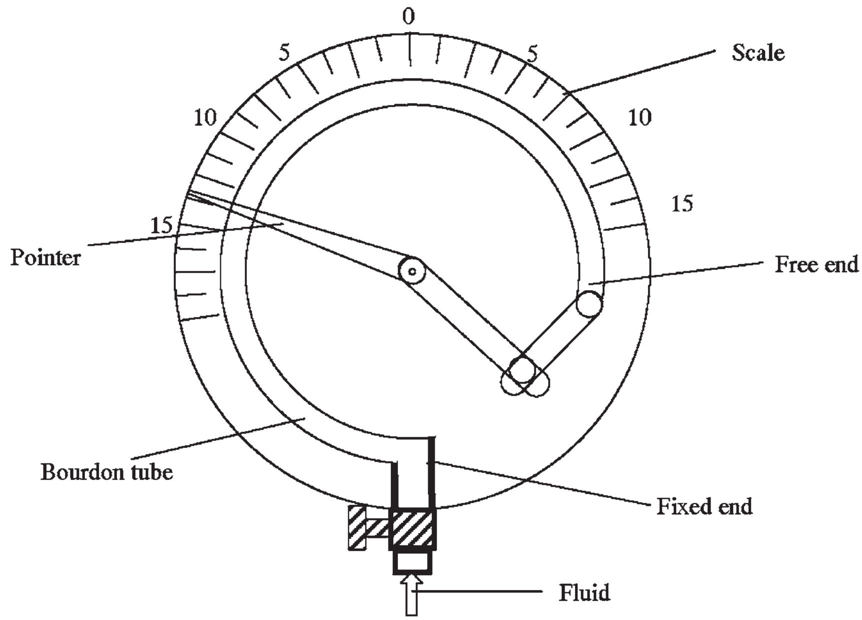
(1) Diaphragm pressure gauge
(2) Bourdon tube pressure gauge
(3) Dead weight pressure gauge
(4) Bellow pressure gauge

13. What are the characteristics of mechanical gauges? Describe Bourdon tube gauge.

The characteristics of mechanical gauges are:

(1) They can measure high pressures above the atmospheric pressure
(2) They give direct pressure reading
(3) They are easily portable instruments.

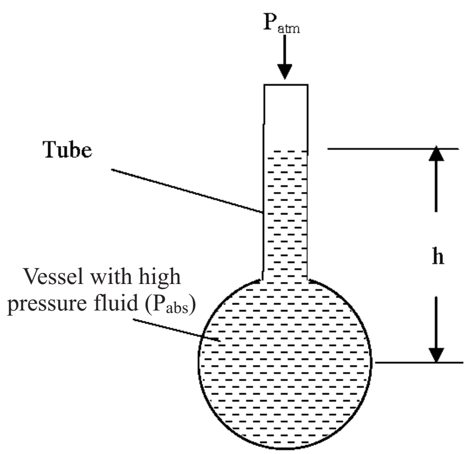
Bourdon Tube: It is a most commonly used mechanical gauge. It consists of a metallic tube bent into a circular shape as shown in the figure. One end of the tube is fixed and other end is free to move inward or outward. A pointer is fixed to the free end which moves over the circular scale graduated in pressure unit (bar) with the help of linkage when the free end moves inward and outward. The fluid whose pressure to be measured is connected to the fixed end of the Bourdon tube and the free end of the Bourdon tube moves due to the pressure of the fluid. The Bourdon tube measures the gauge pressure of the fluid as per the position of the pointer on the scale.



Bourdon Pressure Gauge

14. What is a pressure tube or piezometer? What does it measure?

The manometers work on the relationship between the pressure and the column of the fluid balanced by it. The simplest form of manometers is the pressure tube or piezometer as shown in the figure. The pressure tube consists of a single vertical tube open at the top connected to the vessel or pipe containing the fluid under pressure. Due to pressure of the fluid above the atmospheric pressure, the fluid rises in the tube to a height depending upon its pressure. If fluid rises h meter, then we have gauge pressure of the fluid as:



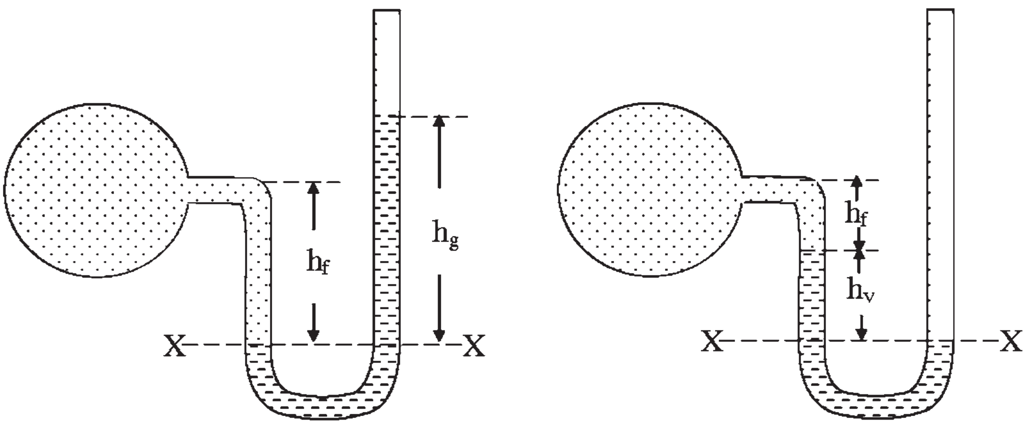
Pressure Tube

$$P_{abs} = P_{atm} + \rho gh$$
$$P_{abs} - P_{atm} = \rho gh$$
$$P_{gauge} = \rho gh$$

Hence pressure tube measures the gauge pressure of the fluid.

15. What is U tube manometer? What are the desired properties of the liquid filled in the manometer?

U tube manometer consists of a glass tube in U shape with one end of it is connected to the vessel whose pressure is to be measured and other end is open to the atmosphere as shown in the figure. The tube contains mercury or any other liquid which has following properties: (1) immiscible with the fluid in the vessel and (2) density of the liquid is greater than density of the fluid in the vessel. If the pressure in the vessel is more than atmospheric pressure, the liquid of the manometer has higher level in open limb than in the closed limb i.e. hg is the rise of the liquid in the open limb. The height hg is called the gauge pressure head of the manometer. On other hand, if pressure in the vessel is less than atmospheric pressure, then liquid rises in the closed limb i.e. h_v is the rise of the liquid in the closed limb. The height h_v is called the vacuum gauge pressure of the manometer. Considering the equilibrium from line $x - x$ for the manometer:



Gauge Pressure　　　　Vacuum Pressure

(a) Gauge Pressure:

$$P_{abs} + \rho_f gh_f = \rho_l ghg + P_{atm}$$
$$P_{abs} - P_{atm} = \rho_l ghg - \rho_f gh_f$$
$$P_{gauge} = \rho_l ghg - \rho_f gh_f$$
$$If \;\; \rho_l >> \rho_f$$
$$P_{gauge} = \rho_l gh_g$$

(b) Vacuum Pressure:

$$P_{abs} + h_f \rho_f g + h_v \rho_l g = P_{atm}$$

$$P_{atm} - P_{abs} = h_f \rho_f g + h_v \rho_l g$$

$$P_{vacuum} = h_f \rho_f g + h_v \rho_l g$$

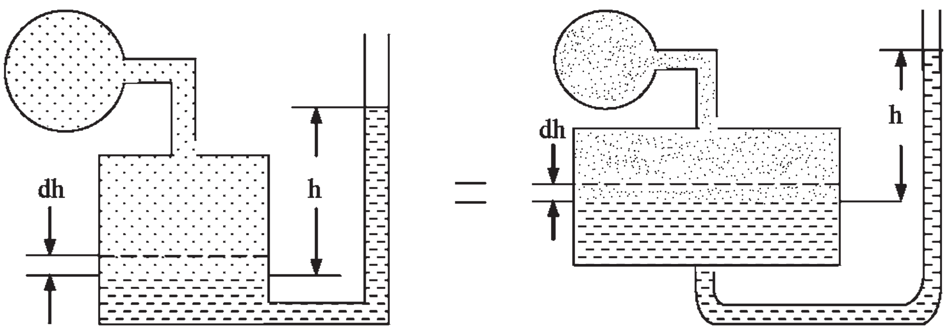
$$If \ \rho_l >> \rho_f$$

$$P_{vacuum} = h_v \rho_l g$$

16. What are the micrometers? Why are they called sensitive manometers?

Micrometers are modified form of double column manometer in which the cross-sectional area of the limb connected to vessel is so large with respect to other limb opened to the atmosphere that any change in the level in the large limb can be neglected. Hence the gauge pressure inside the vessel is directly proportional to the level in the open limb. In micrometers, either one limb is made much larger or a reservoir of large cross-section area is introduced in one of the limbs. In case the cross-sectional area of larger diameter limb (a_{large}) is 100 times than the cross-sectional area of smaller diameter limb, then volume moving from one limb to other has to be same.

∴ Change of volume of large diameter limb = Change of volume in small diameter limb



MICROMETER

$$dh \times a = h \times a_{small}$$

$$\frac{dh}{h} = \frac{a_{small}}{a_{large}} = \frac{1}{100} \to 0$$

As

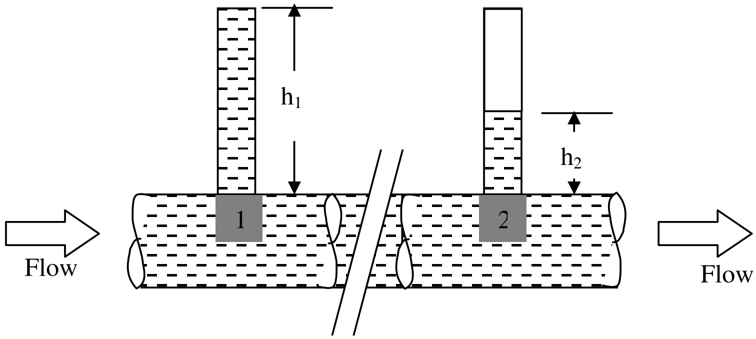
$$P_{gause} = \rho gh - \rho g dh$$

$$= \rho g\left(h - \frac{dh}{h}\right)$$

$$= \rho g(h - 0)$$

$$= \rho gh$$

difference between the points is equal to the difference of the levels of the liquid in the two tubes i.e. $dp = h_1 - h_2$.



Two Piezometer Tubes Method

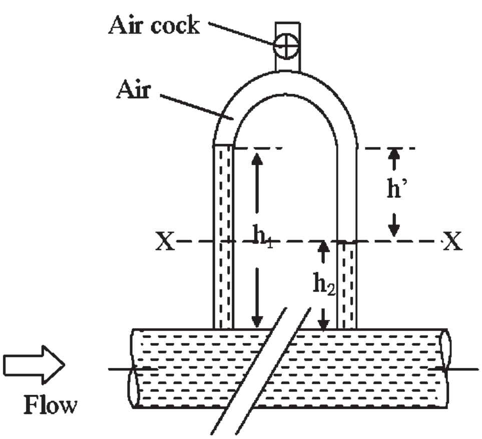
2. Inverted U Tube Manometer: In case two piezometer tubes are joined at top, they form an inverted U tube manometer. An opening at the top of the inverted U tube is provided to drive away air present in the U tube. The upper part of the manometer can also be filled with a lighter measuring fluid, which is lighter than the fluid flowing through the pipeline. This helps in restricting the height of the limbs.

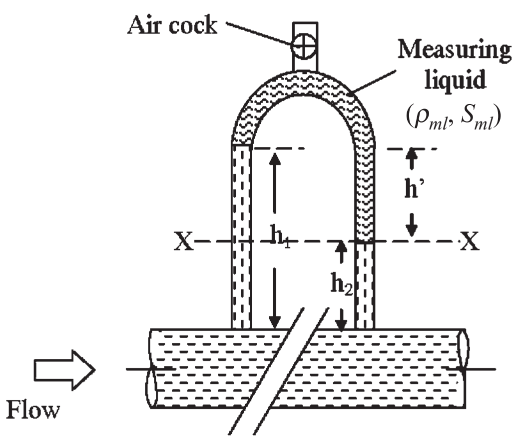
$$\frac{\Delta P}{\rho g} = h_1 - h_2 = h \quad or \quad \Delta P = \rho g h$$

For manometer having measuring liquid (ml)

$$\Delta P = h^1 \rho g \left(1 - \frac{\rho_{ml}}{\rho}\right)$$

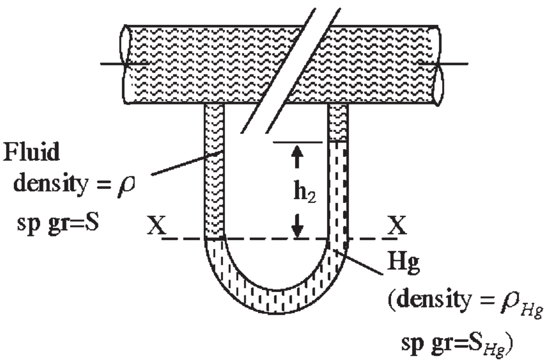
$= h^1 \rho g(1 - s_{ml})$ if water is flowing, then $S_{ml} = S_p$ gravity of measuring liquid.



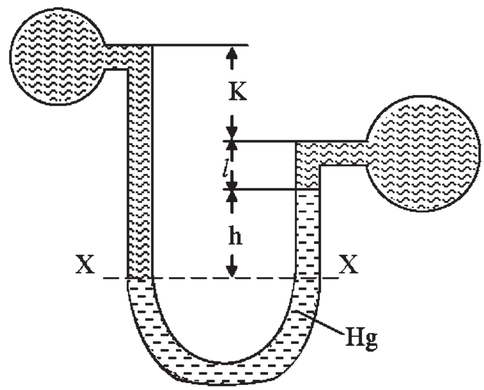


Inverted U Tube Differential Manometer **Differential Manometer with Lighter Fluid**

3. U Tube Differential Manometer: In case pressure difference between two points is high, then length of piezometer tubes and inverted U tube differential manometer is long and unsuitable. In such cases. U tube differential manometer is used to measure the pressure difference. The measuring liquid (generally mercury) used in such manometer is heavier than fluid flowing through the pipe or pipes. The pressure difference can be given as:



U Tube Differential Manometer: One Pipe



U Tube Differential Manometer: Two Pipes

(a) One Pipe Line

$$dP = h\rho_{Hg}g - h\rho g$$

$$= h\rho g\left(\frac{\rho_{Hg}}{\rho} - 1\right) = h\rho g\left(S_{Hg} - 1\right)$$

(b) Two Pipe Lines

$$\Delta P = h\rho g\left(\frac{\rho_{Hg}}{\rho} - 1\right) - k\rho g$$

$$= \rho g\left[h\left(S_{Hg} - 1\right) - k\right]$$

19. What are differences between simple and differential manometer?

Simple Manometer	Differential Manometer
1. It is used to measure pressure at a point.	1. It is used to measure pressure between two points
2. One limb is connected to the point and other is open to air.	2. Limbs are connected to the two points
3. The pressure at a point is obtained in terms of difference of level of fluid flowing through pipe.	3. The difference of pressure is obtained in term of measuring the difference of the levels of manometer liquid.

$$\therefore \qquad dh = \frac{a}{A} \times \frac{h}{2} \to 0$$

The pressure above the line $z - z$ is equal in both limbs.

$$P_1 + \rho_f \times g(h_1 + dh) + \rho_z \times g\left(h_2 + \frac{h}{2} - dh\right)$$

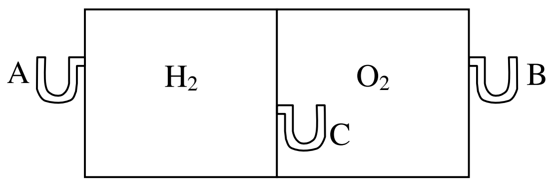
$$= P_2 + \rho_1 \times gh + \rho_2 \times g\left(h_2 - \frac{h}{2} + dh\right) + \rho_f \times g(h_1 - dh)$$

$$P_1 - P_2 = \rho_1 gh - \rho_2 gh$$

$$\frac{P_1 - P_2}{\rho g} = h\left(\frac{\rho_1}{\rho} - \frac{\rho_2}{\rho}\right) \quad \text{where } \rho \text{ density of water}$$

$$= h(S_1 - S_2)$$

22. If atmospheric pressure is 100 kpa and pressure gauge *A* reads 140 kpa, pressure gauge *B* reads –60 kpa, then find the absolute pressure of H_2 and O_2. Also find the reading of pressure gauge *C*.



The pressure gauge *A* is reading gauge pressure of H_2 as its one end is open to atmosphere. The absolute pressure of H_2 is –

$$P_{abs} = P_{gauge} + P_{atm}$$
$$= 140 + 100 = 240 \text{ kpa}$$

Hence absolute pressure of H_2 is 240 kpa.

The pressure gauge *B* is reading the vacuum pressure of O_2 as its one end is open to atmosphere. The absolute pressure of O_2 is –

$$P_{abs} = P_{atm} + P_{vacuum}$$
$$= 100 - 60$$
$$= 40 \text{ kpa}$$

Hence absolute pressure of O_2 is 40 kpa.

The pressure language *C* is reading gauge pressure of H_2 w.r.t. O_2. If it is reading P_{gauge}, then –

$$P_{H_2} = P_{gause} + P_{O_2}$$
$$\therefore \qquad P_{gause} = P_{H_2} - P_{O_2}$$
$$= 240 - 40$$
$$= 200 \text{ kPa}$$

The pressure gauge *C* will show 200 kpa.

23. If the readings of a barometer at top and bottom of a hill are 660 and 760 mm of Hg, then find the height of the hill. Take specific weight of the air (γ) = 12 N/m³.

P_1 = Pressure of top of hill = $h_1 \times \rho_{Hg} \times g$

$$= \frac{760}{1000} \times 13.6 \times 10^3 \times 9.81$$
$$= 101.4 \text{ KN/m}^2$$

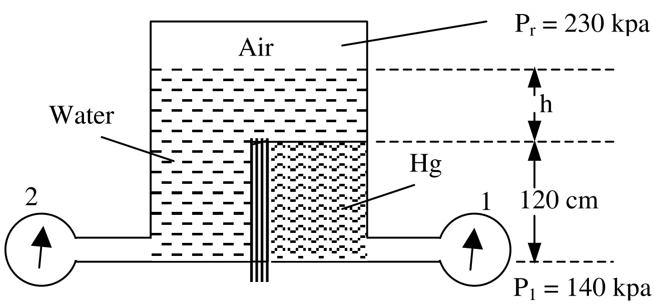
P_2 = Pressure at bottom of hill = $h_2 \times \rho_{Hg} \times g$

$$= \frac{660}{1000} \times 13.6 \times 10^3 \times 9.81$$
$$= 88.06 \text{ KN/m}^3$$

Now $\quad P_1 - P_2 = \gamma h$

or $$h = \frac{(101.4 - 88.06) \times 10^3}{12}$$
$$= \frac{13.34 \times 10^3}{12}$$
$$= 1112 \text{ m}$$

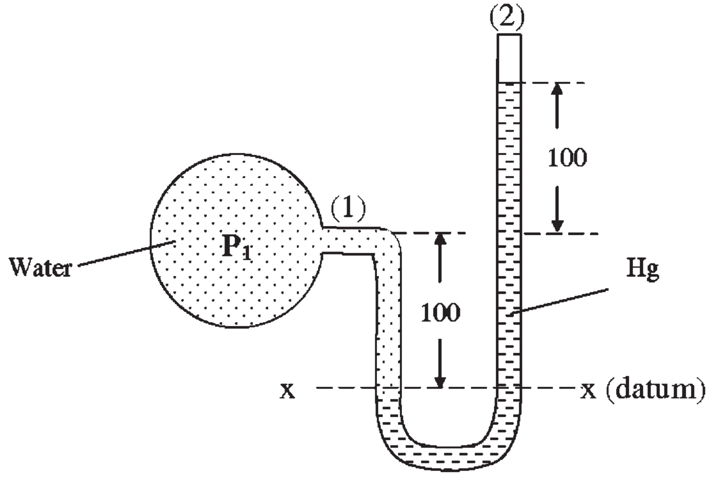
24. A tank is filled with water and mercury with air at top as shown in the figure. If gauge 1 reads 400 kpa (absolute), find (1) height of the water and (2) reading of gauge 2.



The pressure read by gauge 1 is P_1 = 140 kpa. Now pressure P_1 will be equal to pressure exerted by 1.20 cm of Hg, height h of water and air. Therefore is equilibrium –

$$P_1 = 1.20 \times \rho_{Hg} \times g + h \times \rho \times g + Pair$$

$$400 = \frac{1.20 \times 13.6 \times 10^3 \times 9.81}{10^3} + \frac{h \times 10^3 \times 9.81}{10^3} + 230$$
$$= 160 + 9.81 \times h + 230$$
$$h = \frac{10}{9.8} = 1.02 \text{ m}$$



Gauge Pressure

Let P_1 = absolute pressure of the water in the pipeline during equilibrium –
Pressure in the left limb from datum = Pressure in the right limb from datum

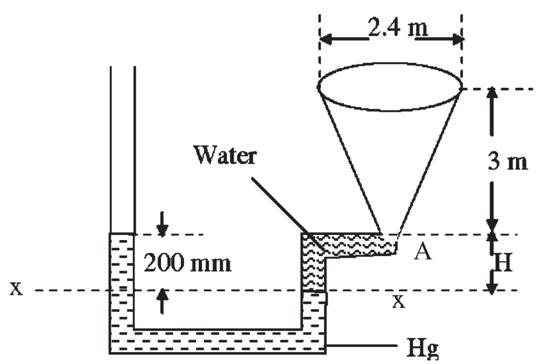
$$P_1 + \frac{100}{1000} \times \rho_w \times g = \left(\frac{100+100}{1000}\right) \times \rho_{Hg} \times g + P_{atm}$$

$$P_1 - P_{atm} = \frac{0.2 \times 13.6 \times 10^3 \times 9.81}{1000} - \frac{0.1 \times 10^3 \times g \times 9.81}{1000}$$

$$P_{gauge} = 26.68 - 0.981$$

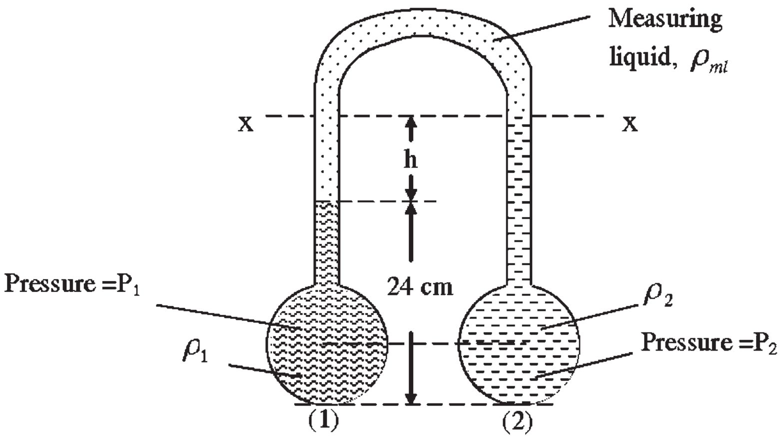
$$P_{gauge} = 25.7 \text{ kpa}$$

27. The figure shows a conical vessel having a U tube manometer attached to its outlet at *A*. When the vessel is empty the reading of the manometer is given in the figure. Find the reading of manometer when the vessel has been completely filled with water.

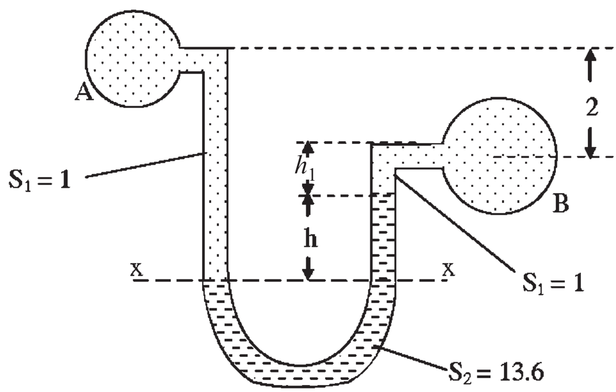


When the vessel is empty, the pressure in both limbs is same from datum line $x - x$
Pressure left limb = Pressure in right limb

$$0.24 \times 1.2 \times 10^3 \times g + h \times 0.8 \times 10^3 \times g = (0.24 + h)1 \times 10^3$$
$$0.288 + 0.8h = 0.24 + h$$
$$0.2h = 0.078$$
$$h = 0.39 \text{ m}$$
$$= 390 \text{ mm}$$



29. Liquids of sp gravity 1.0 flows through pipes *A* and *B* at 0.5 and 0.2 bar respectively. Pipe *A* is 2 m higher than pipe *B*. What would be the difference of levels of U-tube manometer connected to *A* and *B* in case sp gravity of manometer liquid is 13.6 and when liquid is at higher level in the limb connected to *B*.

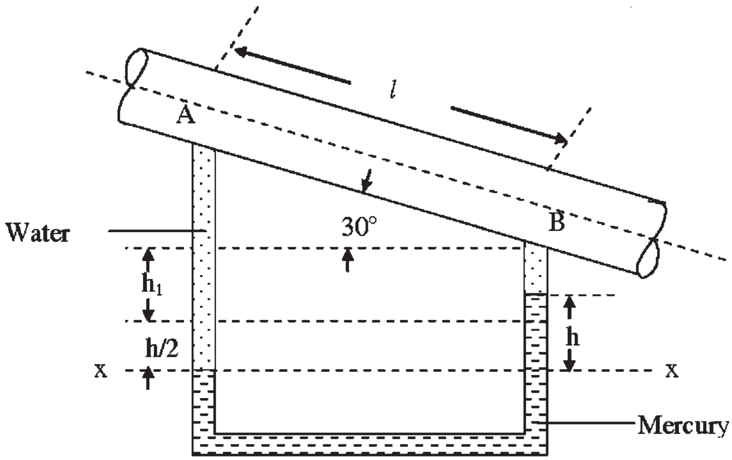


Equating pressure on left and right limb from datum line $x - x$

$$P_a \times (2 + h + h_1)\rho_1 \times g = P_b + h\rho_2 g + h_1\rho_1 g$$
$$0.5 \times 10^5 + (2 + h) \times 10^3 \times 9.8 = 0.2 \times 10^5 + h \times 13.6 \times 10^3 \times 9.8$$
$$0.3 \times 10^2 + (2 + h)9.81 = 13.6 \times 9.81 \times \text{h}$$

$$30 + 19.62 + 9.81h = 133.4h$$
$$123.6h = 49.62$$
$$h = \frac{49.62}{126.6} = 0.401 \text{ m}$$

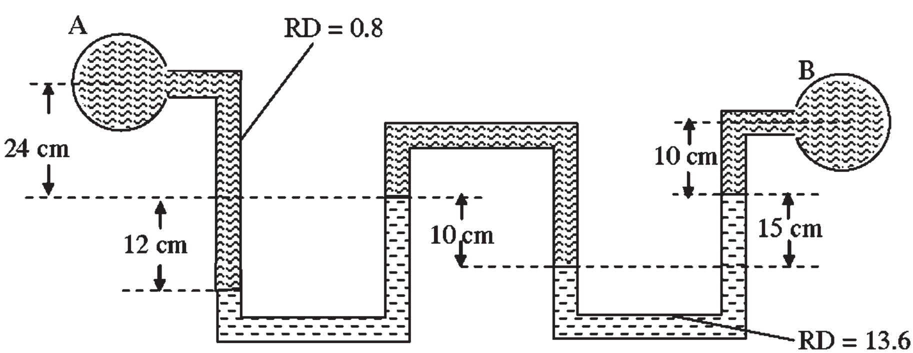
30. Water flow downward in a pipe inclining at 30° with the horizontal as shown in the figure. Find the pressure difference between point *A* & *B* if *l* and *h* = 0.14 m. The manometer fluid is mercury (*S* = 13.6).



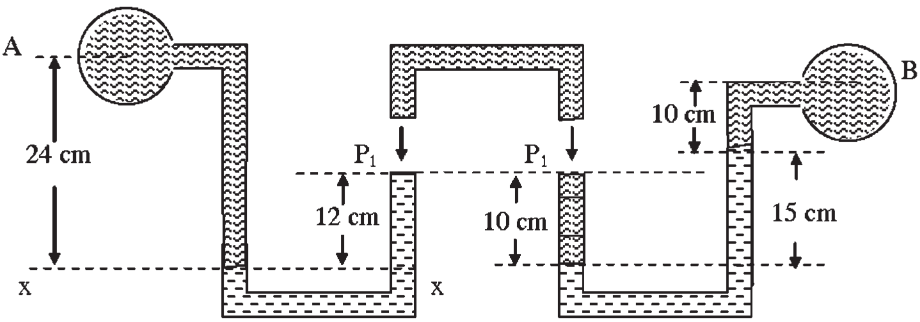
During equilibrium, the pressure in left and right limb is same from datum line $x - x$

$$P_A + 1\times10^3 \times 9.81\left(\ell \sin 30 + h_1 + \frac{h}{2}\right) = 13.6\times10^3 \times h \times 9.81 + 1\times10^3 \times \left(h_1 - \frac{h}{2}\right)\times 9.81 + P_B$$
$$P_A - P_B = 10^3 \times 9.81(13.6\times.14 - 1 - 0.14)$$
$$= 9.83\times10^3(0.764)$$
$$= 7.5 \text{ kpa}$$

31. Determine the pressure difference ($P_A - P_B$) when the reading of manometer is as shown in the figure.



Guidance: To simplify such multi tubes problem, it is best to divide the setup in two or more parts as shown below



Taking equilibrium from datum $x - x$ for left part and datum $y - y$ for right part

$$P_A + 0.24 \times 0.8 \times 10^3 \times g$$
$$= 0.12 \times 13.6 \times 10^3 \times g + P_1$$
$$or\ P_1 = P_A + 1.88 \times 10^3 - 16 \times 10^3$$
$$= P_A - 14.12 \times 10^3$$

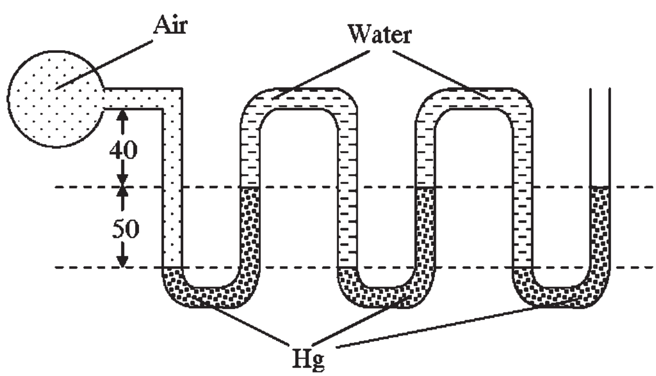
$$P_1 + 0.10 \times .8 \times 10^3 \times g$$
$$= 0.15 \times 13.6 \times 10^3 \times g$$
$$+ 0.10 \times 0.8 \times 10^3 \times g + P_B$$
$$or\ P_1 = P_B + 20.01 \times 10^3$$

Now $\quad P_1 = P_1$

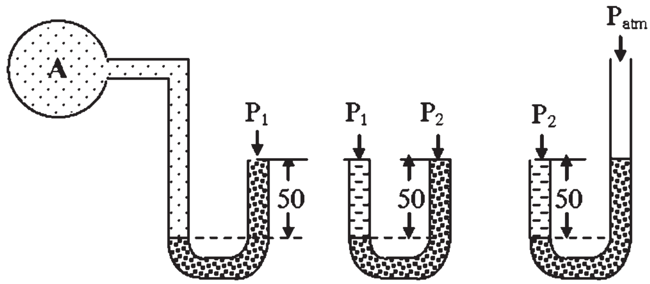
$\therefore \quad P_A - 14.12 \times 10^3 = P_B + 20.01 \times 10^3$

$\therefore \quad P_A - P_B = 34.13$ kpa

32. A multi tube manometer is employed to determine the pressure in a pipeline. The levels inside the tubes are as shown in the figure. What would be the length of single mercury U-tube to record this pressure?



Guidance: Divide the tubes in three parts to simplify the problem as shown below:



$P_A = 0.5 \times 13.6 \times 10^3 \times g + P_1$

$P_A = 66.7 \times 10^3 + P_1$

$= 66.7 \times 10^3 + P_{atm} + 123.6 \times 10^3$

$= 200.3 \times 10^3 + P_{atm}$

$P_1 + .5 \times 1 \times 10^3 \times g = .5 \times 13.6 \times 10^3 \times g + P_2$

$P_1 = 61.8 \times 10^3 + P_2$

$= 61.8 \times 10^3 + P_{atm} + 61.8 \times 10^3$

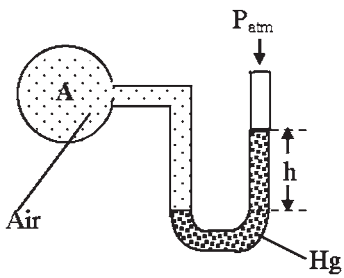
$P_1 = P_{atm} + 123.6 \times 10^3$

$P_2 + 0.5 \times 1 \times 10^3 \times g = P_{atm} + .5 \times 13.6 \times 10^3 \times g$

$P_2 = P_{atm} + 61.8 \times 10^3$

$P_A - P_{atm} = P_{gauge} = 200.3 \text{ kPa}$

In case we have ample U tube manometer, then,



$$P_A - P_{atm} = h\rho_{Hg} \times g$$

$$200.3 \times 10^3 = h \times 13.6 \times 10^3 \times 9.81$$

$$h = \frac{200.3}{13.6 \times 9.81} = 1.5 \text{ m}$$

33. Find the pressure difference between pipe *A* and *B* for the reading of differential manometer as shown below

$P_A = h_1\rho_2 \times g + P_1$

from eqn (1)

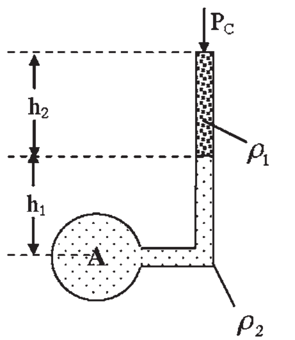
$P_B + \rho_2 hg = \rho_1 hg + P_1$

$P_1 = P_B + hg(\rho_2 - \rho_1) -$ eqn (1)

$P_A = h_1\rho_2 \times g + P_B + hg(\rho_2 - \rho_1)$

$P_A - P_B = (\rho_2 - \rho_1)gh + \rho_2 h_1 g$

For finding $P_A - P_C$ consider the part of the tubing as shown below



$$P_A + h_1\rho_2 g + h_2\rho_1 \times g = P_C$$

$$P_A - P_C = -(h_1\rho_2 g + h_2\rho_1 g)$$

35. Two pressure points in a water pipe are connected to manometer which has the form of an inverted U-tube the space above the water in two limbs of manometer is filled with toluene (sp gr 0.875). If the difference of level of water columns in two limbs reads 12 cm, what is the corresponding difference of pressure?

(AMIE Mech. Engg.)

Guidance: Consider two parts for simplifying

$P_A + (h_1 + h) \times 1 \times 10^3 \times g = P_1$ $\quad$ $P_B + h_1 \times 1 \times 10^3 \times g + h \times .875 \times 10^3 \times g = P_1$

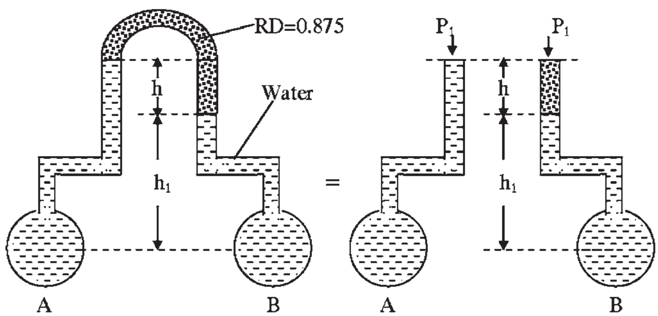
$$P_1 = P_1$$

$$P_A + (h_1 + h) \times 1 \times 10^3 \times g = P_B + h \times .875 \times 10^3 \times g + h_1 \times 1 \times 10^3 \times g$$

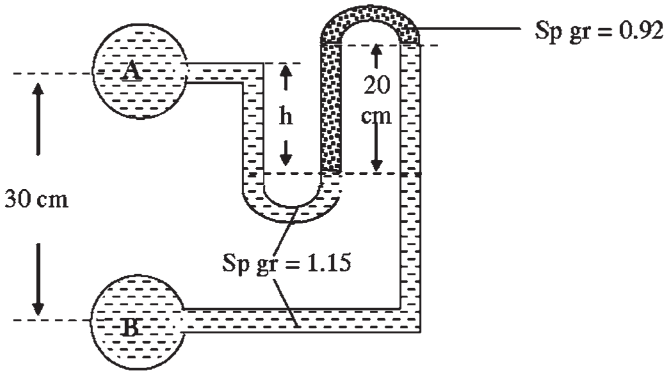
$$\therefore \quad P_A - P_B = h \times 10^3 \times g(1 - 0.875)$$

$$= 0.12 \times 10^3 \times 9.81 \times 0.125$$

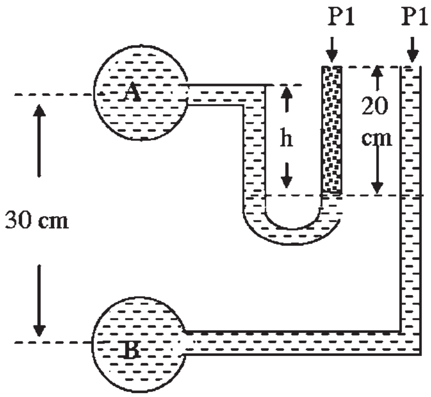
$$= 0.147 \text{ kPa}$$



36. Compute the pressure difference between A & B in the figure.



Guidance: The tubing system can be divided in two parts as shown below



$$P_A + h \times 1.15 \times 10^3 \times g$$
$$= P_1 + .20 \times 0.92 \times 10^3 \times g$$

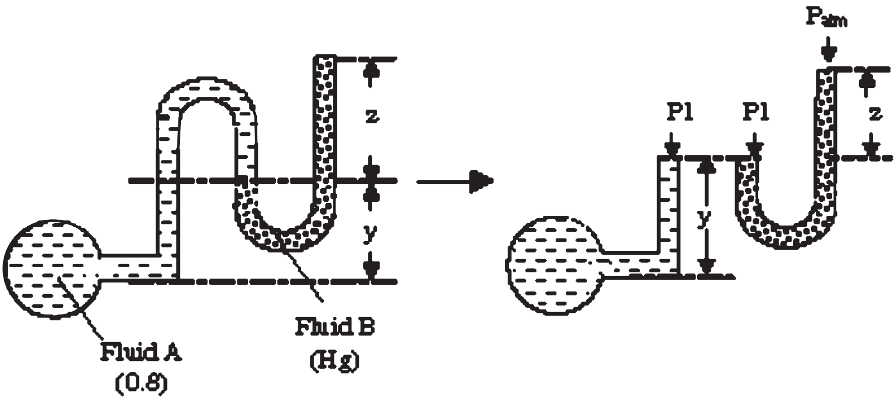
$$P_1 + (.50 - h) \times 1.15 \times 10^3 \times g = P_B$$

Now $P_1 = P_1$

$$P_A + h \times 1.15 \times 10^3 \times g - 0.2 \times 0.92 \times 10^3 \times g$$
$$= P_B - (.5 - h) \times 1.15 \times 10^3 \times g$$

$$P_A - P_B = 0.2 \times 0.92 \times 10^3 \times g - 0.5 \times 1.15 \times 10^3 \times g$$
$$= 1.8 \times 10^3 - 5.64 \times 10^3$$
$$= -3.84 \times 10^3 \text{ kPa}$$
$$= -\frac{3.84 \times 10^3}{1 \times 10^3 \times 9.81}$$
$$= -391 \text{ mm of water}$$

37. The pressure at the point *m* in the figure is given below is to be measured by the open manometer as shown. Fluid *A* is oil (sp gr 0.80) and fluid *B* is mercury. Height y = 75 cm, z = 25 cm. (Punjab University)



Guidance: Divide the tubing in to two system as shown above:

$$P_M = P_1 + Y \times 0.8 \times 10^3 \times g \qquad P_1 = P_{atm} + Z \times 13.6 \times 10^3 \times g$$

$$= P_1 + 0.75 \times 0.8 \times 10^3 \times 9.81 \qquad P_1 = P_{atm} + 0.25 \times 13.6 \times 10^3 \times 9.81$$

Now $P_1 = P_1$

$$P_M - 0.75 \times 0.8 \times 10^3 \times 9.81 = P_{atm} + 0.25 \times 13.6 \times 10^3 \times 9.81$$

$$P_M - P_{atm} = P_{gause}$$

$$P_{gause} = 0.25 \times 13.6 \times 10^3 \times 9.81 + 0.75 \times 0.85 \times 10^3 \times 9.81$$

$$= (33.35 + 6.25) \times 10^3$$

$$= 39.6 \times 10^3 \ Pa$$

$$P_{gause} = 39.6 \text{ kPa}$$

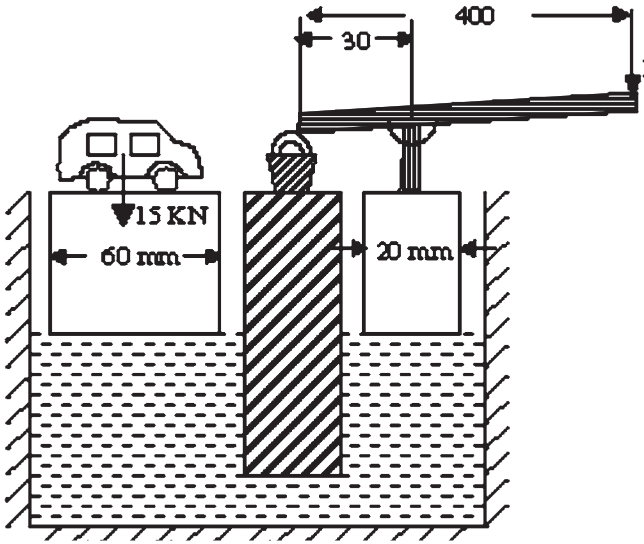
$$P_{anspite} = P_{gause} + P_{atm}$$

$$= 39.6 + 101$$

$$= 140.6 \text{ kPa}$$

38. Show that $P_A - P_B = \left(1 + \frac{a}{A}\right)(r_2 - r_1)gh$ for the manometer as shown below

41. A hydraulic jack consists of a handle cum lever of 40 cm long and various dimensions of assembly as shown in the figure. A vehicle of 15 KN is supported by jack by exerting force *P*. Find *P* if distance between plunger and fulcrum of the lever is 3 cm.



A = area of cross-section of larger piston

$$= \frac{\pi \times (0.06)^2}{4} = 0.0028$$

Pressure developed in the fluid by load = 15 kN is –

$$P = \frac{15 \times 1000}{0.0028} = 5.36 \times 10^6 \text{ N/m}^2$$

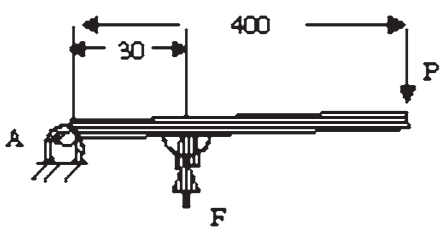
$$a = \text{area of plunger} = \frac{\pi}{4} \times (0.02)^2$$

$$= .000314$$

$$\text{Force on the plunger} = a \times P$$

$$= .000314 \times 5.36 \times 10^6$$

$$F = 1684\ N$$



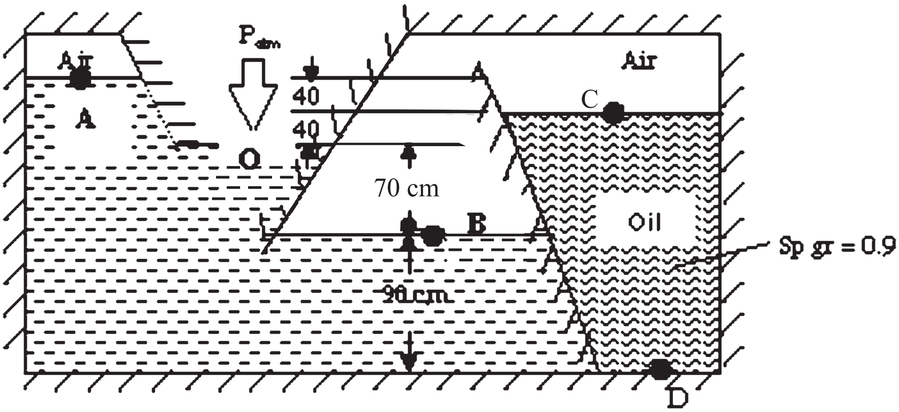
Taking moment w.r.t. point '*A*'

$$F \times 30 = P \times 400$$

$$P = \frac{F \times 30}{400} = \frac{1684 \times 30}{400}$$

$$= 126.3\ N$$

42. Calculate the gauge pressure at point *A*, *B*, *C* and *D* in the water and oil storage arrangement as shown in the figure.



Guidance: The atmospheric pressure is acting on the level point 0. Any level above this level will have vacuum gauge pressure and below this level will have gauge pressure.
At point A, pressure is –

$$P_A = -\rho gh$$

At point *A*, $h = -80$ cm $= -0.8$

$$P_A = -1 \times 10^3 \times 9.81 \times 0.8$$

$$= -7.85 \frac{\text{kN}}{\text{m}^2}$$

At point *B*, $h = +70$ cm

$$\therefore \quad P_B = 0.7 \times 1 \times 10^3 \times 9.81$$

$$= 6.867\ \text{kN/m}^2$$

At point *C*, $P_C = P_B$ as air column will already change the pressure. Now at point *D*, we have

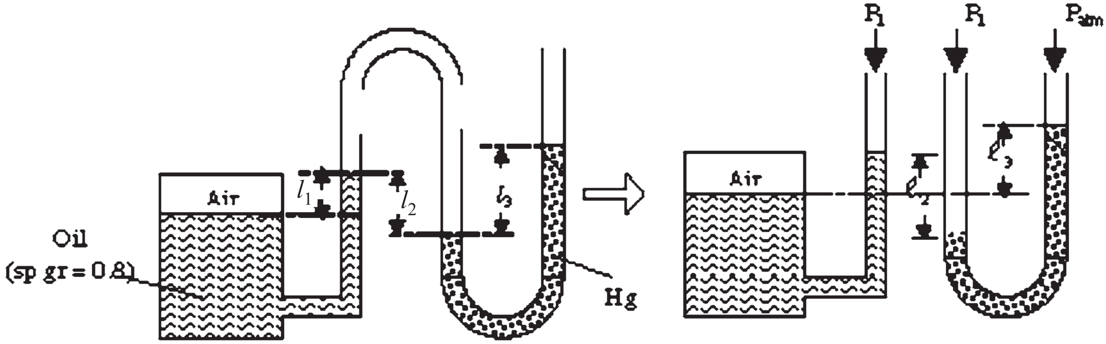
$$P_D = \rho gh + P_B$$

$$= \frac{0.9 \times 10^3 \times 9.81 \times 2.0}{10^3} + 6.867$$

$$= 17.658 + 6.87$$

$$= 24.528 \frac{\text{kN}}{\text{m}^2}$$

43. What is the pressure of air in tank as shown in the figure if l_1 = 40 cm, l2 = 120 cm and l_3 = 90 cm?



Guidance: The system can be divided in two parts as shown in the figure.

$$P_{air} = \ell_1 \times \rho \times g + P_1$$

$$= \frac{0.4 \times 0.8 \times 10^3 \times 9.81 + P_1}{10^3}$$

$$= 3.14 + P_1$$

$$= P_{atm} + 120.07$$

$$P_{atm} + \ell_3 \rho_{Hg} g = P_1$$

$$P_1 = P_{atm} + \frac{0.9 \times 13.6 \times 10^3 \times 9.81}{10^3}$$

From eqn (1) put the value of P_1

$$P_{air} = 3.14 + 120.07 + P_{atm}$$

$$P_{air} - P_{atm} = 12321 \text{ kPa}$$

$$P_{gause} = 123.23 \text{ kPa}$$

44. In inclined manometer was used to find the pressure of water in a pipeline as shown in the figure. Find the absolute water pressure in the pipe in case the inclination of limb to the horizontal is 30°. The ratio of area of reservoir to the tube is 20. The sp gravity of measuring liquid is 1.6.

Flow of volume from reservoir to tube remains constant

$dh \times A = l \times a$ $\quad$ A = area of reservoir

a = area of tube

$$\Delta h = \frac{a}{A} l = \frac{1}{20} \times 0.8 = 0.04 \text{ m}$$

$$h = l \sin\theta = 0.8 \times \frac{1}{2}$$

Applying the condition of equilibrium on datum line $x - x$

$$-dpdA = 1000\left[1+\frac{y}{500}+\left(\frac{y}{1000}\right)^2\right]dy \times dA \times g$$

or
$$-\int_1^2 dp = 1000\,g\int\left[1+\frac{y}{100}+\left(\frac{y}{1000}\right)^2\right]dy$$

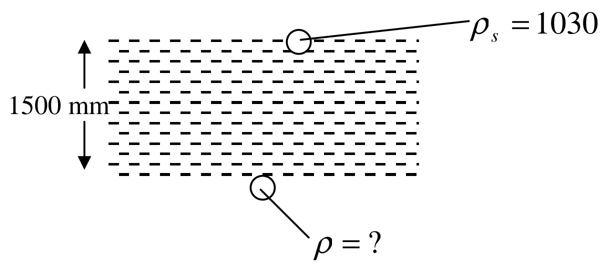
$$P_1 - P_2 = 1000 \times 9.81\left[y+\frac{y^2}{200}+\frac{y^3}{3\times10^6}\right]_0^{80}$$

$$= 9810\times\left[80+\frac{6400}{200}+\frac{512\times10^3}{3\times10^6}\right]$$

$$= 9810[80 + 32 + 0.171]$$

$$= 1100.4 \text{ kPa}$$

46. If the density of sea water at the surface is 1030 kg/m^3 and the average bulk modulus is 2.4×10^9 N/m^2, find the density of sea water at a depth of 1500 m.



$$\text{Bulk modulus } k = \frac{dP}{\dfrac{d\rho}{\rho}}$$

or
$$\frac{d\rho}{\rho} = \frac{dP}{K} \quad \text{or} \quad d\rho = \rho\frac{dP}{K}$$

Now change of pressure $dP = \rho gh$

$$= \frac{(1030)\times 9.81\times 1500}{2.4\times10^9}$$

$$= \frac{1.06\times10^6\times9.81\times1.5\times10^3}{2.4\times10^9}$$

$$= 6.5 \text{ kg/m}^3$$

∴ Density at 1500 = $Ps + dP$ = 1030 + 6.5

$$= 136.5 \text{ kg/m}^3$$

(a) Flow direction is A to B and $P_A - P_B = 20$ kPa
(b) Flow direction is B to A and $P_A - P_B = 1.4$ kPa
(c) Flow direction is A to B and $P_B - P_A = 20$ kPa
(d) Flow direction is B to A and $P_B - P_A = 1.4$ kPa

(GATE – 2005)

As per continuity equation–

$$A_A \times V_A = A_B \times V_B \qquad \text{where } A = \text{area and } V = \text{velocity}$$

As $A_A > A_B$

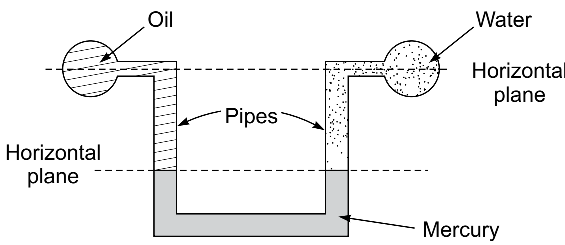
$\therefore$ $V_B > V_A$

As velocity head at 'B' is more than at 'A', hence $P_A > P_B$ and the flow takes place from 'A' to 'B'.

Now
$$P_A - P_B = \rho \cdot g \cdot \Delta B$$
$$= 13.6 \times 10^3 \times 9.81 \times 150 \times 10^{-3}$$
$$= 20 \text{ kPa}$$

Option (a) is correct.

49. The manometer shown in the given figure connects two pipes, carrying oil and water respectively. From the figure one.



(a) can conclude that the pressure in the pipes are equal
(b) can conclude that the pressure in the oil pipe is higher
(c) can conclude that the pressure in the water pipe is higher
(d) cannot conclude the pressure in the two pipes for want of sufficient data.

(IES 1996)

Now balancing the liquid columns and pressure in both tubes with respect to horizontal plane, we have–

$$P_{oil} + \rho_o g_{\cdot H} = P_{water} + \rho_w g_{\cdot H}$$

or $P_{oil} - P_{water} = (\rho_w - \rho_o) g H$

As $\rho_o < \rho_w$

$\therefore$ $P_{oil} > P_{water}$

Option (b) is correct.

50. A differential manometer is used to measure the difference in pressure at point A & B in terms of sp weight of water (W). The sp gravities of liquids X, Y and Z are

Taking base as datum, we have

$$3h \times \rho + 1.5h \times 2\rho + h \times 3\rho = H \times 3\rho$$

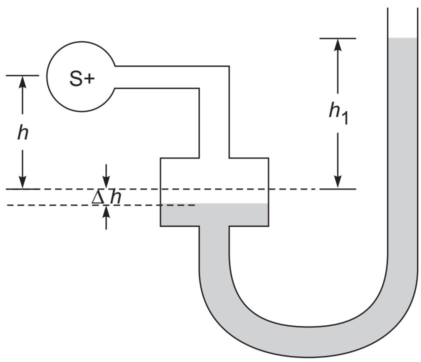
or $$3h + 3h + 3h = 3H$$

or $$3h = H$$

or $$\frac{H}{h} = 3$$

Option 'c' is correct.

54. To measure the pressure head of the fluid of specific gravity '*S*' flowing through the pipeline, a simple micro-manometer containing a fluid of specific gravity 'S_1' is connected to it. The readings are as indicated in the diagram. The pressure head in the pipeline is–



(a) $h_1S_1 - hS - \Delta h(S_1 - S)$
(b) $h_1S_1 - hS + \Delta h(S_1 - S)$
(c) $hS - h_1S_1 - \Delta h(S_1 - S)$
(d) $hS - h_1S_1 + \Delta h(S_1 - S)$

(IES 2003)

Balancing both limbs, we have–

$$P_S + (h + \Delta h)S = (h_1 + \Delta h)\, S_1$$

or $$P_S = h_1S_1 - hS + \Delta h(S_1 - S)$$

Option (b) is correct.

55. The balancing column shown in the diagram contains 3 liquids of different densities ρ_1, ρ_2 and ρ_3. The liquid level of one limb is 'h_1' below the top level and there is a difference of '*h*' relative to that in the other limb. What will be the expression of '*h*'?